Inverse problem for a wave equation

Shitao Liu *

I would like to present an inverse problem about the wave equation with time independent potential and Neumann boundary data. More precisely, let $\Omega \subset \mathbb{R}^n$, $n \geq 2$, be a bounded domain and its boundary $\Gamma = \partial\Omega$ be of class C^2, consisting of the closure of two disjoint parts: $\Gamma = \partial\Omega = \overline{\Gamma_0 \cup \Gamma_1}$, $\Gamma_0 \cap \Gamma_1 = \emptyset$. We consider the following wave equation:

$$\begin{cases} w_{tt}(x,t) = \Delta w(x,t) + q(x)w(x,t) & \text{in } Q = \Omega \times [0,T] \\ w(\cdot, \frac{T}{2}) = w_0(x) & \text{in } \Omega \\ w_t(\cdot, \frac{T}{2}) = w_1(x) & \text{in } \Omega \\ \frac{\partial w}{\partial \nu}(x,t)|_\Sigma = g(x,t) & \text{in } \Sigma = \Gamma \times [0,T] \end{cases}$$

and the following **nonlinear inverse problem**:

Let $w = w(q)$ be a weak solution to the equation, under geometrical conditions on Γ_0, is it possible to retrieve the potential $q(x)$, $x \in \Omega$, from measurement of $w(q)$ on $\Sigma_1 = \Gamma_1 \times [0,T]$?

In my study, I first prove the uniqueness of $q(x)$ by using an appropriate Carleman estimate from [3] and an idea given by V.Isakov in [1]; then I prove a stability result by using a continuous observability inequality.

Similar problem has been studied by V.Isakov and M.Yamamoto in [2], here we give a different proof for uniqueness and we improve the stability result to L_2 norm, thanks to the Carleman estimate in [3]. We can also mention that similar problems for Riemann wave equation and Schrödinger equation could also be done.

Main references

[1] V. Isakov, *Inverse Problems for Partial Differential Equations*, Second Edition, Springer, New York, 2006.

[2] V. Isakov and M. Yamamoto, Carleman estimate with the Neumann boundary condition and its application to the observability inequality and inverse hyperbolic problems, *Contemp. Math.*, **268**(2000), 191-225.

[3] I. Lasiecka, R. Triggiani and X. Zhang, Nonconservative wave equations with unobserved Neumann B.C.:global uniqueness and observability in one shot, *Contemp. Math.*, **268**(2000), 227-325.

*Department of Mathematics, University of Virginia, USA. Email: sl3fa@virginia.edu

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