Explosive bubbles in house prices? Evidence from the OECD countries[*]

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February 2014

Abstract

We conduct an econometric analysis of bubbles in housing markets in the OECD area, using quarterly OECD data for 18 countries from 1970 to 2011. We pay special attention to the explosive nature of bubbles and use econometric methods that explicitly allow for explosiveness. First, we apply the univariate date-stamp procedure of Phillips et al. (2011) to pin down the periods where prices were explosive. Next, we use Engsted and Nielsen's (2012) co-explosive VAR framework to test for bubbles while at the same time allowing prices to be cointegrated with fundamentals. Our main finding is that house prices in many countries were explosive, thus supporting the bubble hypothesis. However, in several countries fundamentals also contributed to the boom. In general there are large cross-country differences in the dynamics of house prices.

JEL Classification:
    Keywords:

1 Introduction

Many countries have experienced dramatic movements in house prices over the past 15-20 years, with large increases during the 1990s and first half of the 2000s followed by price drops since 2006-2007. This pattern has been especially pronounced in countries such as Spain, Ireland, Denmark, Italy, the Netherlands, the UK, and the US, see Figure 1. Understanding these developments is important, not least because the recent international financial crisis to a large extent originated from the housing markets, e.g. the subprime mortgages in the US and the overinvestment in housing in many European countries.

In general, changes in house prices must be due to one of two causes (or a combination of them), either changing 'fundamentals' or speculative bubbles. In the literature, *rents* are usually considered an important part of fundamentals for house prices, see e.g. Hamilton and Schwab (1985), Meese and Wallace (1994), Himmelberg et al. (2005), Gallin (2006), Ghysels et al. (2012), Brunnermeier and Julliard (2008), Campbell et al. (2009), Plazzi et al. (2010), Cochrane (2011), Engsted and Pedersen (2013), and Gelain and Lansing (2013). For the owner of a house who also lives in the house, rents can be seen as a proxy for the unobservable housing service flow and thus are the equivalent to the dividends that an owner of a stock obtains in the equity market.

However, the recent boom-bust developments in real estate markets have generated a heated discussion of whether speculative bubbles could be a major factor in house price movements in addition to changing fundamentals. During the boom period several observers, most notably Shiller (2005), raised the possibility that a bubble was driving US house prices, while others, e.g. Himmelberg et al (2005), McCarthy and Peach (2004), and Krainer and Wei (2005), argued that the US housing market was not inflated by a bubble.

After the end of the boom period a few studies have investigated the bubble hypothesis for the US housing market using formal econometric tests. Phillips and Yu (2011) basically use the econometric methods from Phillips et al. (2011), which rely on forward recursive regressions coupled with right-sided unit root tests, to document explosive behavior in US house prices. Kivedal (2013) uses the co-explosive vector-autoregressive (VAR) methodology from Engsted and Nielsen (2012), and he also finds US house prices to be explosive. Thus, both these recent studies find evidence in support of the bubble hypothesis for the US.

To our knowledge, however, there has not been any systematic econometric analysis

of explosiveness in house prices outside the US. In this paper we fill this gap in the literature. We conduct a thorough econometric analysis of bubbles in housing markets in the OECD area, using quarterly OECD data for 18 countries from 1970 to 2011. We pay special attention to the explosive nature of bubbles and use econometric methods that explicitly allow for explosiveness. First, we apply the univariate date-stamp procedure of Phillips et al. (2011) to pin down the periods where prices were explosive. Next, we use Engsted and Nielsen's (2012) co-explosive VAR framework to test for bubbles while at the same time allowing prices to be cointegrated with fundamentals.

The appealing feature of the Engsted-Nielsen approach is that it allows prices to contain both an explosive component - coming from the bubble - and an I(1) component coming from the non-stationary part of fundamentals, i.e. prices and fundamentals may 'cointegrate' despite the explosive root in prices. This is an important feature of traditional bubble models that is often neglected in empirical bubble studies although emphasized in Diba and Grossman (1988) and Engsted (2006). The drawback of the Engsted-Nielsen approach is that it assumes that the bubble period can be identified á priori; in principle the method does not allow for bursting or partially bursting bubbles during the sample period. Thus, the sample period needs to end before or at the peak of the bubble.

By contrast, the Phillips et al. procedure is explicitly designed to capture bursting bubbles and to date-stamp the beginning and end of the bubble. Thus, this procedure can handle a sample period that contains both bubble and non-bubble subperiods. The drawback of the procedure is that it does not allow for both an explosive root and a unit root and, hence, it does not allow for the estimation of the cointegrating relationship between prices and fundamentals.

We suggest to combine the Phillips et al. and Engsted-Nielsen procedures in the following way. First, the recursive unit root tests are applied to date-stamp the period where prices are explosive. Next, a co-explosive VAR model for prices and fundamentals (rents) is estimated on that period. Thereby we pin down the bubble period while at the same time estimating the effect from rents to prices.

Our main finding is that house prices in many countries were explosive, thus supporting the bubble hypothesis. However, in several countries fundamentals also contributed to the boom. In general there are large cross-country differences in the dynamics of house prices.

The rest of the paper is organized as follows. In the next section the bubble model

is described. Section 3 describes the econometric methodologies. That section also contains a comparison of our approach with earlier bubble tests. In section 4 we present the empirical results using data from the OECD countries. Section 5 concludes.

2 The bubble model

We start by considering the standard model for asset price determination for an informationally efficient market with homogeneous and rational agents. Let P_t denote the house price and X_t denote the service flow, i.e. the housing rents, both at time t. Given a constant and positive expected one period return, $0 < R \in \mathbb{R}$ [1], the model is given by

$$P_t = \frac{1}{1+R} E_t \left(P_{t+1} + X_{t+1} \right). \tag{1}$$

The expectation operator, E_t, is conditioned on information available at time t, that is current and past prices and service flows, $\{P_s, X_s\}_{s \leq t}$.

The general solution to the model is identified as the present value of future service flows, see e.g. Cochrane (2008). When we do not impose a transversality condition on the present value of house prices in the infinite future[2] and let B_t denote the rational bubble component of the price process, then the model has the solution

$$P_t = \sum_{s=0}^{\infty} \frac{E_t X_{t+s}}{(1+R)^s} + b B_t,$$

where $b \in \mathbb{R}$. To avoid arbitrage, c.f. Diba and Grossman (1988) , B_t should satisfy the homogeneous expectation equation

$$B_t = \frac{1}{1+R} B_{t+1} + \xi_{t+1}, \tag{2}$$

where $E_t \xi_{t+1} = 0$. Note that the restriction $R > 0$ implies that any rational bubble must have an explosive nature.

