Physics 1021

Fall 2010-8b

ConcepTest 12.1 Seesaw

You and your friend want to play on the seesaw, but you are much heavier than your friend. If your friend sits in the middle (like the girl on the left), where should you sit?

1. There is no position where you can balance the seesaw
2. Sit close to the end
3. Directly at the pivot point
4. Sit close to the pivot point
5. Sit at the same distance as your friend

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Linear and Rotational Motion

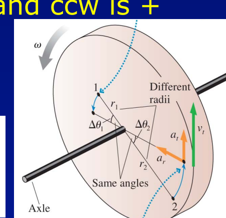
Translation	Rotation
m	I
$\vec{\mathbf{F}}$	τ
$\vec{\mathbf{a}}$	α
$\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$	$\Sigma\tau = I\alpha$
Δx	$\Delta\theta$
$W = F_x\Delta x$	$W = \tau\Delta\theta$
$\vec{\mathbf{v}}$	ω
$K = \frac{1}{2}mv^2$	$K = \frac{1}{2}I\omega^2$
$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$	$L = I\omega$
$\Sigma\vec{\mathbf{F}} = \lim_{\Delta t\to 0}\frac{\Delta\vec{\mathbf{p}}}{\Delta t}$	$\Sigma\tau = \lim_{\Delta t\to 0}\frac{\Delta L}{\Delta t}$
If $\Sigma\vec{\mathbf{F}} = 0$, $\vec{\mathbf{p}}$ is conserved	If $\Sigma\tau = 0$, L is conserved

Every aspect of linear motion has a rotational analogue, as can be seen in this table

We will focus on torques and equilibrium and simple kinematics: cw is – and ccw is +

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$



Torque

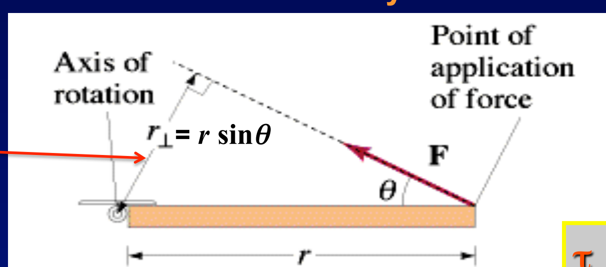
New Topic

PHYS 1: Chap. 9, Pg 5

Two ways to think about torque: Use the one that works most easily

$\tau = F\,d\sin\theta$

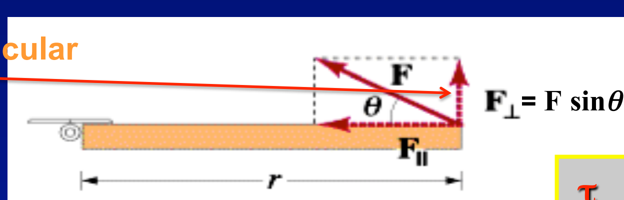
Lever arm



$\tau = F\,r_\perp$

$$\tau = F\cdot(r\sin\theta)$$

Perpendicular force



$\tau = F_\perp\,r$

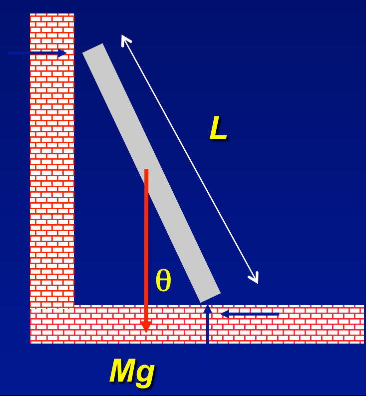
$$\tau = (F\sin\theta)\cdot r$$

PHYS 1021: Chap. 12, Pg 6

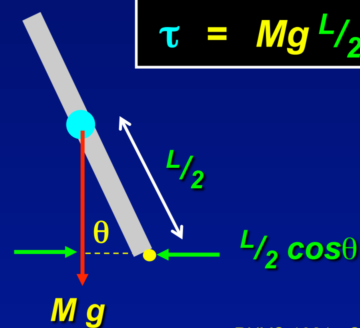
Calculating Torque using lever arm

- A plank of length L and mass M leans against a wall at an angle θ with the ground. What is the torque exerted on the plank by gravity about an axis through its bottom end ?

