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SUSTAINABILITY AND RISK ANALYSES OF BRAZILIAN STATE’S DEBT REFINANCING LAW ON STATE OF RIO GRANDE DO SUL

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1. Introduction

2. Brazilian state’s refinancing law
   2.1 Background
   2.2 Main elements

3. Data and information of Rio Grande do Sul debt
   3.1 Outstanding debt
   3.2 Debt payment
   3.3 Data and current situation of refinancing law

4. Sustainability analysis of refinancing law
   4.1 Sustainability theory
   4.2 Sustainability dynamic of equations on RGS debt
      4.2.1 Basic equation
      4.2.2 Sustainability analysis on equations
      4.2.3 Ratio analysis
   4.3 Practical application of sustainability theory and equations on RGS refinancing law

5. Risk Analysis
   5.1 Market risk
   5.2 Refinancing or roll over risk

6. Final considerations

References
1. Introduction

Sound practice in public debt is unquestionably important for all governments. It is common to witness public entities faced with serious problems in their fiscal policy due irresponsible management of debt. All financial markets are sensitive to shocks and that directly affect public debt behavior, either on servicing payment or stock. In this sense, previous analyses about debt tendency and even how market oscillations impact debt become indispensable.

On those considerations, this paper aims to analyze the sustainability and risks of the public debt of the State of Rio Grande do Sul (RGS), especially under the context of Refinancing Law of the Brazilian states’ governments. It must be clear that the target is to analyze only the refinancing law debt sustainability, as it composes about 90% of the total debt amount of RGS.

The following chapter contains a brief explanation of background and main elements of the Brazilian States’ refinancing law called “Law 9496 - Program to Support States’ Restructuring and Fiscal Adjustment.”
The third chapter will present data and information of RGS debt, mainly those that will be essential for this paper’s objective. Chapter IV contains a brief theory related to debt sustainability and its application through particular equations on RGS public debt. Additionally, in the fourth chapter, significant recommendations will be suggested, as the need for adjustments to refinancing law seems obvious. This topic is quite polemic as even the current situation of the states’ debt is better in comparison to the previous the refinancing law period. However, Brazilian states continue to experience fiscal difficulties and a risky debt profile associated with the debt burden established by the law.

In the fifth chapter, the debt’s risks are explored and adapted according to the particularities of RGS debt. Although Brazilian states’ refinancing law was a significant deal in the late 1990s, some important components must be questioned as financial market variables change rapidly: this fact means the burdens and general condition established by the law outdated.

General approaches will also be used in this writing with the goal of demonstrating reasons for the revision of refinancing law as potentially substantial, not only for the unit’s federation but also for the federal government. In this sense, changes on the state’s debt burden would also bring widespread positive impacts by improving the federal debt profile.
2. Brazilian’s states refinancing law

2.1 Background

The refinancing law known as “Law 9496 - Program to Support Restructuring and Fiscal Adjustment of States” was negotiated in 1998. The previous period, in which the federal government started implementing the economy stabilization plan called “Plano Real”, was characterized by high interest rates in order to confront high inflation, which caused ballooning Federal and State debts, including in the State of Rio Grande do Sul (Graphic 1). Because the States owned bonds carrying high costs and a variable interest rate, they consequently were faced with serious fiscal problems caused by the significant level of expenditures committed to debt servicing payments.

Graph 1: HISTORICAL OF RIO GRANDE DO SUL DIRECT ENTITIES STOCK DEBT (prices 2009)
This context led the federal government to refinance state debt, assuming their bonds debt, contracting renewed debt burdens with them and therefore becoming the states’ creditor. On one hand, the “Law 9496” benefited states, including the State of RGS, since it changed the risk of high cost interest rates for fixed rate. On the other hand, the law prevented all Brazilian states from issuing new bonds and further required careful budgeting and increased primary surplus from the states.

2.2 Main elements

The agreement designed the Price System with new burdens for the States: indexing of debt by a monetary adjustment based on an inflation index called IGP-DI (Index general Prices – Internal availability) and carrying a fix annual real interest rate of 6%. The related index takes into
account internal price variations that directly affect the economic activities. Other aspects of this index will be detailed in later chapters as the knowledge of its behavior and variations are essential for development of the paper’s proposal.

The debt is amortized on a monthly basis during 360 months (that is, 30 years). The amount of amortization is based on a dual system of a benchmark installment and a fiscal revenue-related installment. As we will see below, this dual system gives rise to two types of debt.

The initial benchmark installment, C0, was determined by dividing the outstanding stock of new debt by the total number of installments, 360 months. The subsequent installment would change each month due to an indexation factor also based on the producer price index. The formulas below typify this.

\[ C_0 = \frac{D_0}{360} \]
\[ C_t = (1 + p_t)C_{t-1} \]

Where the sub-index t stands for the current month, p is the inflation as measured by the producer price index, and Do is the new debt at the inception of the refinancing law.

The fiscal revenue-related installment is equal to 13% of a given definition of the RGS’s fiscal revenues, that is, 0.13 x R. The relevant fiscal
revenue, \( R \), is defined as the Net state revenues after transfers from the Federal Government and other adjustments.