In general the 'spread' defined as $S_t \equiv P_t - X_t/R$ by Campell and Shiller (1987), is a cointegrating relation that eliminates the common stochastic trend between prices and service flows. To see this, we utilize the general solution to the model, by inserting for

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[1]We will follow the bubble literature and assume a constant expected return, even though many asset price models relax this assumption.

[2]The condition $\lim_{T \to \infty} (1+R)^{-T} E_t[P_{t+T}] = 0$ would rule out bubbles a priori.

P_t and get

$$S_t = \frac{1+R}{R} \sum_{s=1}^{\infty} \frac{E_t\left(\Delta_1 X_{t+s}\right)}{(1+R)^s} + bB_t. \tag{3}$$

Here we see that the stochastic behaviour of prices depends only on X_t and B_t. Hence, if $\Delta_1 X_t$ is a stationary process, then S_t will be the cointegrating relation between prices and service flows. Note that S_t can still exhibit non-stationary behaviour due to the explosive bubble component.

Now, we can rewrite equation (1) in order to facilitate the econometric tests of later sections. First we define

$$M_t \equiv P_t + X_t - (1+R)P_{t-1} \tag{4}$$

and note that from equation (1) we have $E_{t-1}M_t = 0$. Therefore M_t is a martingale difference sequence. To express this martingale difference in terms of non-integrated components we add and subtract RP_t and identify the spread

$$M_t = (1+R)\Delta_1 P_t - RS_t. \tag{5}$$

Alternatively this can be expressed in terms of the service flow growth[3]

$$M_t = \Delta_{1+R}S_t + (1+R^{-1})\Delta_1 X_t. \tag{6}$$

Here, $\Delta_{1+R} \equiv (1 - (1+R)L)$. If X_t is $I(1)$, then S_t must be stationary to ensure that that M_t is a martingale difference. Hence, Δ_{1+R} is sufficient to eliminate the potential explosiveness in prices and S_t must indeed be the cointegrating relation that removes the stochastic trend of prices and service flows.

3 The econometric methodology

3.1 The Phillips et al. procedure

Our econometric approach begins by date-stamping periods in which prices have experienced explosive behaviour. This procedure follows Phillips and Yu (2011) who conduct

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[3]Add and subtract $\frac{1+R}{R}\Delta_1 D_t$ to M_t

recursive right-sided augmented Dickey-Fuller (ADF) tests for explosiveness.

This is done by estimating the following econometric model for prices, P_t

$$P_t = \mu_p + \delta P_{t-1} + \sum_{j=1}^{J} \phi_j \Delta P_{t-j} + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2), \tag{7}$$

where $\mu_p, \delta, \phi_j \in \mathbb{R}$. For each regression we determine the lag-length, J, by the lowest number of lags that sufficiently removes autocorrelation from the error term, ε_t, evaluated by Portmanteau tests at the 5% critical value. The null hypothesis of a unit root, $H_0 : \delta = 1$, is tested against the right-sided alternative, $H_1 : \delta > 1$. Let $w_0 \in [0, 1]$ denote some lower fraction of the sample and for a total sample of T price observations let $\tau_0 = \lfloor w_0 T \rfloor$ denote the corresponding sample size[4]. From here we recursively add another observation and re-estimate (7) for sample sizes $\tau = \lfloor wT \rfloor$ when $w_0 \leq w \leq 1$. For every estimation we calculate the ADF t-statistic, which we denote ADF_w. To date stamp the bubble period we hold the time series of ADF_w statistics against the appropriate right-tailed critical values from the standard Dickey-Fuller distribution. By denoting the collapse of the bubble by τ_c we use a last occurrence strategy, which differs slightly from the procedure in Phillips et al (2011).

$$\tau_c = sup_{s \geq w_0} \left\{ s : ADF_s > cv_{\beta_T}^{adf}(s) \right\}$$

Following Phillips et al (2011) we set the critical values by $cv_{\beta_T}^{adf}(s) = \log(\log(sT))/100$. We do not pursue the identification of the emergence of bubble periods as our model explains only bubbles that has been present for the infinite past.

As a robustness check, we also do date stamping based on a forward rolling window such that the sample size $w_w = w_0$ for all estimations.

3.2 The Engsted-Nielsen procedure

The co-explosive testing procedure applied in e.g. Engsted and Nielsen (2012) is comparable to e.g. Campbell and Shiller (1987) and Johansen and Swensen (1999). We consider

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[4]We use $\lfloor \cdot \rfloor$ to denote integer values of its argument

the vector $V_t = (P_t, X_t)'$ and assume that it follows the k'th order vector autoregression

$$M : \qquad V_t = \mu_v + \sum_{j=1}^{k} A_j V_{t-j} + \epsilon_t, \quad \epsilon \sim NID_2(0, \Omega), \tag{8}$$

where $A_j \in \mathbb{R}^{2 \times 2}$ and $\mu_v \in \mathbb{R}^2$

Following Nielsen (2010) this vector autoregression can be reparameterized in the error correction form

$$\Delta_1 \Delta_\rho V_t = \mu_v + \Pi_1 \Delta_\rho V_{t-1} + \Pi_\rho \Delta_1 V_{t-1} + \sum_{j=1}^{k-2} \Phi_j \Delta_1 \Delta_\rho V_{t-j} + \epsilon_t. \tag{9}$$

Here we use the notation that $\Delta_\rho = (1 - \rho L)$ and we have $\Pi_1, \Pi_\rho, \Phi_j \in \mathbb{R}^{2 \times 2}$ and $\rho \in \mathbb{R}$. The VECM representation is linked to the VAR representations through the standard error correction form, that follows, from Grangers representation theorem

$$\Delta_1 V_t = \mu_v + \Pi V_{t-1} + \sum_{j=1}^{k-2} \Gamma_j \Delta_1 V_{t-j} + \epsilon_t. \tag{10}$$

and the following set of identities.

$$\Pi_1 = \frac{\Pi}{1 - \rho}, \quad \Pi_\rho = -\rho \left(I_p + \Pi_1 + \sum_{j=1}^{k-1} \rho^{-j} \sum_{l=j+1}^{k} A_l \right), \quad \Phi_j = - \sum_{l=j+1}^{k-1} \rho^{j-l} \sum_{i=l+1}^{k} A_i. \tag{11}$$

where $\Pi, \Gamma_j \in \mathbb{R}^{2 \times 2}$.

In order to analyse the presence of rational bubbles we make the assumption that V_t has one unit root and an explosive root, $\rho > 1$. Hence, we have the following reduced rank restrictions $\Pi_1 = \alpha_1 \beta_1'$ and $\Pi_\rho = \alpha_\rho \beta_\rho'$, where $\alpha_1, \beta_1, \alpha_\rho, \beta_\rho \in \mathbb{R}^2$. Here β_1 has the common interpretation of a cointegrating relation of V_t equivalent to the one identified by the spread, S_t. Therefore we have the model implied restriction $\beta_1 = (1, -1/R)'$. Secondly, β_ρ is interpreted as a co-explosive vector that counteracts explosiveness. Therefore non-explosiveness of rents, X_t can be tested by the restriction $\beta_\rho = (0, 1)'$.