Torque = Force × perpendicular distance



$$\tau = Mg\,{}^{L}/_{2}\cos\theta$$

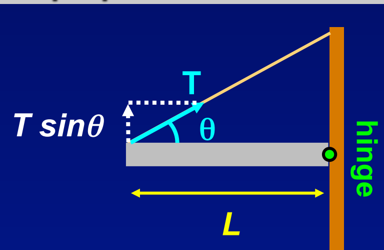


PHYS 1021: Chap. 12, Pg 7

Calculating Torque using perpendicular force

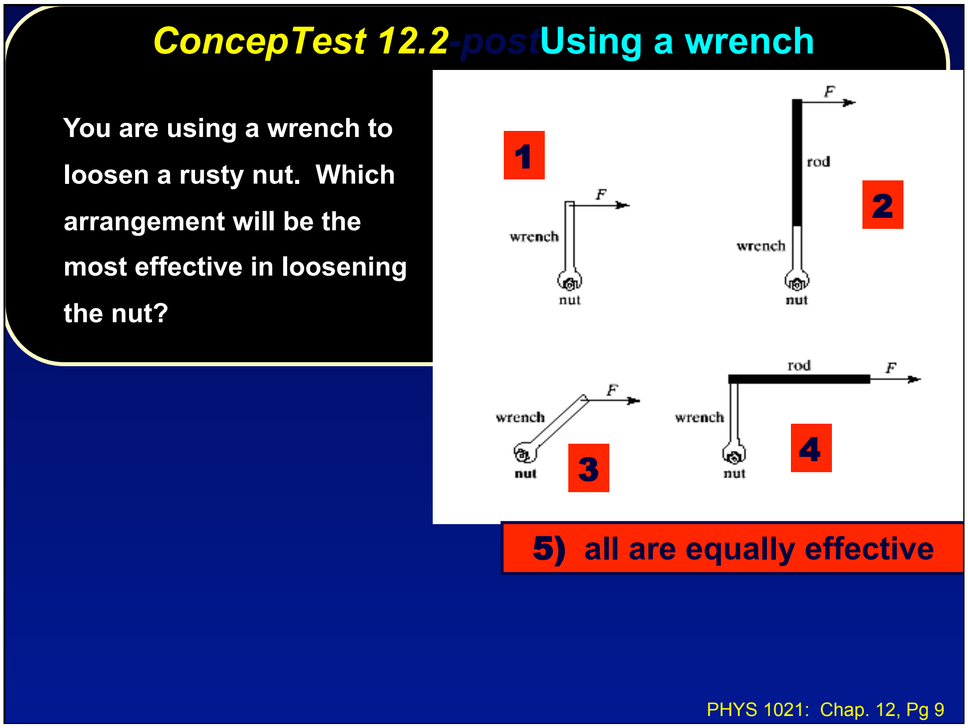
- A beam of length L is supported by a cable which has tension T. What is the torque produced by the tension in the cable about the hinge?

