PSMB 1021

Stats

Statistics

New Topic

ConcepTest stats coin flips 1a

You flip a coin two times in a row. How much more likely is it to get one and and one tail than to get 2 heads?

1) Equally
2) Half as likely
3) Twice as likely
4) 4 times as likely

ConcepTest stats coin flips 1b

You flip a coin two times in a row. How much more likely is it to get one and and one tail than to get 2 heads?

1) Equally
2) Half as likely
3) Twice as likely
4) 4 times as likely

Statistics in Physics

Statistics must be used to model physics systems that have large numbers of particles.

Why is this??

- We can only follow one or two or at best 3 particles at a time mathematically
- All is no lost, for if we can understand the dynamics of two particles at a time, we can consider these to be average values for a much larger number particles
- Because we have a large number of objects, we can use the law of large numbers to make accurate predictions of average or expected values.
- Today we will see that expectation values of expected averages and standard deviations have statistical,that is actual, real significance.

Probability

New Topic

Probability, the starting point

We ask the question, if there are m ways to get the desired state, and there are n ways not to get it

What is the probability to get the desired state?

- First, we change the question. Let L = m + n, and the question becomes one of what are the chances to get the desired state out of L possibilities.
- Then, just by counting:

$$P_m = \frac{m}{L}$$

Probability, more details

If we have more than one desired outcome?

What is the probability to get several desired states simultaneously?

$$\mathrm{P}_{\mathrm{ABCD}} = P_A P_B P_C P_D$$

Some examples:

- Getting heads 5 times in a row …

We have a ½ chance to get heads on one flip. If we flip twice, then only half of the heads we flip on the first one will be heads the second time, half of those the third time, etc.

$P_{5H} = (½)^5$

- Now, in general, if there is the probability to get an outcome one time is P, the probability to achieve this m times is

$P_m = P^m$

Probability, example

Imagine that in front of you are 4 boxes, in each is 4 balls: red, green, blue, and orange.

- If you reach into one and randomly pull a ball out, what is the probability to pull out a red ball:

$$P = m/L = ¼ = 25\%$$

- What is the probability to pull a red ball out of all 4 boxes?

$$P = (P_1)^n = (¼)^4 = 0.0039 = 0.39\%$$

- What is the probability to pull a red, then a green, then a second green, then a blue ball (that is RGGB) out of 4 consecutive boxes?

$$P = (P_1)^n = (¼)^4 = 0.0039 = 0.39\%$$

- Does it matter that you pulled 2 G's in a row?

Ponderable: Flipping coins

1. If you flip a coin one time, what is the probability to get heads? To get tails? Plot the probabilities as a histogram
2. Now if you flip a coin twice, what is the probability to get
   1. two heads
   2. two tails? Ok, this is easy
   3. one head and one tail? Careful here, you could get H or T first
   4. Plot the probabilities as a histogram
3. Do the same for 4 coins.
   1. List the possible arrangements (e.g. 4H0T, 3H1T, etc)
   2. What is the probability to get each arrangement … we will also call these states
   3. Plot the probabilities as a histogram

Ponderable: Flipping more coins

1. If you flip a coin 10 times. What is the probability to get 10 heads in a row?
2. If you flip a coin 10 times
   1. What is the probability to get HTTTTTTTTT = H+(9 tails)?
   2. How ways can you get 1 head and 9 tails? (1H9T)
   3. What is the probability to get 1H9T

Multiplicity

New Topic

Multiplicity, more than one way?

- Most times there is more than one way to get a given state. In the previous example, you can see that there is more than one way to get 2 Gs, an R, and a B.
- The number of ways is the multiplicity, and it is calculated by:

$$W_{AB} = \frac{(A+B)!}{A!B!}$$, for two elements

- That is when there are A elements of one kind and B elements of another.
- The can be extended to three distinct elements:

$$W_{ABC} = \frac{(A+B+C)!}{A!B!C!}$$

- Or even more

$$W_{ABCDE...} = \frac{(A+B+C+D+E+...)!}{A!B!C!D!E!...}$$

Example: Multiplicity, example

- How many ways are there to get 2 Gs, an R, and a B (that is 2G1R1B)?

Remember that there are 4 balls, so we are really asking about the number of ways to get 2G1R1B0O (zero orange!)

- The number of ways is the multiplicity, and it is calculated by:

$$W_{2G1R1B0O} = \frac{(2+1+1+0)!}{2!1!1!0!} = \frac{4!}{2!} = 12....(0!=1)$$

- What is the probability to get 2G1R1B?

There are now 4 boxes, each of which has 4 different balls in it:

L = the number of possible arrangements = (#of balls/box)$^{(\text{\# of boxes})}$

L = 4^4 and P_{2G1R1B} = 12/4^4= 0.047 = 4.7%

- Remember RGGB? P_{RGGB} = 0.39%, it is a lot smaller

Ponderable: Flipping coins II

1. Imagine that you flip a coin 4 times in a row, list the possible combinations of heads and tails (e.g. 2H2T. 1H3T, ...)? What is the multiplicity for each of the combinations? What is the probability for each? --- you may want to make a table.
2. Which is most likely?
3. Sum the multiplicities? Does this add up to 2^4?
4. For 10 flips, what is the probability to have 10 heads
5. For 10 flips, what is the number of outcomes with 6 heads, with 4 tails?
6. What is the probability to have 6 heads, with 4 tails?
7. What is the probability to have 5 heads, with 5 tails?
8. For 100 flips, compare the multiplicities for having 50 heads and 50 tails with 49 heads and 51 tails (google math can do the factorials).