Now, a combination of these restrictions along with those imposed by the VAR model and the martingale difference equation (??) imply the following set of all restrictions (see

Engsted and Nielsen (2012) for details)

$$\iota'\alpha_1 = -1, \qquad \beta_1 = (1, -1/R)', \quad \rho = 1 + R, \qquad \iota'\mu_v = 0 \qquad (12)$$
$$\iota'\alpha_\rho = -(1+R)^2/R, \qquad \beta_\rho = (0,1)', \qquad \qquad \iota'\Phi_j = 0$$

where $\iota' = (1,1)$.

3.2.1 The cointegration restriction

The starting point in the estimation procedure is to estimate the unrestricted model, M, found in equation (8). Here we compute the characteristic roots of the companion matrix, and if the largest root is larger than unity, $\hat{\rho}_0 > 1$, this serves as an indication of explosiveness. Hereafter, we apply the sequential rank test of Johansen (1995). This corresponds to the hypothesis

$$H_1: \qquad (\Pi_1, \mu_v) = \alpha_1(\beta_1', \zeta_1), \quad \Pi_\rho = \alpha_\rho \beta_\rho' \qquad (13)$$

where $\zeta_1 \in \mathbb{R}$ is a constant that is restricted to the cointegrating space. If we use M_1 to denote M when restricted by H_1 then

$$M_1: \qquad \Delta_1\Delta_\rho V_t = \mu_v + \alpha_1\beta_1'\Delta_\rho V_{t-1} + \alpha_\rho\beta_\rho'\Delta_1 V_{t-1} + \sum_{j=1}^{k-2}\Phi_j\Delta_1\Delta_\rho V_{t-j} + \epsilon_t. \qquad (14)$$

and we calculate the updated characteristic root, $\hat{\rho}_1$. If this characteristic root is found to be strictly lager than unity we proceed through the subsequent steps of the estimation procedure.

3.2.2 The non-explosiveness of rents

Under the assumption of H_1 we will next test the hypothesis of non-explosive rents, that is imposing the restriction

$$H_X: \qquad \beta_\rho = (0,1)'$$

This additional hypothesis restricts the model to

$$M_{1X}: \qquad \Delta_1\Delta_\rho V_t = \mu_v + \alpha_1\beta_1'\Delta_\rho V_{t-1} + \alpha_\rho\Delta_1 X_{t-1} + \sum_{j=1}^{k-2}\Phi_j\Delta_1\Delta_\rho V_{t-j} + \epsilon_t.$$

The likelihood of this model is determined by a numerical profile argument. Specifically, we apply a grid search over a range of values for ρ. The likelihoods are then determined by reduced rank regressions of $\Delta_1 \Delta_\rho V_t$ on $\Delta_\rho V_{t-1}$, where we correct for lagged rent growth, $\Delta_1 X_{t-1}$, and differences, $\Delta_1 \Delta_\rho V_{t-j}$. The Likelihood-ratio test of H_X under the model M_1 is asymptotically $\chi^2(k-1)$ (see Engsted and Nielsen (2012)), where p denotes the lag length of the unrestricted VAR model.

3.2.3 The restriction on the spread

Suppose that H_1 and H_X are both not rejected. In this case we can set up the restriction on the spread by

$$H_S: \qquad \beta_1 = (1, -1/R)', \quad \rho = 1 + R.$$

This restriction implies that the model takes the form

$$M_{1XS}: \qquad \Delta_1 \Delta_\rho V_t = \mu_v + \alpha_1 \Delta_\rho S_{t-1} + \alpha_\rho \Delta_1 X_{t-1} + \sum_{j=1}^{k-2} \Phi_j \Delta_1 \Delta_\rho V_{t-j} + \epsilon_t.$$

where the spread is inserted for $S_t = P_t - \frac{1}{\rho-1} X_t$. The maximization of the likelihood is done along the same numerical lines as in the previous step, whereby we obtain $\hat{\rho}_{1XS}$. C.f. Engsted and Nielsen (2012) we see that the likelihood ratio test statistic of H_S in M_{1X} is asymptotically $\chi^2(1)$.

3.2.4 The efficient market hypothesis

Previous papers have devoted some attention to the empirical test of the martingale difference in equation (**??**), also known as the classical *Efficient Market Hypothesis* (EMH). We will abstract from this, as it seems unlikely that the housing market should be efficient, given the huge transaction cost (e.g. on average 6% of property values are paid in realtor fees in the US), the degree of asymmetric information, and geographical heterogeneity. Furthermore, when tested, there is not a single market in our sample in which we do not reject the EMH.

3.3 Comparison with earlier bubble tests

West (1987) developed an often cited specification test for rational stochastic bubbles. The test compares two sets of estimates of the underlying asset pricing model. The first set of estimates is consistent both with and without a bubble, while the second set is only consistent in the absence of a bubble. Equality of the two sets of estimates is then tested using a Hausman (1978) type specification test. The null hypothesis is no bubble, while the presence of a bubble should lead to rejection of the hypothesis. A problem with this procedure (noted by West himself in West (1985)) is that the test is not consistent. Under the alternative hypothesis that a bubble is present, the probability that the test will reject the null does not go to unity asymptotically. This is a direct consequence of the explosiveness of prices under the alternative. The Engsted and Nielsen (2012) procedure that we apply in this paper does not face this problem because in this procedure the null hypothesis explicitly involves a bubble.

Diba and Grossman (1988) proposed to test for rational bubbles by using Bhargava's (1986) von Neumann-like statistic to test the null hypothesis of a unit root in prices against the explosive alternative. They also tested for cointegration between prices and fundamentals arguing that with a constant discount factor cointegration precludes bubbles while no cointegration would be consistent with the presence of a bubble. By using Bhargava's (1986) test for explosiveness the Diba and Grossman methodology assumes that the variables are at most a first-order autoregressive process, and the discount factor cannot be estimated but must be specified a priori. The Engsted and Nielsen (2012) procedure extends Diba and Grossman's procedure by specifying a general VAR for the variables that allows for an explosive root in addition to a possible common stochastic I(1) trend (i.e. cointegration) between prices and fundamentals. In addition, the procedure allows estimation of the discount factor instead of prefixing it a priori as in the Diba-Grossman procedure.