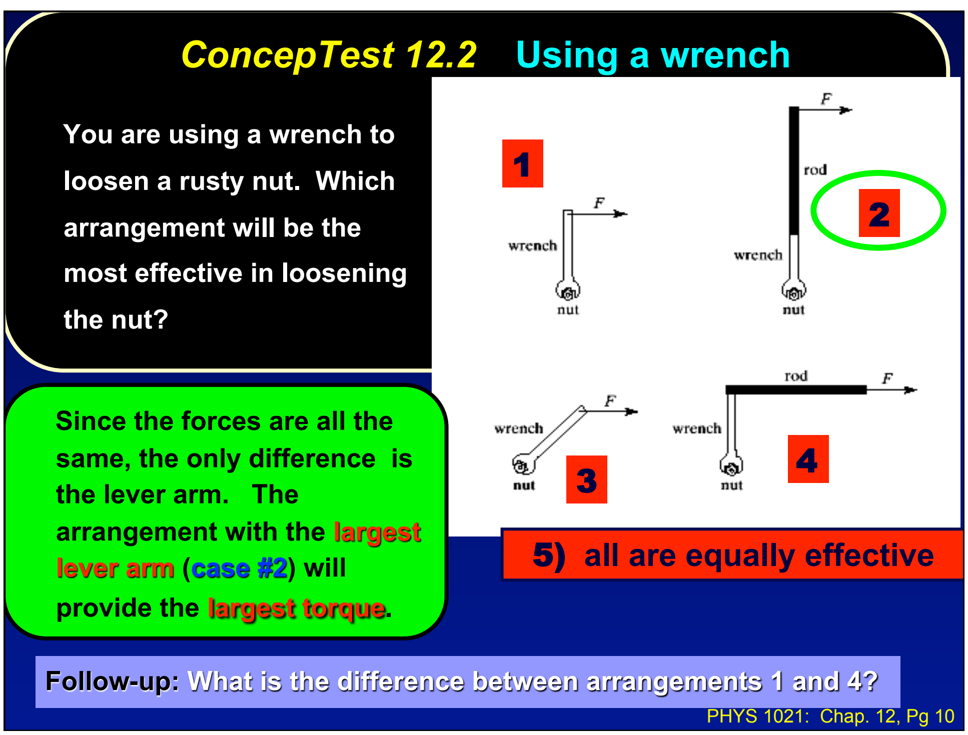
Torque = perpendicular force × distance



$$\tau = F_{\perp} \times \text{distance} = L\,T\sin\theta$$

PHYS 1021: Chap. 12, Pg 8


ConcepTest 12.2-post Using a wrench
You are using a wrench to loosen a rusty nut. Which arrangement will be the most effective in loosening the nut?
1
F
wrench
nut
F
rod
2
wrench
nut
F
wrench
nut
3
rod
F
wrench
nut
4
5) all are equally effective
PHYS 1021: Chap. 12, Pg 9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You are using a wrench to loosen a rusty nut. Which arrangement will be the most effective in loosening the nut?
1
F
wrench
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F
rod
2
wrench
nut
F
wrench
nut
3
rod
F
wrench
nut
4
5) all are equally effective
Since the forces are all the same, the only difference is the lever arm. The arrangement with the largest lever arm (case #2) will provide the largest torque.
Follow-up: What is the difference between arrangements 1 and 4?
PHYS 1021: Chap. 12, Pg 10

ConcepTest 12.3-post Cassette Player

When a tape is played on a cassette deck, there is a tension in the tape which applies a torque to the supply reel. Assuming the tension remains constant during playback, how does this applied torque vary as the supply reel becomes empty?

1) torque increases
2) torque decreases
3) torque remains constant

ConcepTest 12.3 Cassette Player

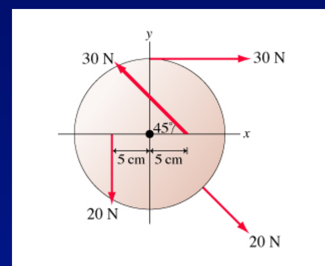
When a tape is played on a cassette deck, there is a tension in the tape which applies a torque to the supply reel. Assuming the tension remains constant during playback, how does this applied torque vary as the supply reel becomes empty?

1) torque increases
2) torque decreases
3) torque remains constant

As the supply reel empties, the **lever arm decreases** because the **radius of the reel (with tape on it) is decreasing**. Thus, as the playback continues, the **applied torque diminishes**.

Example: Torque on a disk

- The 20-cm-diameter disk in the figure can rotate on an axle through its center. What is the net torque about the axle?
- What is the angular acceleration, α.
- After two seconds, how far will the disk have rotated?



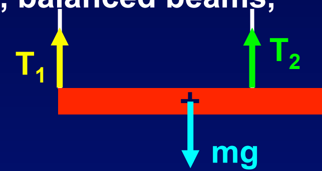
- See page 347 for moments of inertia

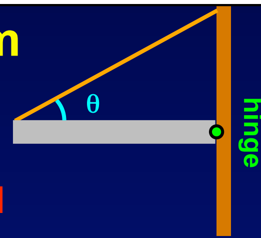
Static Equilibrium

New Topic

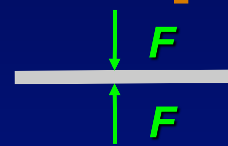
Static Equilibrium

- Ladders, sign-posts, balanced beams, bridges, etc...



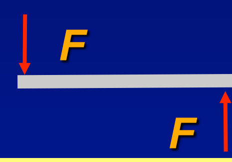


- ***Translational*** **equilibrium:** $\Sigma F = 0$



  - But point of application matters !!
  - Rotation is possible !

- ***Rotational*** **equilibrium:** $\Sigma \tau = 0$



Note: rotation axis is arbitrary. When choosing axes to calculate torque, be clever to make the problem easy....

Example: What is wrong with this arm?

- **Balance torques about the rotation axis:**
- **(9.9*35+18*16.50) = 5*F**

F~ (10*7+20*3) = 130N

To pick up 1 L of milk !!!!

What is the evolutionary advantage here? What are arms good for? Throwing, hammering! Convert force to torque to angular acceleration

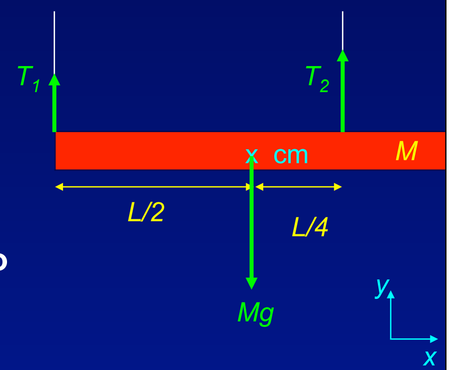
Estimate weight of a projectile that is easy to throw. A baseball has a mass of 145g => 1.45 N

Using Torque

- Now consider a plank of mass *M* suspended by two strings as shown. We want to find the tension in each string.