Except for the first three months or so, the amount paid by RGS to the BFG was 0.13 \( R \) which has fallen short of the benchmark installment: 0.13 \( R < C \). One could think of the benchmark installment as an accrual payment, and the revenue-related installment as a cash or actual payment.

The dual system of amortization, gives rise to two types of debt, namely, the actual debt amortized with the 13% of revenues; and the residual debt, which accumulates due to the shortfall of the actual payment vis-à-vis the benchmark payment. This debt will accumulate until the year 2028 when it will be refinanced again for 10 years.

Thus, assuming that every month \((C_t - 0.13 R_t) > 0\) (as has been the case since the beginning of the restructuring, except for the three first months), the dynamic of the outstanding stock of actual and residual debt at the end of each month are given by equations 1 and 2, respectively;

\[
\begin{align*}
[1] & \quad D_t^a \equiv (1 \cdot (C_t - 0.13 R_t)) D_{t+1}^a \equiv 0.13 R_t \\
[2] & \quad D_t^r \equiv (1 \cdot (C_t - 0.13 R_t)) D_{t+1}^r \equiv (C_t - 0.13 R_t)
\end{align*}
\]
Equation 1 shows the dynamic of the outstanding stock of actual debt at the end of month $t$. The stock increases due to the monthly indexation and the monthly real annual interest rate ($r = 0.06 / 12$); and it decreases due to the monthly amortizations of 13% of the fiscal revenue.

Equation 2 shows the dynamic of the residual debt. This stock increases, in addition to indexation and interest, due to the gap between the benchmark and actual payments, that is $(C_t - 0.13 R_t) > 0$. We will analyze the sustainability under the assumption that for each month (or for most of the months) this assumption holds.1

Equation 3 is a useful auxiliary equation that encapsulates in one variable, $a$, the actual and benchmark payments. Thus, if the actual and benchmark payments coincide, $a = 1$; if the actual payment falls short of the benchmark, $a < 1$; and if the benchmark payment falls short of actual payment, $a > 1$. And equation 4 defines the total debt as the sum of both types of debt.

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1 If the revenue-related installment turned out to be larger than the benchmark installment, $(C_t - 0.13 R_t) < 0$, equation 1 has to slightly modified, use $C$ instead of $0.13 R$; that is the actual payment now is the benchmark installment. Equation 2 remains the same except that the now term $(C_t - 0.13 R_t)$, being negative, is not an accumulation factor but an amortization factor.
All those demonstrated equations will be extremely useful to analyze the sustainability dynamic of debt on later chapters.

3. Debt’s data and information of the State of Rio Grande do Sul

Before the debt analysis of sustainability starts, a general view of the whole state debt numbers is essential. So, this chapter aims to describe debt profile of RGS.

3.1 Outstanding debt

As it was mentioned in the last chapter, the State has eliminated all debt bonds which were then replaced by a single debt agreement with Federal Government. This change can be identified on the graph below.

GRÁFICO 2 – COMPOSITION OF DEBT ON THE STATE OF RIO GRANDE DO SUL (Dezember 2009 values)
The graph above demonstrates RGS’s debt performance, and the shift since 1998 due the Law 9496 is notable. While in 1994 State bonds were used to represent around 76% of the total debt, years later those were virtually eliminated.

The dominance of internal debt over external debt is also a relevant aspect. The chart below details the total debt, R$ 36.9 billion, and where it was distributed at the end of 2009. It implies Federal Government as the major creditor of RGS’s debt, representing 95% of the State’s total debt.

<table>
<thead>
<tr>
<th>R$</th>
<th>2009</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Law 9496/97</td>
<td>33.759,526,400,70</td>
<td>91.33</td>
</tr>
<tr>
<td>Law 9496/97 Principal</td>
<td>15.824,320,579,65</td>
<td>42.81</td>
</tr>
<tr>
<td>Law 9496/97 Resíduo</td>
<td>12.359,312,810,06</td>
<td>33.44</td>
</tr>
<tr>
<td>PROES Intralimit</td>
<td>4.076,322,522,49</td>
<td>11.03</td>
</tr>
<tr>
<td>PROES Extralimit</td>
<td>1.499,570,488,50</td>
<td>4.06</td>
</tr>
<tr>
<td>Law 8727/83</td>
<td>1.295,520,815,70</td>
<td>3.5</td>
</tr>
<tr>
<td>Law 7976/89</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>DMLP</td>
<td>104,897,183,41</td>
<td>0.28</td>
</tr>
<tr>
<td>Caixa Econômica Federal</td>
<td>26,973,195,64</td>
<td>0.07</td>
</tr>
<tr>
<td>BNDES</td>
<td>704,850,56</td>
<td>0</td>
</tr>
<tr>
<td>INSS</td>
<td>97,567,814,74</td>
<td>0.26</td>
</tr>
<tr>
<td>Total Debt with Federal Government</td>
<td>35,285,190,260,75</td>
<td>95.46</td>
</tr>
<tr>
<td>Other domestic debts</td>
<td>17,176,534,64</td>
<td>0.05</td>
</tr>
<tr>
<td>Total Domestic Debt</td>
<td>35,302,366,795,39</td>
<td>95.51</td>
</tr>
<tr>
<td>External Debt</td>
<td>1,660,815,357,51</td>
<td>4.49</td>
</tr>
<tr>
<td>TOTAL DEBT AMOUNT</td>
<td>36,963,182,152,70</td>
<td>100</td>
</tr>
</tbody>
</table>

Other important observations must be made about the previous chart: there is another refinancing law created by the federal government, Law
8727. However this will not be included as part of the analysis as it is not representative. As cited before, it is possible to observe that in the State of RGS, the Residual count is substantial, R$ 12.36 billion. This suggests that fiscal efforts to pay debt servicing have not been sufficient to pay the benchmarking (or original) installment designed by the Law 9496.