Using a linear VAR for prices and fundamentals requires that the discount factor is constant. Most previous bubble studies in fact assume that the discount factor is constant. In the empirical finance literature this assumption is controversial since returns are often found to be predictable (see e.g. Cochrane (2008)). However, Engsted et al. (2012) show that a rational bubble may make returns appear predictable even when expected returns (and thereby the discount factor) is constant. In addition, even if expected returns are time-varying, Craine (1993) and Timmermann (1995) show that unless expected returns are highly persistent, the cointegrating relationship between prices and

fundamentals implied by the constant discount factor present value model will also hold approximately when the discount factor is time-varying.[5]

Evans (1991) showed in a simulation study that in a finite sample unit root and cointegration based tests will often not identify the explosive component of periodically collapsing rational bubbles (see also Hall et al. (1999)). Thus, the Engsted and Nielsen (2012) framework may not work well in that situation. By contrast, the Phillips et al. (2011) recursive procedure is explicitly designed to account for the periodically collapsing nature of the Evans type bubbles. However, as we mentioned in the Introduction, the Phillips et al. procedure does not allow for the estimation of the cointegrating relationship between prices and fundamentals because it does not allow both a unit root and an explosive root.

In fact, based on Diba and Grossman (1988) many earlier empirical bubble studies have claimed that cointegration between prices and fundamentals rules out bubbles. For example, Phillips et al. (2011, p.206) state: "In the presence of bubbles, p_t [price] is always explosive and hence cannot co-move or be cointegrated with d_t [fundamental] if d_t is itself not explosive. Therefore, an empirical finding of cointegration between p_t and d_t may be taken as evidence against the presence of bubbles." This statement is at best incomplete. In fact, as shown above the stochastic I(1) trend in d_t will be part of p_t also in the presence of an explosive bubble, and the multivariate cointegrated VAR analysis based on reduced rank regressions will capture this feature. However, it is true that univariate regression based cointegration analysis will not be able to accomodate the common I(1) trend in p_t and d_t if p_t also involves an explosive trend because the regression residuals will always be non-stationary in that case.

Combining the Phillips et al. (2011) and Engsted and Nielsen (2012) procedures, as we have suggested in the present paper, makes it possible to take the best from both approaches thereby making it possible to date-stamp the bubble period and estimating both the explosive root and the common I(1) trend between prices and fundamentals within that period.

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[5]Craine (1993) shows that with a time-varying (but stationary) discount factor the *ratio* between prices and fundamentals will be stationary under no bubbles. Thus, testing for explosiveness of this ratio is robust to the assumption about the discount factor. However, no other testable restrictions follow from Craine's approach.

4 Empirical results

4.1 The Data

The described method is applied to a large dataset consisting of 21 OECD countries. That is Australia, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, the United Kingdom, Greece, Ireland, Israel, Italy, Japan, Korea, the Netherlands, Norway, New Zealand, Sweden, and the United States. The dataset has previously been used in Engsted and Pedersen (2013). The dataset contains real and nominal prices along with the price-rent ratio for housing markets and are reported from national statistical sources. The series are provided on a quarterly basis and the average over the observations in 2005 are indexed to 100. Most of the series contain observations from 1970Q1 to 2011Q4 except for 8 countries that have later starting points. That is Australia (1972Q3), Belgium (1976Q2), Spain (1971Q1), Greece (1997Q1), Israel (1998Q1), Korea (1990Q1), Norway (1979Q1), Sweden (1980Q1). Due to the short sample sizes we discard Greece, Israel and Korea from our analysis, leaving us with 18 OECD countries.

For every country we use the real house prices, P_t, and the price-rent ratio, to back out the real rent series, X_t. This implies that the average rent values for 2005 observation are indexed to 1, due to the indexation of the price-rent ratio. Figure 1 and figure 2 show plots of the price and rent series respectively. Likewise we show first differences of prices and rents in figure 3 and figure 4.

For most countries house prices grow modestly from the beginning of the 1970s and until the mid 1990s. From this point on the prices seem to experience explosive behaviour, which is the motivation for applying the co-explosive vector autoregression framework to this particular dataset. However, some countries behave very different from the majority. These countries include Germany and Japan who both have experienced somewhat declining house prices since the beginning of the 1990s. Furthermore, a group of countries have shown dramatic increases and subsequent declines in house prices around 1990 or earlier in the sample. That is e.g. what we see in Switzerland, Finland and Japan. As seen in Figure 2 the explosive behaviour of prices is also a visible feature of the first differences of prices, which supports the use of the co-explosive framework. This pattern is particularly visible for Denmark, Spain, France and the United States. After 2007 most housing markets have seen stabilizing or drastically decreasing prices. The exceptions are Norway and Germany where prices have increased a lot over the post-crisis years

The pattern for rents is much more stable than what is seen in the prices. Except for a few sudden increases in Swiss, German, and Swedish rents, these do not seem to have explosive behaviour, which is essential for our analysis. Instead the series seem to follow two patterns, either they have a U-shape, which is particularly evident in countries like Spain and Ireland, or they are moderately trending, like what is seen in the United Kingdom and Japan. From the first differences, it is also clear that the rent series do not behave explosively.

4.2 The Results

Table 1 contains results from our estimations. For each of the 18 OECD countries we test three samples, where the first is the entire sample, then we test the sample selected by the forward recursive (FW) Phillips et al. (2011) test and lastly we use the Phillips et al. test to a rolling window (Win). In the cases of Germany, Finland, Italy, and Japan the forward recursive strategy does not identify a bubble period, in which case we neglect this selection. In the cases of Belgium, Canada, and Norway the forward recursive strategy identifies the entire samples as the bubble period.

We commence the estimation by estimating a bivariate vector auto-regression for $V_t = (P_t, X_t)'$. We investigate the characteristics of the residuals and determine the optimal lag length, $k \geq 2$, of the model by ensuring that there is no residual autocorrelation in the model[6]. From the unrestricted model, we compute the characteristic roots of the companion matrix and in the third column of table 1, under $\hat{\rho}_0$, we report modulus of the roots. We notice two cases, either $\hat{\rho}_0$ is close to unity suggesting that the system contains at least one stochastic trend, or the characteristic root is strictly larger than unity. The last case is considered as preliminary evidence of explosiveness in the system. This explosive behaviour is mainly featured in the sub samples identified by the Phillips et al. tests. Countries with no preliminary evidence of explosiveness cover Switzerland, Germany, Finland, the United Kingdom, Italy, and the Netherlands.

Next, we determine the cointegration rank by Johansen's rank test including a restricted constant. In many cases we are not able to reject the null of zero rank against the alternative of $r = 1$, implying that prices and rents have no common stochastic trend, but are both non-stationary and unrelated in their levels. In none but one of the cases where we reject $r = 0$ we can reject the hypothesis of $r \leq 1$. In these cases prices and rents

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[6]At the moment all samples have $k = 2$, in general this ensures no-autocorrelation. But for some samples this is not satisfied

are cointegrated, which is required for further testing of the theoretic model. Belgium differs by being the only country where we can reject $r \leq 1$, meaning that the companion matrix has full rank and that the vector autoregression is well defined in levels. Even though the test is valid in the explosive case, Cushham (2003) finds that serial correlation seriously deteriorates the size of the Johansen test, which might be the case if the model is subject to moving average components. This does seem to be the case in some of the estimations.

