

ALGEBRA QUALIFYING EXAM SYLLABUS

- Groups, subgroups, normal subgroups, quotient groups, homomorphisms, isomorphism, isomorphism theorems, Cayley's theorem.
- Normal and composition series, Jordan-Holder theorem, solvable groups, simple groups.
- Direct products, structure theorem for finitely generated abelian groups.
- Groups acting on sets, class equation, Sylow's theorems.
- Free groups.
- Elementary properties of rings, ideals, Zorn's lemma, quotient rings, homomorphism and isomorphism theorems, field of fractions of an integral domain, polynomial rings, Noetherian rings, algebras.
- Unique factorization domains, principal ideal domains, Euclidean domains, prime ideals.
- Fundamental properties of modules over a ring, finitely generated modules over a principal ideal domain, chain conditions, tensor product, free modules, direct sum.
- The Jacobson radical of a ring, artinian rings, simple and semisimple rings, idempotent decomposition, Wedderburn-Artin theorems.
- Algebraic and transcendental field extensions, algebraically closed fields and algebraic closure of a field, splitting fields, normal extensions, separable and inseparable extensions, Galois theory of finite extensions, radical extensions, solvability by radicals.

References:

- Larry Grove, *Algebra*, Dover Publications, 2004.
- Thomas W. Hungerford, *Algebra*, Graduate Texts in Mathematics, Vol. 73, Springer Verlag, 1974.
- Nathan Jacobson, *Basic Algebra*, second edition, W. H. Freeman and Company, volume I, 1985, volume II, 1989.