ConcepTest stats gas in a bottle 2a

You have an evacuated glass jar into which you release 10^6 N_2 molecules. Compare the is probability to have 10 molecules in the upper half and 999,990 in the lower half with a 500,000/500,000 split.

1) Way bigger
2) Way smaller
3) A little smaller
4) About the same

ConcepTest stats gas in a bottle 2b

You have an evacuated glass jar into which you release 10^6 N_2 molecules. Compare the is probability to have 10 molecules in the upper half and 999,990 in the lower half with a 500,000/500,000 split.

1) Way bigger
2) Way smaller
3) A little smaller
4) About the same

ConcepTest stats gas in a bottle 3a

You have an evacuated glass jar into which you release 10^6 N_2 molecules. Compare the is probability to have 500,100 molecules in the upper half and 499,900 in the lower half with a 500,000/500,000 split.

1) Way bigger
2) Way smaller
3) A little smaller
4) About the same

ConcepTest stats gas in a bottle 3b

You have an evacuated glass jar into which you release 10^6 N_2 molecules. Compare the is probability to have 500,100 molecules in the upper half and 499,900 in the lower half with a 500,000/500,000 split.

1) Way bigger
2) Way smaller
3) A little smaller
4) About the same

Ponderable: Let's make a deal: Probablility depends upon the rules

- Let's play let's make a deal
- Here are three envelopes, one contains a dollar, the other two contain a piece of colored paper
- Pick one of them
- Now I will show you that one of the others has the colored paper in it. Do you want to switch?

ConcepTest stats lets make a deal 4a

Based on the probabilities, should she switch?

1) Definitely yes
2) Definitely no
3) maybe
4) It doesn't matter

ConcepTest stats lets make a deal 4b

Based on the probabilities, should she switch?

1) Definitely yes
2) Definitely no
3) maybe
4) It doesn't matter

- Now you have a 1/3 = 33% chance that the dollar is in the envelope, your first pick, so if you stay with #1, you have a 1/3 chance of winning.
- If you switch, your odds are 100% - 33% = 67%

Example: calculate P by counting

- Let's play let's make a deal
- Here are three envelopes, one contains a dollar, the other two contain a piece of colored paper
- Pick one of them
- Now I will show you that one of the others has the colored paper in it. Do you want to switch?

Ponderable: Now there are 4 envelopes

- Let's play let's make a deal
- Here are four envelopes, one contains a dollar, the other three contain a piece of colored paper
- Pick one of them
- Now I will show you that two of the others has the colored paper in it. Do you want to switch?
- Calculate the probability for getting the dollar by switching and by not switching by the two methods just introduced.

Expectation Values

New Topic

Calculating Physical Quantities—expectation values

- If we assign a numerical value to certain outcomes, then we can calculate the predicted score when all outcomes are sampled randomly.

$$\bar{x} = \langle x \rangle = \sum_{i=1}^{N} x_i P_i$$

- How do we use this? We need rules that assign scores to each possible combination, and then we can calculate the expected value of the score we would get. That is the value we would measure if we made a large number of samples:

Calculating Physical Quantities more generally

- The expectation value of any quantity that can be calculated as a function of x can be calculated, if the probabilities for every value of x are known:

$$\overline{f} = \langle f(x) \rangle = \sum_{i=1}^{N} f(x_i) P_i$$

- We just finished discussing and working examples for f(x) = x. Another very important quantity is $f(x) = x^2$

$$< x^2 >= \sum x_i^2 P_i$$

- From this can be calculated the standard deviation:

$$\sigma = \sqrt{\left\langle \left(x - \overline{x}\right)^2 \right\rangle} = \sqrt{\sum_{i=1}^{N} x_i^2 P_i - \left(\overline{x}\right)^2}$$

ConcepTest stats dice 5a

If roll a single die many times, what is the expectation value of the score (based on the average face value of each roll)

1) 1
2) 3
3) 3.5
4) 5
5) 6

ConcepTest stats dice 5b

If roll a single die many times, what is the expectation value of the score (based on the average face value of each roll)

1) 1
2) 3
3) 3.5
4) 5
5) 6

Calculating expectation values: example

In a game of dice, the person who rolls gets the number of points (1-6) on the face of the die. So rolling a single die, what is the expectation value of the score?

- There are 6 possible values, and each has equal probability, 1/6. We can make a table:

S (score)	P_s	sP_s
1	1/6	1/6
2	1/6	2/6
3	1/6	3/6
4	1/6	4/6
5	1/6	5/6
6	1/6	6/6
Sums	1	21/6=3.5

- As expected, the sum of the probabilities is 1 and <s> = the average dice score