We report the unrestricted cointegration relation, denoted β'. There are huge differences in the estimates, not only across countries, but also intra country for the different sample sizes. In many cases the sign of the cointegration relation is 'wrong' in the sense that either equilibrium prices or dividends should be negative. However, there is no sub sample in which we can reject the hypotheses that $\beta_1 = (1,0)'$ or $\beta_1 = (0,1)'$. Furthermore, there is no clear interpretation of the coefficients in β_1 as in Engsted and Nielsen (2012) as our rent series are implicitly indexed to unity in 2005.

From the restricted model M_1 imposing a unit root, $r = 1$, we determine the largest characteristic root and its modulus, which is reported as $\hat{\rho}_1$ in table 1. In many cases we find that the largest characteristic root is equal to unity, even in the cases that indicated explosive behaviour in the unrestricted vector auto-regression. In these cases we do not proceed with the formal tests of non-explosive dividends and the spread-restriction.

If the data still behave explosively after imposing the cointegration, then we proceed by first testing the H_X hypothesis. That is whether $\beta_\rho = (0,1)'$ and hence that dividends are non-explosive. Given this restriction, we report the largest characteristic root of M_{1X} under $\hat{\rho}_{1X}$. By a likelihood ratio test this hypothesis is tested, noting that the test is $\chi^2(1)$ distributed. We find that in the cases of Spain, France, the United Kingdom, and the United States, we can reject this hypothesis at the 5% level, but in 9 countries this is not rejected.

Finally, we would like to consider whether the spread condition is a reasonable restriction by imposing $H_S : \beta_1 = (1, -1/R)'$ along with the relation between the one period return and the explosive root, $\rho = 1 + R$. However, the indexation of prices and rents has an implications for the size of the one period return, R, in the spread $S : t = P_t - 1/Rx_t$. However, the explosive root in the price process, ρ, is independent of this indexation and hence, we cannot formally test the spread restriction given our data. From previous investigations of the likelihood function in the dimension of β_1, which appears to be very flat, we do not expect that we, given non-indexed data, would be able to reject the spread

condition. This is supported as the indexation implies a 4% annual return on housing which seem to be in the reasonable range.

Overall, there is only one sub sample in which we can identify an explosive root, cointegration of prices and rents and where we do not reject the non-explosiveness of rent. That is Irish sample selected by the fixed window Phillips et al. test.

In general we find that the Phillips et al. test with the rolling window is better to date stamp the bubbles, in the sense that we find an explosive characteristic root of the M_1 model more often than with the forward recursion. This follows partially as the forward recursion has a tendency to include a few observations from the post-peak period of the samples, which seriously affects the co-explosive framework.

5   Concluding remarks

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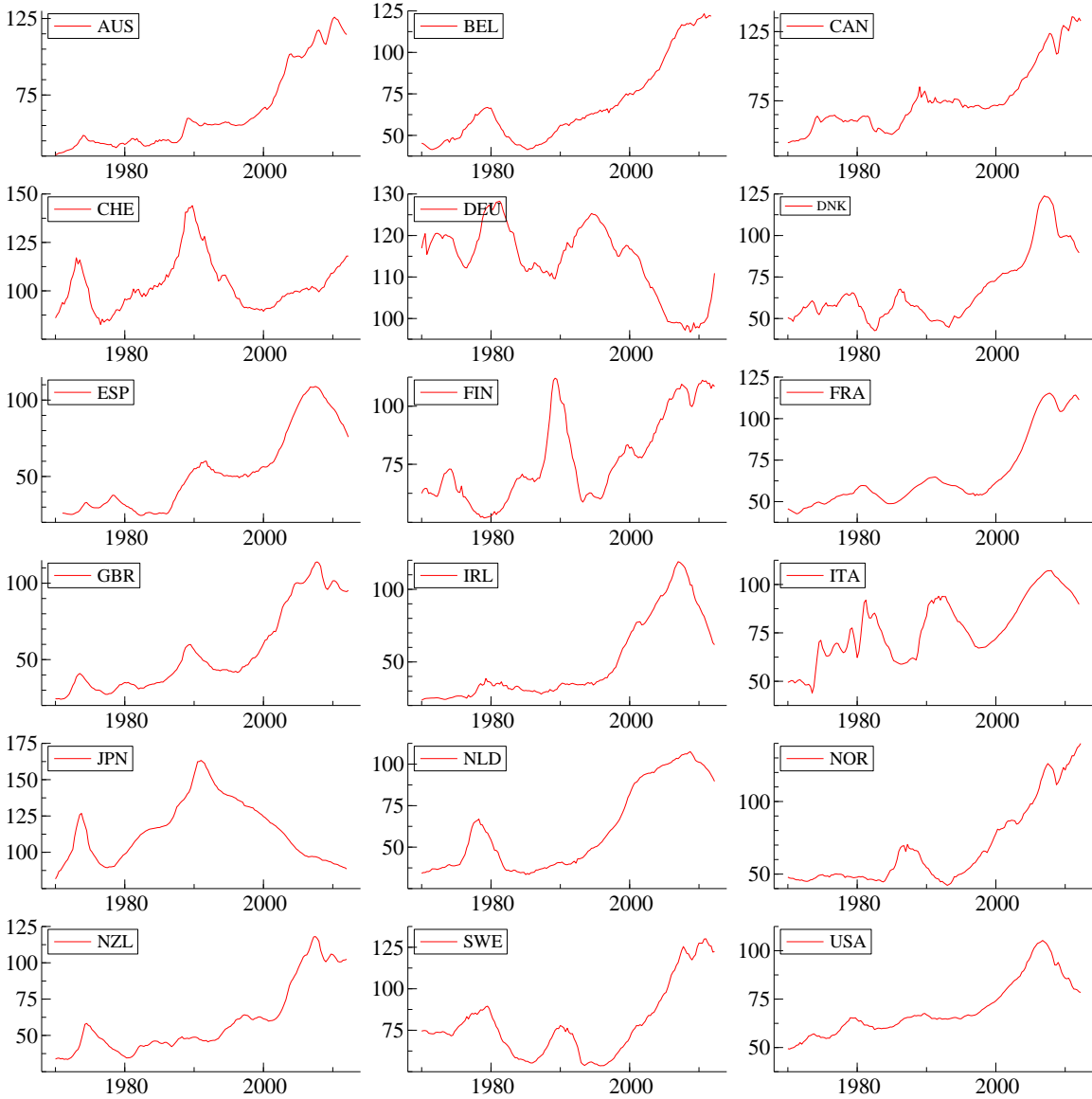# A Empirical findings