Debt amount composition per index is described on the next chart. The amount of public debt indexed by IGP-DI connects more than 90% of RGS’s total debt. This illustrates the directly proportional relationship between RGS debt and the index behavior. This explains the reason why most analyses will surround the index.

<table>
<thead>
<tr>
<th>Index</th>
<th>2009</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price Index (IGP-M e IGP-DI)</td>
<td>34,024,119</td>
<td>92.1</td>
</tr>
<tr>
<td>Exchange Index (US$, Ien, Cesta Moedas)</td>
<td>1,787,956</td>
<td>4.8</td>
</tr>
<tr>
<td>TJLP - Long Term Interest Rate</td>
<td>113,071</td>
<td>0.3</td>
</tr>
<tr>
<td>TR - Reference Rate</td>
<td>1,011,334</td>
<td>2.7</td>
</tr>
<tr>
<td>Selic (Prime Rate)</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>Outros</td>
<td>26,703</td>
<td>0.0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>36,963,182</td>
<td>100</td>
</tr>
</tbody>
</table>

The absence of Brazilian financial market official interest rate represented by SELIC, on the composition of RGS’s outstanding debt, is striking.
3.2 Debt payment

This section introduces briefly RGS’s debt payment numbers, and its ratio in relation to debt revenue. The latter ratio is an important reference as it establishes the payment of 13% net revenue as stated by law 9496. This percentage that exceeds the limit corresponds to payments of other debt loans.

<table>
<thead>
<tr>
<th>Year</th>
<th>Total debt servicing</th>
<th>Debt Servicing/ Net Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>1994</td>
<td>598</td>
<td>4.87%</td>
</tr>
<tr>
<td>1995</td>
<td>781</td>
<td>8.10%</td>
</tr>
<tr>
<td>1996</td>
<td>854</td>
<td>7.16%</td>
</tr>
<tr>
<td>1997</td>
<td>708</td>
<td>5.62%</td>
</tr>
<tr>
<td>1998</td>
<td>1.466</td>
<td>11.15%</td>
</tr>
<tr>
<td>1999</td>
<td>1.835</td>
<td>14.16%</td>
</tr>
<tr>
<td>2000</td>
<td>1.808</td>
<td>16.71%</td>
</tr>
<tr>
<td>2001</td>
<td>1.886</td>
<td>16.79%</td>
</tr>
<tr>
<td>2002</td>
<td>2.038</td>
<td>18.27%</td>
</tr>
<tr>
<td>2003</td>
<td>1.874</td>
<td>17.66%</td>
</tr>
<tr>
<td>2004</td>
<td>1.861</td>
<td>17.55%</td>
</tr>
<tr>
<td>2005</td>
<td>2.063</td>
<td>19.04%</td>
</tr>
<tr>
<td>2006</td>
<td>2.089</td>
<td>17.23%</td>
</tr>
<tr>
<td>2007</td>
<td>2.061</td>
<td>16.64%</td>
</tr>
<tr>
<td>2008</td>
<td>2.015</td>
<td>16.30%</td>
</tr>
<tr>
<td>2009</td>
<td>2.100</td>
<td>15.36%</td>
</tr>
</tbody>
</table>

According to the previous chart, the rise of the ratio among debt servicing and net revenue, with the settlement of the brazilian states’ refinancing law, is striking. This makes sense as larger fiscal efforts by Brazilian states were committed under the law. In other words, it was the
contribution from the states, through a higher primary surplus, that helped the macroeconomic stabilization of the country. Taking a look at Brazil’s public sector primary surplus since 2002, the fiscal cooperation of the states is clearly evidenced, as proven on the next chart given by the Brazil Central Bank.

**DEMONSTRATION OF BRAZIL PUBLIC SECTOR PRIMARY SURPLUDES**

<table>
<thead>
<tr>
<th></th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central Government</td>
<td>2.2</td>
<td>2.3</td>
<td>2.7</td>
<td>2.6</td>
<td>2.2</td>
<td>2.3</td>
<td>2.5</td>
<td>1.4</td>
</tr>
<tr>
<td>States and Municipalities</td>
<td>0.7</td>
<td>0.8</td>
<td>0.9</td>
<td>1.0</td>
<td>0.8</td>
<td>1.2</td>
<td>1.1</td>
<td>0.7</td>
</tr>
<tr>
<td>States</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
<td>0.8</td>
<td>0.7</td>
<td>1.0</td>
<td>0.9</td>
<td>0.6</td>
</tr>
<tr>
<td>Municipalities</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Public Companies</td>
<td>0.7</td>
<td>0.8</td>
<td>0.6</td>
<td>0.8</td>
<td>0.8</td>
<td>0.5</td>
<td>0.2</td>
<td>0.0</td>
</tr>
<tr>
<td>Total</td>
<td>3.6</td>
<td>3.9</td>
<td>4.2</td>
<td>4.4</td>
<td>3.9</td>
<td>4.0</td>
<td>3.5</td>
<td>2.1</td>
</tr>
</tbody>
</table>

The states’ support for the success of macroeconomic consistency in this recent period is undeniable. The primary surplus from the States reached an average of more than 20% of Brazil’s total primary surplus in the last 8 years.