- First use $\Sigma F = 0$

  $T_1 + T_2 = Mg$



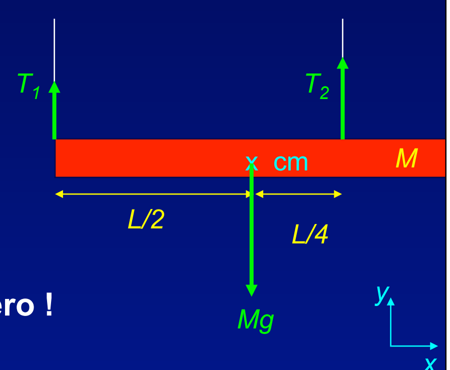
- This is no longer enough to solve the problem !
  - 1 equation, 2 unknowns

- **We need more information !!**

Using Torque...

- We *do* have more information:
  - We know the plank is not rotating ! -- has no angular acceleration.

$$\Sigma\tau = 0$$



- The sum of all torques is zero !

- **This is true about any axis we choose !**

Example: Using Torque...

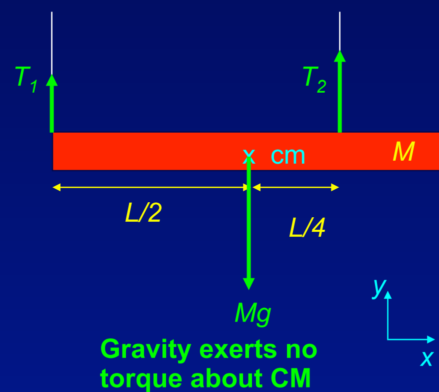
- Choose the rotation axis to be at the center of mass:

- Torque due to the string on the **right** is **counter-clockwise**:

$$\tau_2 = T_2 \frac{L}{4}$$

- Torque due to the string on the **left** is **clockwise**:

$$\tau_1 = -T_1 \frac{L}{2}$$

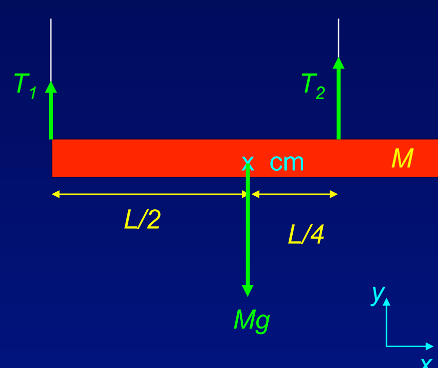


Using Torque...continued

$$\Sigma\tau = 0$$

$$T_2 \frac{L}{4} - T_1 \frac{L}{2} = 0$$

$$T_2 = 2T_1$$



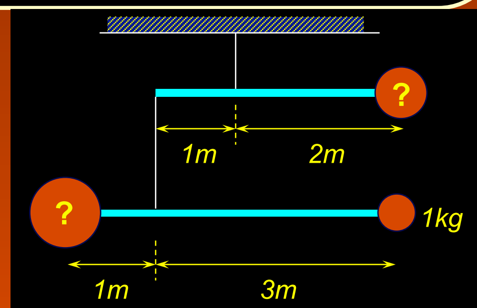
We already found that: $T_1 + T_2 = Mg$

$$T_1 = \frac{1}{3} Mg \qquad T_2 = \frac{2}{3} Mg$$

ConcepTest 12.4-*post* mobile

A (static) mobile hangs as shown below. The rods are massless and have lengths as indicated. The mass of the ball at the bottom right is *1kg*. What is the total mass of the mobile ?

1) 5 kg
2) 6 kg
3) 7 kg
4) 8 kg
5) 9 kg



ConcepTest 12.4 mobile

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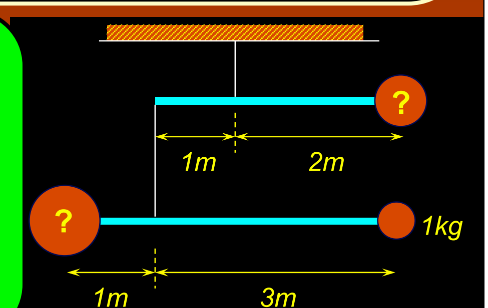
(1) 5 kg
(2) 6 kg
(3) 7 kg
(4) 8 kg
(5) 9 kg

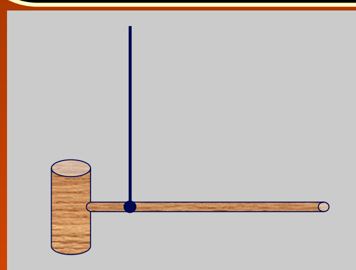
Use torques in two steps:
(1) find the big mass on the bottom left (lower rod only)
(2) use the entire lower rod assembly (with two masses) to find the mass on top right
Finally add up all the masses.