A.1 Data
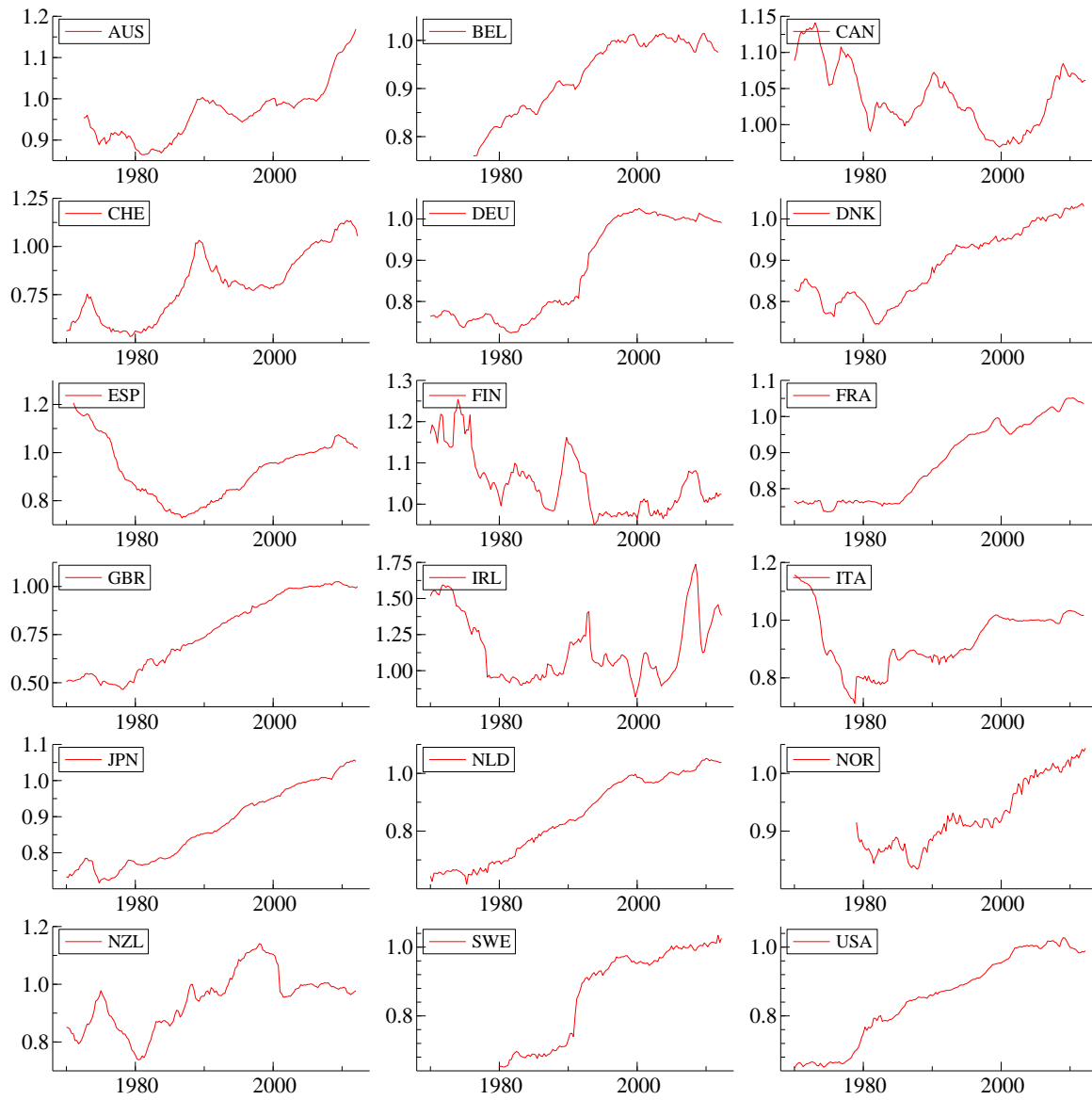

Figure 1: Plots of prices for the 18 OECD countries.

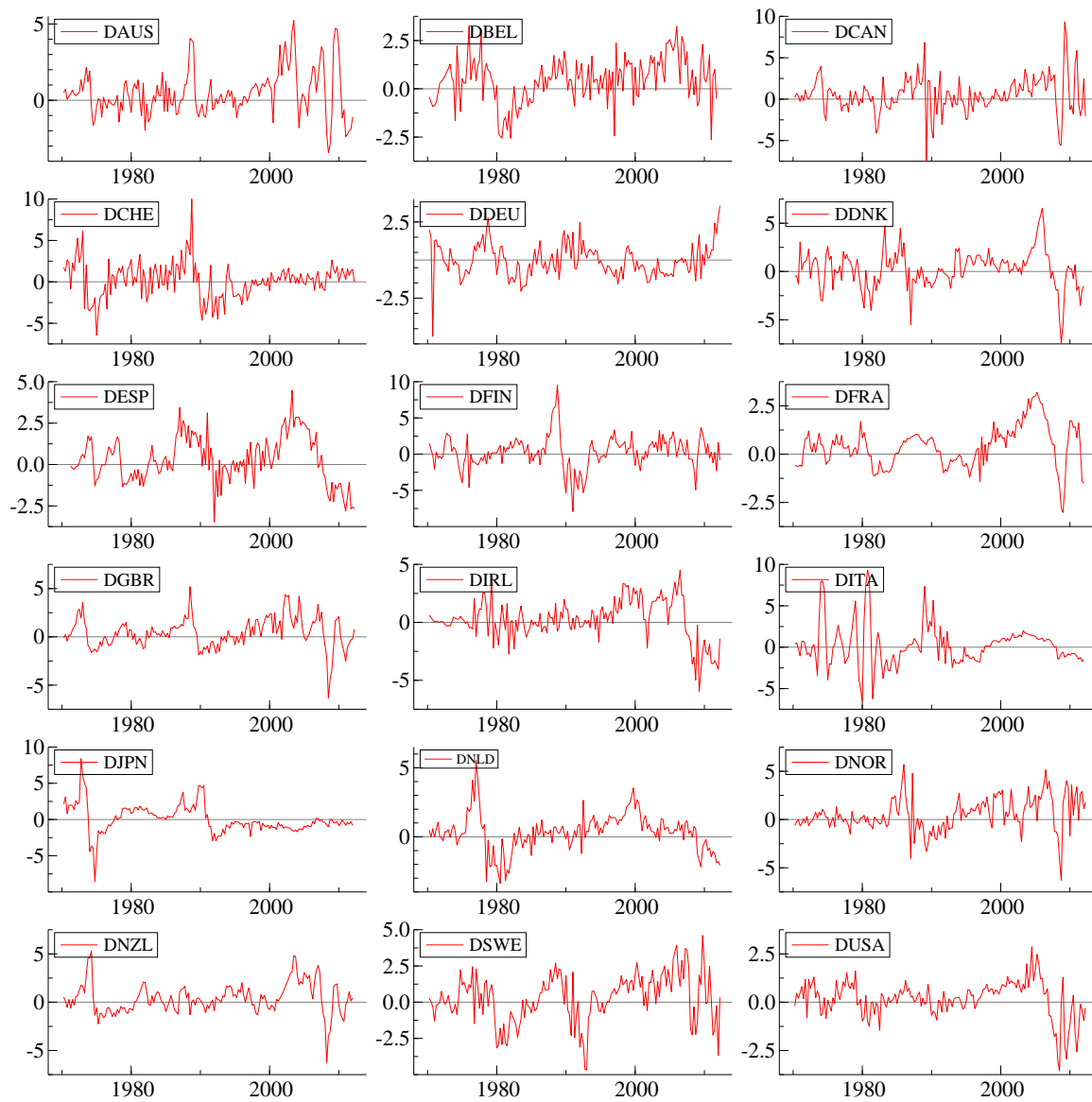Figure 2: Plots of rent for the 18 OECD countries.