3.3 Data and current situation of refinancing law in the State of RGS

Currently, debt payment under 13% of real net revenue is not enough to cover benchmarking installments. Therefore the ratio a (according to equation 3) is less than 1. This difference goes to the residual count which
will be refinanced for 10 years from 2028, as outstanding debts will probably still exist. Although it will be possible to refinance residual, it has actually increasing significantly due to currency devaluation in 1999, which affected the chief index of State debt, IGP. The gap between the benchmark installment and debt payment was not balanced until now.

If this circumstance continues with no corrective measures, the problem to refinance the residual in 2028 is going to get worse. Not only is the state going to be damaged, but also federal government will have difficulties recalling this debt. Actually, it is important to realize that the residual increases are not only due to related gap, but also because debt burden continues with no payment (as demonstrated by equation 2). This situation is one of the central questions that this paper aims to assess carefully, both under sustainability theory and risky concepts related with public debt.

The chart below exposes total debt path under the law 9496 on state of RGS (including the Residual). The huge nominal growth of Residual remains systematic, from 10% to 40% of the total in the last 8 years. Moreover, the raise within years 1998 to 1999 is evident and the reasons are related with the index behavior.
<table>
<thead>
<tr>
<th>Year</th>
<th>Law 9496</th>
<th>Residual</th>
<th>Total</th>
<th>Residual/Total</th>
<th>% Nominal Residual Growth per year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>9.244.197.901</td>
<td>322.058.765</td>
<td>9.566.256.666</td>
<td>3.37%</td>
<td></td>
</tr>
<tr>
<td>1999</td>
<td>10.907.777.423</td>
<td>873.261.287</td>
<td>11.781.038.710</td>
<td>7.41%</td>
<td>171.1%</td>
</tr>
<tr>
<td>2000</td>
<td>11.426.071.492</td>
<td>1.098.681.604</td>
<td>12.524.753.196</td>
<td>8.77%</td>
<td>25.8%</td>
</tr>
<tr>
<td>2002</td>
<td>15.128.053.742</td>
<td>2.785.197.980</td>
<td>17.913.251.722</td>
<td>15.55%</td>
<td>62.3%</td>
</tr>
<tr>
<td>2003</td>
<td>16.102.045.519</td>
<td>3.842.412.341</td>
<td>19.944.457.860</td>
<td>19.27%</td>
<td>38.0%</td>
</tr>
<tr>
<td>2004</td>
<td>17.562.222.626</td>
<td>5.216.711.406</td>
<td>22.778.934.032</td>
<td>22.90%</td>
<td>35.8%</td>
</tr>
<tr>
<td>2006</td>
<td>18.475.073.565</td>
<td>7.746.866.498</td>
<td>26.221.940.063</td>
<td>29.54%</td>
<td>20.4%</td>
</tr>
<tr>
<td>2007</td>
<td>19.246.845.229</td>
<td>9.280.004.839</td>
<td>28.526.850.068</td>
<td>32.53%</td>
<td>19.8%</td>
</tr>
<tr>
<td>2008</td>
<td>20.847.210.584</td>
<td>11.485.284.579</td>
<td>32.332.495.163</td>
<td>35.52%</td>
<td>23.8%</td>
</tr>
<tr>
<td>2009</td>
<td>19.900.643.102</td>
<td>12.359.312.810</td>
<td>32.259.955.912</td>
<td>38.31%</td>
<td>7.6%</td>
</tr>
</tbody>
</table>

4. Sustainability analysis of refinancing law

4.1 Sustainability theory

After this general explanation about the refinancing law 9496 and how it has been applied to the state of RGS, debt sustainability concepts should now be applied according to all relevant data. Several approaches to evaluate debt sustainability are available. However, according to refinancing law particularities and present situation, this section will describe a theory that better relates the relationship between primary surplus (represented by the limit on the state of RGS debt) and interest rate; seeking how the relationship impacts debt path.
Superficially, studies take the Debt/GDP ratio to evaluate debt sustainability, showing how correlated variables may impact debt path across time to evaluate tendencies. Although GDP is typically used as the denominator for the ratio, in this paper, Revenue is going to be used as the denominator. Basing our analysis on Revenue would be more appropriate and useful to the Brazilian state, as it is incorporated in the state’s refinancing law.

Taking a look in this relation on graph below, the debt path rose since the beginning of refinancing law. The ratio started with 1.98 and hit the peak of 2.76 in 2002. Although the ratio dropped on the year of 2009, a clear reason exists: exceptionally IGP-DI was negative what impacted positively the ratio. Tracing a line, the upward trajectory of the debt pay is evident and this indicates sustainability problems.
According to theoretical principles, debt may be considered sustainable if the present value of future primary surpluses equals or surpasses current outstanding public debt. Future revenues must be sufficient to cover primary expenditures plus the interest rate charges originated by outstanding debt. If the condition is not met, the market may not be willing to finance public deficit.

