

APPLIED MATHEMATICS QUALIFYING EXAM SYLLABUS

Function Spaces

- Normed vector spaces, inner product spaces; function spaces (e.g., $L^p(\Omega)$, $C^k(\Omega)$, $C^{k,\alpha}(\Omega)$, etc.); norm of an operator; compact operators; contraction mapping; Fredholm alternative theorem; Arzelà-Ascoli theorem.

Ordinary Differential Equations

- Dimensional analysis.
- Regular and singular perturbation theory; inner and outer expansions; boundary layer analysis.
- Calculus of variations; Euler-Lagrange equations; classical harmonic oscillator, the pendulum, minimal surfaces; Dirichlet and Neumann boundary conditions. Variational problems with constraints.
- One-dimensional boundary value problems; eigenvalues and eigenfunctions. Sturm-Liouville theory, comparison theorems; Green's functions, integral equations.
- Stability and bifurcation for systems of ODEs; linear stability and Lyapunov functional; classification of bifurcation points; exchange of stability.
- Numerical methods for ordinary differential equations. Runge-Kutta method; shooting method; convergence, stiffness and stability.

Partial Differential Equations

- First order equations, characteristics, and shock waves; failure of the characteristic method, weak solutions.
- The wave and the heat equations in the entire space. 1-D wave equation (d'Alembert's formula), 3-D wave equation (spherical means method, Kirchhoff's formula); the 2-D wave equation, the method of descent; heat equation, Fourier transforms, heat kernel.
- The wave and heat equations in a bounded domain; separation of variables; Laplace operator on a bounded region; Poisson's equation; maximum principles; Green's functions.
- Schrödinger's equation; quantum harmonic oscillator; hydrogen atom; spherical harmonics.
- Numerical methods for elliptic, parabolic, and hyperbolic partial differential equations. Finite difference methods; accuracy, convergence, and stability.

References:

The first reference listed in each category is the primary reference.

Applied Mathematics

- J. D. Logan, *Applied Mathematics*, Wiley-Interscience, 1997.
- V. I. Arnold, *Mathematical Methods of Classical Mechanics*, GTM, Springer-Verlag, 1980.
- R. Courant & D. Hilbert, *Methods of Mathematical Physics*, Vol I-II, Interscience, 1962.

Ordinary Differential Equations

- E. A. Coddington & N. Levinson, *Theory of Ordinary Differential Equations*, McGraw-Hill, 1955
- M. W. Hirsch & S. Smale, *Differential Equations, Dynamical Systems, and Linear Algebra*, Academic Press, 1974.

Partial Differential Equations

- W. A. Strauss, *Partial Differential Equations*, Wiley, 1992.
- H. F. Weinberger, *Partial Differential Equations*, Wiley, 1965.
- F. John, *Partial Differential Equations*, Springer-Verlag, 1982.

Function Spaces

- J. B. Conway, *A Course in Functional Analysis*, GTM, Springer, 1985.

Numerical Analysis

- R. L. Burden and J. D. Faires, *Numerical Analysis*, PWS, 1993.