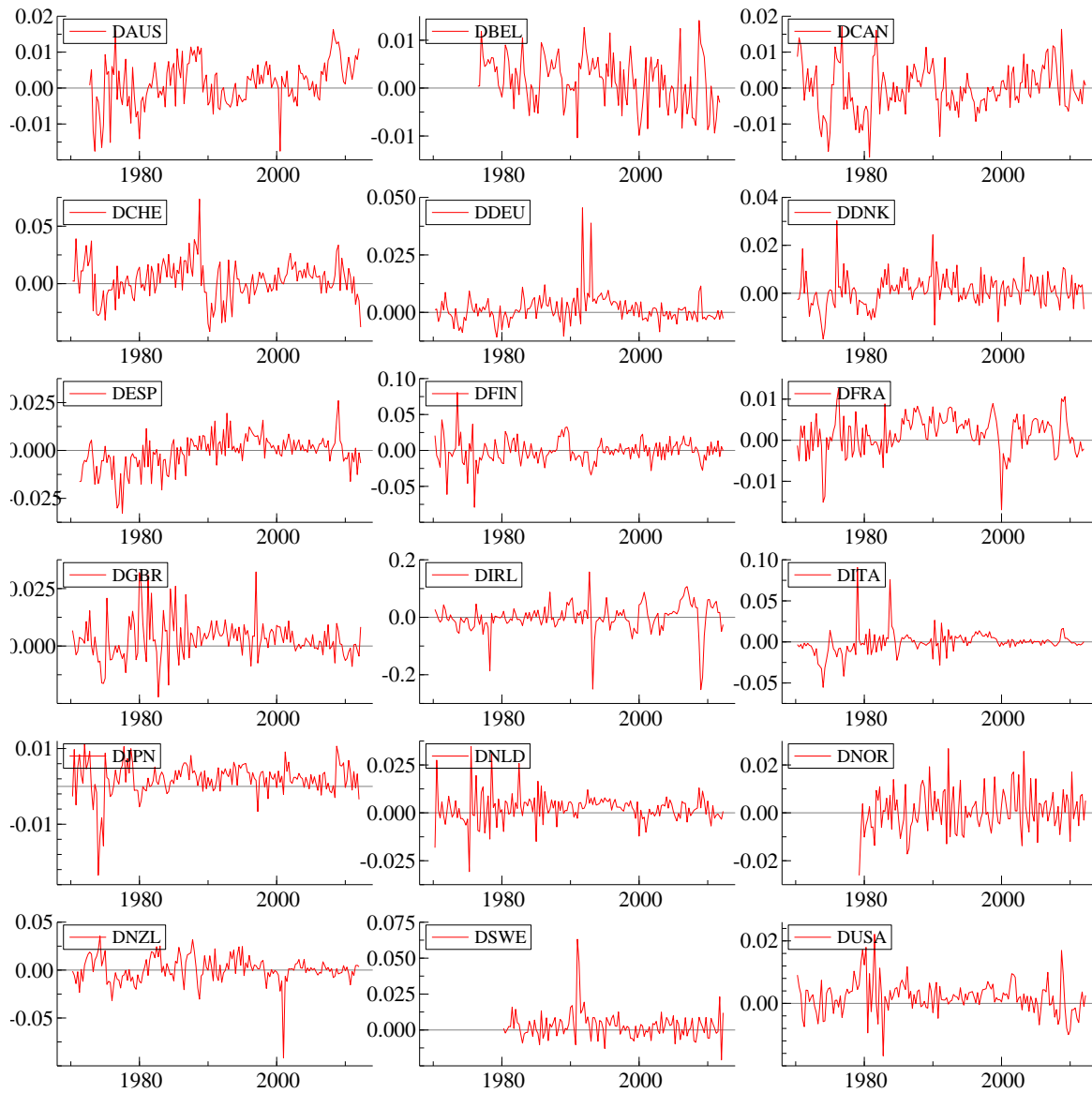Figure 3: Plots of first differences of prices.

Figure 4: Plots of first differences of rents.

A.2 Results

Table 1: Results from the Engsted and Nielsen (2012) analysis.