ConcepTest 12.5-post Croquet Mallet

A croquet mallet balances when suspended from its center of mass. If the mallet is cut in two pieces at this point, which piece has the greater mass?

1) the piece with the head
2) the piece with the handle
3) both pieces are equal
4) can't tell without knowing the mass of the mallet



ConcepTest 12.5 Croquet Mallet

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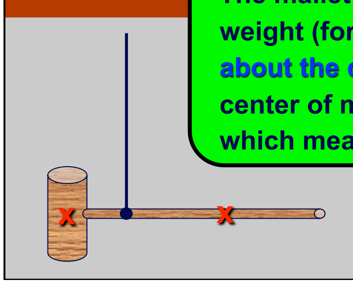
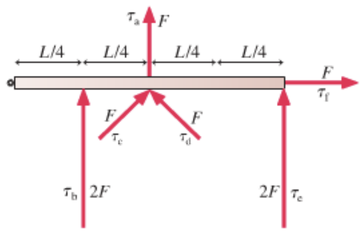
1) the piece with the head
2) the piece with the handle
3) both pieces are equal
4) can't tell without knowing the mass of the mallet

The mallet balances not because the pieces have equal weight (force) but **because the pieces exert equal torques about the center of mass**. Since the **head is closer** to the center of mass than the handle, its **lever arm is smaller**, which means that its **mass must be greater**.



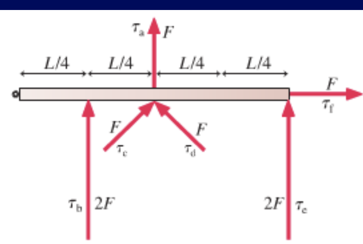
Ponderable: Where is your C.M.

Six forces are applied to the door in FIGURE Q12.8. Rank in order, from largest to smallest, the six torques τ_a to τ_f about the hinge. Explain.



Ponderable: Size of torque

Six forces are applied to the door in FIGURE Q12.8. Rank in order, from largest to smallest, the six torques τ_a to τ_f about the hinge. Explain.



All are positive (ccw)

e, a = b, c = d, f=0

Ponderable: Where is your C.M.

A student gives a quick push to a ball at the end of a massless, rigid rod, as shown in FIGURE Q12.10, causing the ball to rotate clockwise in a *horizontal* circle. The rod's pivot is frictionless.

a. As the student is pushing, is the torque about the pivot positive, negative, or zero?

b. After the push has ended, does the ball's angular velocity (i) steadily increase; (ii) increase for awhile, then hold steady; (iii) hold steady; (iv) decrease for awhile, then hold steady; or (v) steadily decrease? Explain.

c. Right after the push has ended, is the torque positive, negative, or zero?

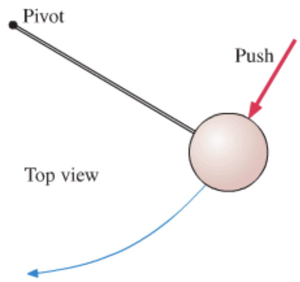


FIGURE Q12.10

PHYS 1: Chap. 11, Pg 27

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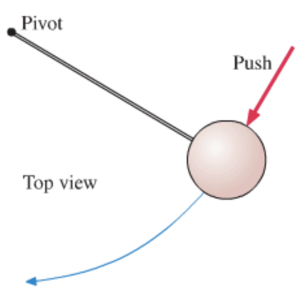


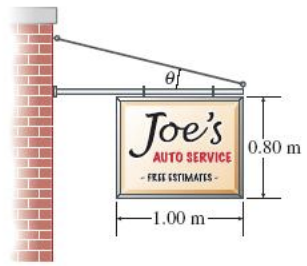
FIGURE Q12.10

a. negative (cw)
b. Constant, net torque is now zero
c. Torque is zero

PHYS 1: Chap. 11, Pg 28

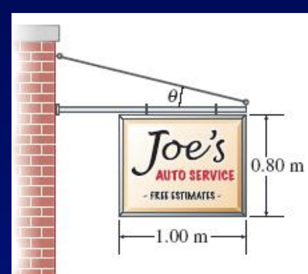
Example: Hang the sign

You are asked to hang a uniform beam and sign using a cable that has a breaking strength of 415 N. The store owner desires that it hang out over the sidewalk as shown. The sign has a weight of 178.0 N and the beam's weight is 37.0 N. The beam's length is 1.86 m and the sign's dimensions are 1.00 m horizontally × 0.80 m vertically. What is the minimum angle θ that you can have between the beam and cable?