A primary surplus may also be a good tool for analyzing debt sustainability, as it offers assurance that debt service can be covered. In this sense, maintaining a primary surplus enough to face debt interests is sufficient condition to achieve fiscal balance. This concept is extremely pertinent when applied to the refinancing law. The law requires from State of RGS a primary surplus (13% of Real Net Revenue), but it does not require that the surplus be large enough to pay interest payments. Therefore, the next step will detail equations which depict this sustainability approach.

The primary deficit is determined as the difference among nominal deficit and debt interest rate, as it is expressed on the equation below.

\[ DP = DT - J = G - T \]

Which considers:
DP – Primary deficit
DT – Nominal deficit
J – Interest Rate
G – Government Expenditures
T – Government Revenues

Thus, fiscal balance is reached through a primary surplus in the same value of debt interest. By this view, the variation on debt amount is defined by the nominal deficit less the share financed by money issues, established by:

\[ D - D_{-1} = DT - (B - B_{-1}) \]

Considering:

- B – Monetary Base
- D – Debt Amount

Regarding interest expenditures as a function of: real interest rate, inflation rate and debt amount on the previous period.

\[ J = K D_{-1} + i (1+K) D_{-1} \]

With,

- K = Inflation Rate
- i = Real interest rate
Joining the previous equations, the result is:

\[ D = (G-T) + J - (B - B_{-1}) + D_{-1} \]

Considering related variables as a share of GDP, the following equation is achieved. For this paper, the GDP is equivalent with Real Net Revenue, as the latter is better to assess the Brazilian states’ debt path.

\[
\begin{align*}
d &= (G-T) + i(1+K) \frac{D_{-1}}{Y} + K \frac{D_{-1}}{Y} - (B - B_{-1}) + D_{-1} \\
&= (G-T) + i(1+K) \frac{D_{-1}}{Y} + K \frac{D_{-1}}{Y} - (s + b) \frac{(1+y)Y}{(1+K)(1+y)Y_{-1}}
\end{align*}
\]

Regarding GDP during a respective year as:

\[ Y = (1 + y) (1 + K) Y_{-1} \]

\[ Y = \text{Real Growth of GDP} \]

With

\[
\begin{align*}
s &= \frac{(T - G)}{Y} \\
b &= \frac{(B - B_{-1})}{Y}
\end{align*}
\]

The result is:

\[
\begin{align*}
d &= \frac{i D_{-1}}{(1+y)Y_{-1}} + \frac{K D_{-1}}{(1+K)(1+y)Y_{-1}} + \frac{D_{-1}}{(1+K)(1+y)Y_{-1}} - (s + b) \frac{(1+y)Y}{(1+K)(1+y)Y_{-1}}
\end{align*}
\]
The next equation defines the sufficient condition that maintains the debt path stable over time. The connections between the variable is also demonstrated.

\[ d = \frac{(1 + i) \cdot d_{1} - (s + b)}{(1 + y)} \]

This equation clarifies how interest rates affect the increase in indebtedness. Another relevant point is the importance and the impacts of a substantial primary surplus to reduce debt ratio.

### 4.2 Sustainability

**Dynamic of equations on RGS debt**

#### 4.2.1 Basics equations

Using the auxiliary equation 3 and the total debt equation 4 as determined previously on chapter 2, the system can be written as:

1') \[ D^e_t \cdot (1 + r_t, r_t, r_t) D^e_{t+1} = 0.13 R_t \]

2') \[ D^r_t \cdot (1 + r_t, r_t, r_t) D^r_{t+1} = \frac{(1 + r_t, r_t) \cdot 0.13 R_t}{r_t} \]

4') \[ D^T_t \cdot (1 + r_t, r_t, r_t) D^T_{t+1} = \frac{(1 + 2, r_t) \cdot 0.13 R_t}{r_t} \]

Using this system, one can make certain inferences regarding the dynamic of the outstanding stock of actual, residual and total debt, by
assuming certain values for the key parameters. For example, keeping our assumption of \( a < 1 \) (actual payment falls short of the benchmark payment), equations 1’, 2’ and 4’ state that the two types of debts, and the total debt, will increase due to inflation and interest rate factors. Additionally, the actual debt will decrease due to the actual payment of 0.13 \( R \); the residual debt will further increase due to the positive gap between the benchmark and actual payment; and the total debt might either increase or decrease depending upon how wide the gap is if \( a < 0.5 \), the total debt further increases; if \( a = 0.5 \), the total debt will decrease on this account.