Example: Hang the sign

Estimate: The sign and pole have less weight than half the breaking force of the cable. Therefore the angle can be small, say less than 45 degrees



Strategy Use $\sum F = 0$ and $\sum \tau = 0$. Choose the axis of rotation at the point where the beam meets the store.

Solution The tension in the cable cannot exceed 415 N. Sum the torques.

$$\sum \tau = 0 = T \sin\theta (1.86\text{ m}) - (37.0\text{ N})(0.93\text{ m}) - (178.0\text{ N})(1.36\text{ m})$$

Solve for θ and substitute 415 N (the breaking strength) for T.

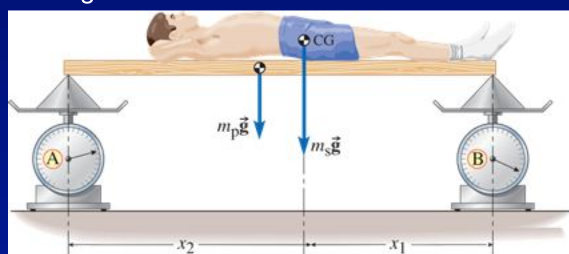
$$\theta = \sin^{-1}\frac{(37.0\text{ N})(0.93\text{ m}) + (178.0\text{ N})(1.36\text{ m})}{(415\text{ N})(1.86\text{ m})} = 21.0°$$

The minimum angle is $\boxed{21.0°}$.

Ponderable: Where is your C.M.

Since humans are generally not symmetrically shaped, the height of our center of gravity is generally not half of our height. One way to determine the location of the center of gravity is shown in the diagram. A 2.2-m-long uniform plank is supported by two bathroom scales, one at either end. Initially the scales each read 100.0 N. A 1.60-m-tall student then lies on top of the plank, with the soles of his feet directly above scale B. Now scale A reads 394.0 N and scale B reads 541.0 N.

- What is the student's weight?
- How far is his center of gravity from the soles of his feet?
- When standing, how far above the floor is his center of gravity, expressed as a fraction of his height?



Ponderable: Where is your C.M.

(a) Strategy The weight is equal to the change in the combined readings of the scales.

Solution Compute the student's weight.

$$W = 362.0\text{ N} + 525.5\text{ N} - 100.0\text{ N} - 100.0\text{ N}$$
$$= \boxed{687.5\text{ N}}$$

(b) Strategy The system is in equilibrium. Choose the axis of rotation at the point of contact between the plank and scale B.

Solution Find x_1.

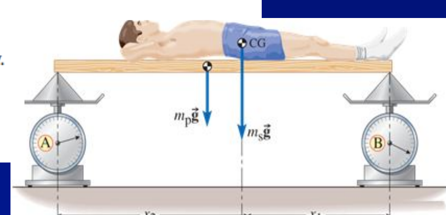
$$\Sigma\tau = 0 = m_s g x_1 - F_A L + m_p g\left(\frac{L}{2}\right), \text{ so}$$

$$x_1 = \frac{F_A L - m_p g\left(\frac{L}{2}\right)}{m\,g} = \frac{(2.10\text{ m})\left[362.0\text{ kg} - \frac{1}{2}(200.0\text{ N})\right]}{687.5\text{ N}}$$
$$= \boxed{0.800\text{ m}}.$$

(c) Strategy The height of the student is $h = 1.45$ m.

Solution Find the height y of the student's center of gravity.

$$y = \frac{x_1}{h}h = \frac{0.800\text{ m}}{1.45\text{ m}}h$$
$$= \boxed{0.552\,h}$$



Rotational Motion

New Topic

PHYS 1: Chap. 9, Pg 33

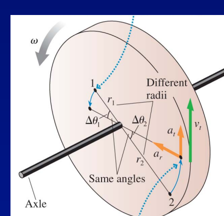
Linear and Rotational Motion

We will focus on
torques and equilibrium
and simple kinematics:

cw is – and ccw is +

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

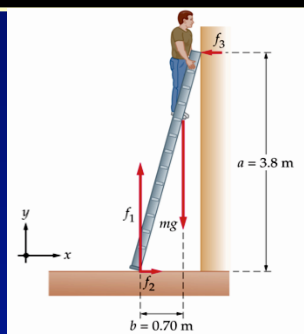


PHYS 1: Chap. 9, Pg 34

ConcepTest 12.6-post Climbing the Ladder

As the person climbs higher and higher up the ladder, does this make the ladder more or less likely to slip?