To study debt sustainability, it is appropriate to study the dynamics of the ratio of the outstanding debt, according to a specific variable. We use the RGS’s fiscal revenue, \( R \), as such variable (alternatively, we could use the RGS’s GDP). We define the ratio as \( d = \frac{D}{R} \). Dividing both sides of equations 1’, 2’ and 4’ by \( R_t \) and multiplying the first term on the right hand side of equations 1’, 2’ and 4’ by \( R_{t-1} / R_t \), and some algebraic manipulation, we obtain the dynamic equations for the ratios of the actual, residual and total debt to fiscal revenue, respectively.

\[
\begin{align*}
d_t^a & = \frac{1}{R_t} \left( R_{t+1} - r_t d_{t+1}^a \right) + 0.13 \\
\end{align*}
\]

[5]
Where \( R \) is growth rate of the nominal fiscal revenue, 
\( \hat{R}_i = (R_i / R_{i+1}) \). Now we can use equations 5, 6 and 7 to analyze debt sustainability; that is, the conditions under which the ratio of the total debt to revenue will remain constant, increase or decrease.\(^2\)

### 4.2.2 Sustainability analysis on equations

**Case A):**

From equation 7, it is logical to obtain the following conclusions:

The total debt ratio will always increase, as long as the following conditions hold true every month:

\[
\frac{(1 + \hat{R}_i)^{t+1} - (1 + \hat{R}_i)^{t}}{1 - (1 + \hat{R}_i)^{t}} \times \frac{1}{1 - (1 + \hat{R}_i)^{t+1}} \geq 0
\]

Or more compactly:

\[
i \hat{R}_i \geq \frac{1}{2} \left( 1 + \hat{R}_i \right)
\]

\(^2\) Note the analogy of these equations with the standard dynamic equation for the debt-to-GDP ratio for a country: 

\[
d_t = \frac{\hat{Y}}{Y} \frac{1}{1 + \hat{R}_i} \frac{ps}{Y}.
\]

Where \( i \) is the nominal interest rate (in our case, the nominal interest rate would be \( p + r \)); \( \hat{Y} \) stands for the rate of growth of nominal GDP for the country, and \( ps \) is the ratio of the primary surplus (if > 0) to GDP (or ratio of primary deficit to GDP if < 0).
Case B):

The total debt ratio will always decrease if the following conditions hold true every month:

\[ r_{t-1} > r_t > i_t > \hat{R}_t \text{ and } \beta > 0.5 \]

Case C):

If \( r_{t-1} > r_t > i_t > \hat{R}_t \text{ and } \beta > 0.5 \), the total debt ratio could remain constant, increase or decrease depending upon the exact values of the key variables shown in those inequalities and the level of indebtedness at the end of the previous month (the level of the ratio). Let us first analyze the case of a constant total debt ratio between t-1 and t; that is \( d^T_t \approx d^T_{t-1} = 0 \).

Taking the first difference of equation 7 and imposing the condition for a constant total debt ratio, obtains:

\[
\begin{align*}
d_t^T - d_{t-1}^T &= \beta_i - \frac{\hat{R}_t}{\hat{R}_{t-1}} d_{t-1}^T \cdot \frac{(2\beta - 1) 0.13}{?} - 0 \\
\text{Therefore,} & \text{ If } d_{t-1}^T \approx d_{t-1}^T \text{ the ratio will remain constant, } d_t^T \approx d_{t-1}^T \\
\text{If } d_{t-1}^T \approx d_{t-1}^T \text{ the ratio will increase, } d_t^T \approx d_{t-1}^T
\end{align*}
\]
If, $d_{t_1}^T \neq d_{t_1}'^T$ the ratio will decrease, $d_t^T \neq d_{t_1}^T$

### 4.2.3 Ratio analysis

The first difference of equation 7 showed above could be used to express the importance of each factor affecting the ratio. For this purpose, it is convenient to show each component explicitly:

$$d_t^T \neq d_{t_1}^T \neq d_{t_1}'^T, \quad ? \neq ?_{1?} \frac{\hat{1}_{t}}{\hat{1}_{t}^T} \neq ?_{1?} \frac{\hat{r}_{t}}{\hat{a}_{t}} \neq ?_{1?} \frac{\hat{d}_{t}}{\hat{a}_{t}} \neq ?_{1?} \frac{\hat{d}_{t}}{\hat{a}_{t}} = \frac{(2^? ? 1) 0.13}{?} \geq 0$$

Note that the last term will contribute to decrease the ratio only if $a > 0.5$ (analogous to a primary surplus in the standard sustainability analysis for one type of debt for a country) and vice-versa. If $a = 1$ (the benchmark and actual payments coincide), the last term is zero, meaning that the amount that amortizes the actual debt is offset by the amount of accumulation of the residual debt. In this case, the total debt ratio will most likely increase, unless the rate of growth of nominal revenues is higher than the sum of the inflation and real interest rates.

The next graph demonstrates ratio’s behavior since the beginning of the refinancing law. Problems on debt sustainability are evident as the benchmark and actual payments do not match each other, resulting in $a < 1$. The share of net revenue devoted to pay the refinancing law is not enough
to cover the original installment; therefore the residual debt tends to increase. This fact coincides with the dynamic of residual equation as demonstrated on equation 2.

4.3 Empirical application of sustainability theory and equations on State of RGS refinancing law

According to the last section, the ratio for a is lower than one. However, for a deeper analysis on debt sustainability it is necessary to review the ratio according to equation no.7, or more precisely with Case B. This case states that the total debt ratio will always decrease if the following condition holds true on every month:
\[ r_{11} > r, \hat{r} > i, \hat{R}, \text{ and } 0.5 \]

The graph below demonstrates that although the ratio is lower than one, it stays above 0.5 at all times, which satisfies the second part of the condition. Graphically, through the following ratio called \( \beta \), the first part of established condition is checked:

\[
\beta = \frac{(1+p+r)}{(1+R)}
\]

Having a \( \beta > 1 \), would indicate that the debt tends to increase, as nominal growth is not sufficient to cover inflation and interest.
In most months the outcome is greater than 1\(^3\), therefore the sustainability condition is not satisfied. The dynamic of debt will not be balanced and total debt is going to increase consequently.

The results of this application on RGS state debt is consistent with all data presented along previous chapters. The analysis explains the increasing trend in the ratios: refinancing law total debt in comparison with net revenue, and even in the growth of residual debt as a share of refinancing law.

The coefficient \( \beta \) is composed by the main variables that affect the debt sustainability dynamic. A detailed approach on this may clarify the influence of each variable. The ratio \( \beta \) will be fractioned (as highlighted) for a particular analyze according following equation shows:

\[
\beta = \left( \frac{1+p + r}{1+R} \right)
\]

Next graph demonstrates the relationship between inflation and net revenue nominal growth. The inflation index, represented by the Producer Index, has a significant influence on debt path.

\(^3\) The average of \( \beta \), during the refinancing law, is higher than 1.
Although this graph is similar with the graph for ratio \( \beta \), the data reveals an average equal to 1. The result can be interpreted that whole net revenue (including the real growth) is used to cover only monthly monetary variations. Others aspects involving the inflation index will be discussed on the next chapter. An important inference must be done: if interest due is not covered by net revenue, overall debt increases. Next graph confirms the effects of unpaid interests.
The graph above demonstrates the interest rate (calculated by the original installment) and the paid interest rate (share of net revenue devoted to pay the debt related with the refinancing law). The savings made by the government of RGS since 1999 were not sufficient to confront the original interest rate. In the last two years, those payments have been balanced because fiscal efforts were made in 2008 to raise the revenue (real growth of more than 10%). However unpaid interests have been accumulating for many more years has been accumulated on the Residual account without being paid.

The Residual is scheduled to be financed in 2028, but if an imbalance holds on with no actions, the share of total expenditures devoted to debt servicing will be higher than the beginning. Predictions point out about 16% of the Real Net Revenue to pay public debt in 2028.
5. Risk analysis

The focus of this chapter is to demonstrate how inflation and interest rates influence RGS’s debt sustainability. The analysis made on the previous chapter sets precedent and suggests that these variables have a substantial role on debt path towards debt sustainability.

In this context, the vulnerability of RGS’s debt will be associated with market oscillations and risks. The main contributors to risk factors will be identified and explained throughout the analysis.

5.1 Market risk

Market changes affect the value of assets and liabilities. Both cash flows and outstanding debt amount are sensitive to many market conditions, including movements in prices, interest rates, inflation indexes and commodity prices. Changes in market prices impact the value of the benchmark portfolio. Accordingly, the size of a portfolio could require a shocking correction.

Market risk is usually calculated with financial models that are widely used to capture the long term impact of market risk. The target of the models is attempting to measure the extent to which government’s debt
servicing costs would be affected by market shocks, such as higher interest rates, exchange rates or inflation indexes.

A vital approach in market risk theory states that the government should seek natural hedges in its asset and liability portfolios, in order to reduce portfolio risk. Natural hedges occur when portfolio exposures can be structured to offset one another, without recourse to purchasing financial devices. An example of a natural hedge is ensuring that interest rate composition of government debt correlates as closely as possible with interest rates of its assets.

The exposure of RGS debt portfolio with a huge concentration into a single inflation index (IGP-DI) is striking. Special attention to this index is essential for its risk management. This price index is an important component of the Producer Price Index group, which presents a higher volatility than the Consumer Price Index. This volatility results in significant oscillations, which may be reflected either in the original installment or in the debt amount. The IGP-DI index also has a close relationship with commodities price. The commodities market presents extremely high volatility, which makes public debt of brazilian states highly exposed to speculation in the commodities market.

The currency exchange rate is another factor contributing to the excessive volatility of the IGP-DI. The high correlation between the
producer index and the currency exchange rate is probably the main factor causing imbalances on the state’s debt since the statement of the refinancing law. Statistics collected from surveys detected that approximately 60% of a shock in exchange rates reflects on IGP on the short term. Since the beginning of the refinancing law, all currency devaluations were absorbed by the IGP. These devaluations have impacted heavily on RGS public debt because the Real Net Revenue did not follow in the increase of debt servicing. Furthermore, the sustainability analysis proved that net revenue growth does not cover all monetary variation. This gap, of unpaid debt servicing, started the residual count, since the first substantial devaluation of the Real in 1998, originated after Russia’s Crisis. Graphs above demonstrate respectively the correlation of unpaid interests and the exchange rate. The interest rate carried by the residual is not included, and therefore the downward tendency on the last 5 years does not mean that the full interest rate has been paid.
Comparing the last graph with the exchange rate trajectory depicted on the graph below, it is clear that the Real’s devaluation had a direct impact in unpaid interest, particularly in 1998 and 2002. The peak was reached in 2003 with almost R$ 1 billion no interest paid.