1) less likely to slip
2) more likely to slip
3) slipping does not depend on how high the person is

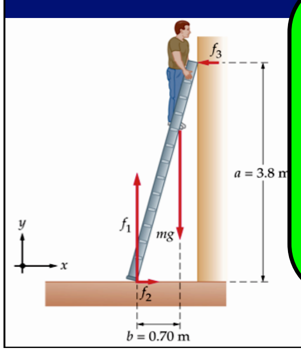


PHYS 1: Chap. 11, Pg 35

ConcepTest 12.6 Climbing the Ladder

As the person climbs higher and higher up the ladder, does this make the ladder more or less likely to slip?

1) less likely to slip
2) more likely to slip
3) slipping does not depend on how high the person is

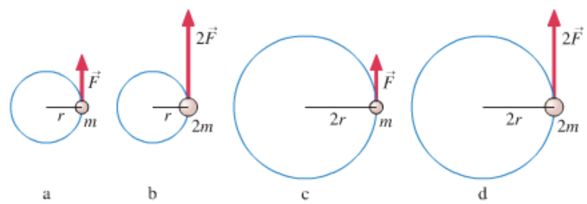


We know that the **friction force f_2 is equal to the normal force on the wall f_3** (based on x-direction forces). But as the person climbs, the **clockwise torque around the bottom will increase**, so that **f_3 must also increase to balance the torques**. Consequently, **f_2 will increase**, up to the **limit of static friction**, after which the **ladder will slip**!

PHYS 1: Chap. 11, Pg 36

Ponderable: Where is your C.M.

Rank in order, from largest to smallest, the angular accelerations α_a to α_d in FIGURE Q12.11. Explain.



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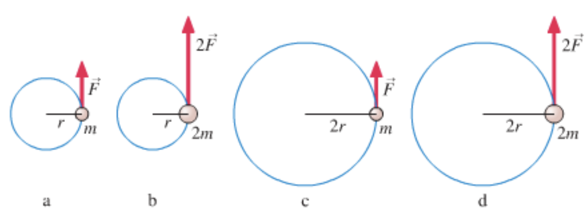
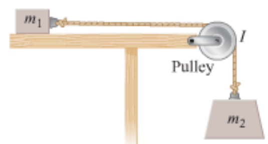


Each linear acceleration is F/m and is the same.
Therefore, using $a/r = \alpha$, a = b > c = d

Example: Pulley is no longer massless

Ch. Ex. 100 - Torque and Angular Momentum

Question



A block of mass m_2 hangs from a rope. The rope wraps around a pulley of rotational inertia I and then attaches to a second block of mass m_1, which sits on a frictionless table. What is the acceleration of the blocks when they are released?

Example: Pulley is no longer massless

For the two blocks, we have

mass 1:

$$\Sigma F_x = T_1 = m_1 a_x = m_1 a \text{ and}$$
$$\Sigma F_y = 0;$$

mass 2:

$$\Sigma F_x = 0 \text{ and}$$
$$\Sigma F_y = T_2 - m_2 g = m_2 a_y = -m_2 a\text{, so}$$
$$T_2 = \boxed{m_2 g - m_2 a}.$$

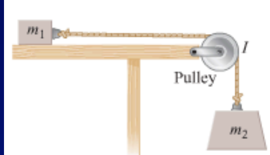
For the pulley, we have

$$\Sigma \tau = -T_1 R + T_2 R = I\alpha = I\frac{a}{R}\text{, so}$$
$$T_1 - T_2 = \boxed{-\frac{Ia}{R^2}}.$$

Find the acceleration of the blocks.

$$T_1 - T_2 = m_1 a + m_2 a - m_2 g = -\frac{Ia}{R^2}\text{, so}$$
$$\left(m_1 + m_2 + \frac{I}{R^2}\right) a = m_2 g \text{ or}$$
$$a = \boxed{\frac{m_2 g}{m_1 + m_2 + \frac{I}{R^2}}}.$$

Momentum



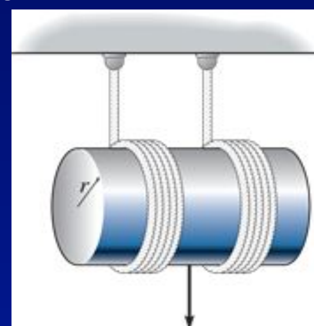
For your estimate, just say that a must be smaller than g b/c of the added inertia of the pulley, and downward pointing.

Ponderable: Roll down the barrel

A uniform cylinder with a radius of 15 cm has been attached to two cords and the cords are wound around it and hung from the ceiling. The cylinder is released from rest and the cords unwind as the cylinder descends.

•What is the acceleration of the cylinder?