The official development index in Brazil is the Consumer Price Index (IPCA), not the Producer Index (IGP). The IPCA is known to be less volatile than the IGP. This suggests that at least a share of RGS debt portfolio should be composed by IPCA. Table below shows the historic between both indexes since 1998.

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<tbody>
<tr>
<td>IGP-DI</td>
<td>1.71%</td>
<td>19.99%</td>
<td>9.60%</td>
<td>10.40%</td>
<td>26.41%</td>
<td>7.67%</td>
<td>12.13%</td>
<td>1.22%</td>
<td>3.79%</td>
<td>7.89%</td>
<td>9.10%</td>
<td>-1.42%</td>
</tr>
<tr>
<td>IPCA</td>
<td>1.66%</td>
<td>8.94%</td>
<td>5.97%</td>
<td>7.67%</td>
<td>12.53%</td>
<td>9.30%</td>
<td>7.60%</td>
<td>5.69%</td>
<td>3.14%</td>
<td>4.45%</td>
<td>5.90%</td>
<td>4.31%</td>
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During this period, the IGP presented 65% greater variation than the IPCA. Focusing on the volatility of both indexes, at 0.079, the standard deviation of the IGP is clearly higher than the IPCA’s 0.030.

The federal government debt management office has already been reducing the share of IGP on debt portfolio. The federal government is redeeming bonds pegged to the producer index and it has not issued more since 2005. These actions made the percentage of Producer Index on federal public debt portfolio fall more than 50%. Therefore, the reduction of the IGP on states debt should also be inserted on the general public debt strategy of the country.

Another point that must be emphasized relates to the inconsistency between assets, represented by flows of real net revenue, and liabilities, represented by refinancing law portfolio. Debt management theory would call an “absence of natural hedge” as detailed before. If the net revenue had followed the IGP, public debt in the state of RGS would have stayed balanced. This fact does not mean that the entire portfolio must be replaced with one entirely linked to net revenue. Nevertheless, the analysis suggests that mixing the debt composition (seeking at least a reduction on the excessive exposure to a single volatile index) would certainly improve the debt profile.
These assessments show that RGS’s debt portfolio is not compliant with the main principles of sound practices in debt management. The sound practices points out a reduction in volatility and predictability on debt portfolio. And those ideas have not been practiced, since debt is coupled with a volatile and risky index.

**5.2 Refinancing or roll over risk**

The refinancing risk is defined as the risk of heavy change on public debt payment profile on the moment that it must be refinanced. Sometimes this situation can lead a government to fail in refinancing a share or totally its debt maturing at any period. Average time of maturity or even percentages of debt maturing on 12 months are indicators examples to measure the refinancing risk. These indicators attempt for cash flows availability to pay debt maturing.

Relating the concept with refinancing law 9496 operation, the states are not exposed to refinancing risk related with debt, since their payment are linked with a percentage of net revenue. On one hand, a significant advantage for states (specifically for RGS government) is undeniable; otherwise the state would be required to serve total original installment of the refinancing law. In this sense, the federal government assumes the roll over risk, because the share of unpaid debt installment is directly refinanced and accumulated on the residual count.
On the other hand, as the state of RGS has not been capable of paying down its entire current debt, the problem has been delayed to be solved in 2028, when the Residual count will have 10 more years to be paid. If nothing else is done, the refinancing risk will cause serious fiscal problems for RGS. In this case, RGS state will have to allocate an extremely high percentage of its expenditures for debt servicing, which will seriously compromise its cash flow availability.

6. Final considerations

The analysis of the refinancing law, specifically on RGS, has produced evident conclusions. RGS’s fiscal efforts have not been sufficient to redirect the state’s debt into a sustainable path. Certainly, federalism issues should also be included in this discussion: the rigid control imposed by the refinancing law on Brazilian states debt was the option chosen by the federal government as a part of the economic stabilization plan of the country. However, the debt sustainability situation, on RGS with the federal government, is a problem that the state may not solve by itself.

The benefits of the refinancing law as a device to improve fiscal responsibility in the late 1990s are evident among Brazilian states, which were used to carrying debt without much concern about indebtedness
levels. The role of federal government on that period, when it recognized the necessity to refinance States’ debt and fix it, is notable. However, financial market has continued changing and therefore the refinancing law burdens, which can be considered reasonable for that period, are overcome. Since the beginning of the refinancing law, no changes were made on debt burdens, while the market conditions and variables are changing every day.

In this sense, adjustments on the refinancing law burdens, which would improve the states’ debt portfolio, seem logical regarding the theoretical aspects explored in this paper. The State’s contribution, through fiscal efforts on the fiscal policy practiced by the federal government aiming the macroeconomic stability, was significant. The contribution of states represents an important share of the primary surplus accomplished by the country.

The data demonstrated the relevance of the inflation index to the sustainability equations. The producer index is highly volatile, as evidenced through the historical average and other statistics.

Perhaps the most important adjustment that must be done is on the debt burden. The adjustment would improve not only the State’s debt portfolio, but also the federal government’s debt burden. Another issue that may become important is related to the real interest rate, fixed at 6%. This
rate is becoming a matter of concern as the real interest rate of the country is falling below this reference.

The federal government debt management has obtained great results for the country. The results could be even better if federal government would align the debt of the states with the debt strategy of the country.
References