•If the mass of the cylinder is 2.6 kg, what is the tension in each cord?



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a. Estimate that a is negative and its magnitude is less than g because of the extra rotational inertia

$\Sigma F = ma$

$2T - mg = ma$ (1)

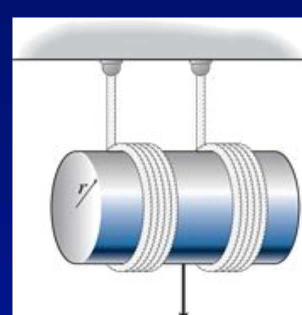
$\Sigma\tau = 2Tr = I\alpha = -Ia/r$ (a positive a at the point where the rope unwinds gives a negative, CW, α)

$2Tr = -\frac{1}{2} mr^2 a/r$

$T = -ma/4$ (2)

Plug into (1)… $-2ma/4 - mg = ma$ … $a = -2g/3$

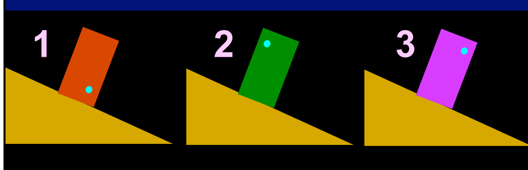
b. Using (2) …. $T = -mg/6$



ConcepTest 12.7 Tipping Over

- A box is placed on a ramp in the configurations shown below. Friction prevents it from sliding. The center of mass of the box is indicated by a blue dot in each case. In which cases does the box tip over ?

(1) all
(2) 1 only
(3) 2 only
(4) 3 only
(5) 2 and 3



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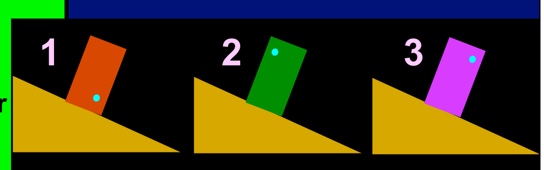
ConcepTest 12.7 Tipping Over

A box is placed on a ramp in the configurations shown below. Friction prevents it from sliding. The center of mass of the box is indicated by a blue dot in each case. In which cases does the box tip over ?

(1) all
(2) 1 only
(3) 2 only
(4) 3 only
(5) 2 and 3

The torque due to gravity acts as if all the mass of an object is concentrated at the CM. Consider the bottom right corner of the box to be a pivot point. If the box can rotate such that the CM is lowered, it will !!



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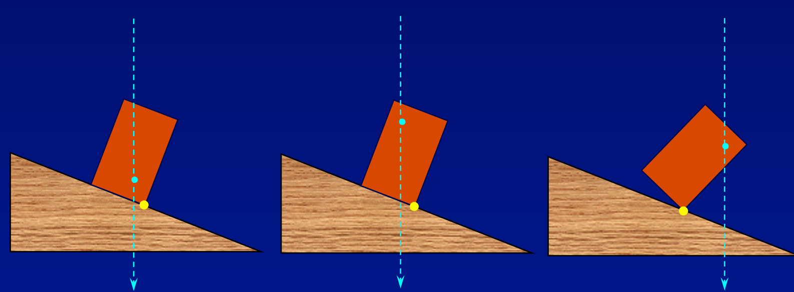
ConcepTest
Solution

- We have seen that the torque due to gravity acts as though all the mass of an object is concentrated at the center of mass.
  - Consider the bottom right corner of the box as a pivot point.
  - If the box can rotate such that the CM is lowered, it will !



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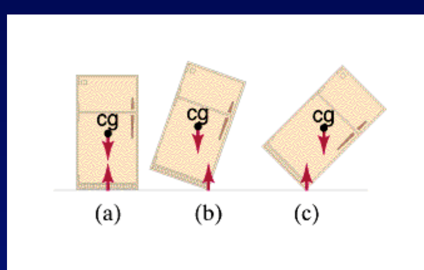
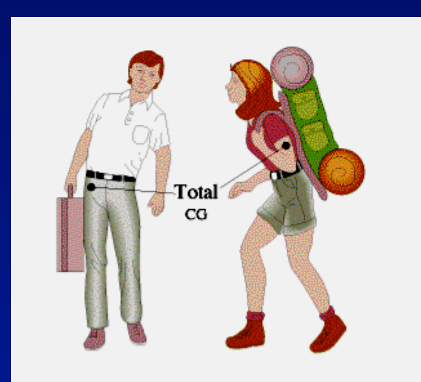


When CM of the refrigerator is no longer over the support point, it will tip over.

People try to keep the CM over their feet, in order to feel stable.



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Time's up … Thanks for your attention and I'll see you next time.
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