Sample	Sel.†	VAR: $\hat{\rho}_0$	Cointegration H_0: r=0	Cointegration H_0: r=1	β'	$\hat{\rho}_1$	$\hat{\rho}_{1x}$	$\hat{\rho}_{1xs}$	LR($M_{1x}\|M_1$) $\chi^2(1)$	LR($M_{1xs}\|M_{1x}$) $\chi^2(1)$	LR($M_{1xs}\|M_1$) $\chi^2(2)$
Australia											
Q1 1973-Q1 2012	All	1.009	[0.152]	[0.672]	(1,-179.13)	1.000	-	-	-	-	-
Q1 1973-Q1 2011	FW	1.013	[0.257]	[0.512]	(1,-2014.8)	1.000	-	-	-	-	-
Q1 1973-Q1 2004	Win	1.033	[0.789]	[0.801]	(1,160.25)	1.026	1.028	1.040	[0.085]	[0.471]	[0.175]
Belgium											
Q4 1976-Q4 2011	All	0.991	[0.003]	[0.009]	(1,2404.8)	1.000	-	-	-	-	-
Q4 1976-Q4 2011	FW	0.991	[0.003]	[0.009]	(1,2404.8)	1.000	-	-	-	-	-
Q4 1976-Q4 2007	Win	1.010	[0.001]	[0.005]	(1,363.11)	1.000	1.016	1.028	[0.157]	[0.028]	[0.033]
Canada											
Q1 1970-Q2 2012	All	1.001	[0.678]	[0.377]	(1,-818.94)	1.000	-	-	-	-	-
Q1 1970-Q2 2012	FW	1.001	[0.678]	[0.377]	(1,-818.94)	1.000	1.003	1.010	[0.339]	[0.524]	[0.517]
Q1 1970-Q1 2008	Win	1.018	[0.288]	[0.338]	(1,-34.77)	1.019	1.018	1.025	[0.483]	[0.653]	[0.706]
Switzerland											
Q1 1970-Q2 2012	All	0.989	[0.831]	[0.929]	(1,3.33)	1.000	-	-	-	-	-
Q1 1970-Q1 1990	FW	0.988	[0.814]	[0.831]	(1,-122.60)	1.000	-	-	-	-	-
Q1 1970-Q2 2012	Win	0.989	[0.831]	[0.929]	(1,3.33)	1.000	-	-	-	-	-
Germany											
Q1 1970-Q2 2012	All	0.995	[0.510]	[0.849]	(1,29.75)	1.000	-	-	-	-	-
Q1 1970-Q2 2012	Win	0.995	[0.510]	[0.849]	(1,29.75)	1.000	-	-	-	-	-
Denmark											
Q1 1970-Q1 2012	All	1.003	[0.752]	[0.597]	(1,-194.97)	1.000	-	-	-	-	-
Q1 1970-Q1 2008	FW	0.995	[0.873]	[0.924]	(1,-343.47)	1.000	-	-	-	-	-
Q1 1970-Q2 2006	Win	1.016	[0.495]	[0.441]	(1,285.62)	1.027	1.031	1.048	[0.463]	[0.178]	[0.308]

Continued on next page

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Table 1 – *Continued from previous page*

Sample	Sel.†	VAR: $\hat{\rho}_0$	Cointegration H_0: r=0	H_0: r=1	β'	$\hat{\rho}_1$	$\hat{\rho}_{1x}$	$\hat{\rho}_{1xs}$	LR($M_{1x}\|M_1$) $\chi^2(1)$	LR($M_{1xs}\|M_{1x}$) $\chi^2(1)$	LR($M_{1xs}\|M_1$) $\chi^2(2)$
Spain											
Q1 1971-Q2 2012	All	0.978	[0.005]	[0.871]	(1,-167.41)	1.000	-	-	-	-	-
Q1 1971-Q1 2009	FW	1.000	[0.000]	[0.742]	(1,-110.40)	1.041	1.030	1.026	[0.000]	[0.070]	[0.000]
Q1 1971-Q4 2006	Win	1.023	[0.000]	[0.196]	(1,-79.22)	1.000	1.053	1.047	[0.000]	[0.197]	[0.000]
Finland											
Q1 1990-Q2 2012	All	0.955	[0.210]	[0.213]	(1,468.16)	1.000	-	-	-	-	-
Q1 1990-Q1 2008	Win	0.952	[0.138]	[0.150]	(1,196.27)	1.000	-	-	-	-	-
France											
Q1 1970-Q1 2012	All	1.002	[0.396]	[0.313]	(1,-364.30)	1.000	-	-	-	-	-
Q1 1970-Q4 2007	FW	1.008	[0.646]	[0.882]	(1,647.33)	1.008	1.001	1.020	[0.011]	[0.091]	[0.010]
Q1 1970-Q1 2007	Win	1.017	[0.374]	[0.856]	(1,33.34)	1.020	1.001	1.020	[0.004]	[0.095]	[0.004]
Great Britain											
Q1 1970-Q2 2012	All	0.996	[0.030]	[0.633]	(1,-115.11)	1.000	-	-	-	-	-
Q1 1970-Q1 2008	FW	0.999	[0.062]	[0.0828]	(1,-137.55)	1.000	1.000	1.030	[0.000]	[0.023]	[0.000]
Q1 1970-Q2 2005	Win	0.999	[0.095]	[0.800]	(1,-123.92)	1.000	-	-	-	-	-
Ireland											
Q1 1970-Q2 2012	All	0.977	[0.080]	[0.572]	(1,-3185.8)	1.000	-	-	-	-	-
Q1 1970-Q1 2009	FW	1.003	[0.074]	[0.308]	(1,-269.76)	1.000	-	-	-	-	-
Q1 1970-Q2 2007	Win	1.024	[0.000]	[0.376]	(1,-17.03)	1.021	1.026	1.024	[0.053]	[0.245]	[0.078]
Italy											
Q1 1970-Q1 2012	All	0.981	[0.225]	[0.884]	(1,-99.90)	1.000	-	-	-	-	-
Q1 1974-Q1 2006	Win	0.980	[0.333]	[0.940]	(1,-59.95)	1.000	-	-	-	-	-
Japan											
Q1 1970-Q1 2012	All	1.001	[0.003]	[0.338]	(1,161.64)	1.000	-	-	-	-	-
Q1 1970-Q2 2006	Win	1.001	[0.014]	[0.385]	(1,123.25)	1.000	-	-	-	-	-

Table 1 – *Continued from previous page*

Sample	Sel.†	VAR: $\hat{\rho}_0$	Cointegration H_0: r=0	H_0: r=1	β'	$\hat{\rho}_1$	$\hat{\rho}_{1x}$	$\hat{\rho}_{1xs}$	$\mathbf{LR}(M_{1x}\|M_1)$ $\chi^2(1)$	$\mathbf{LR}(M_{1xs}\|M_{1x})$ $\chi^2(1)$	$\mathbf{LR}(M_{1xs}\|M_1)$ $\chi^2(2)$
Netherlands											
Q1 1970-Q2 2012	All	0.996	[0.000]	[0.734]	(1,-108.67)	1.000	-	-	-	-	-
Q1 1970-Q4 2008	FW	0.994	[0.000]	[0.606]	(1,-146.65)	1.000	-	-	-	-	-
Q1 1970-Q4 2001	Win	0.997	[0.001]	[0.670]	(1,-87.33)	1.000	-	-	-	-	-
Norway											
Q1 1979-Q2 2012	All	1.013	[0.198]	[0.430]	(1,-1912.1)	1.007	1.001	1.014	[0.087]	[0.087]	[0.053]
Q1 1979-Q2 2012	FW	1.013	[0.198]	[0.430]	(1,-1912.1)	1.007	1.001	1.014	[0.087]	[0.087]	[0.053]
Q1 1979-Q4 2007	Win	1.024	[0.166]	[0.479]	(1,1098.6)	1.020	1.001	1.026	[0.178]	[0.449]	[0.054]
New Zealand											
Q1 1970-Q1 2012	All	0.989	[0.969]	[0.829]	(1,635.23)	1.000	-	-	-	-	-
Q1 1970-Q4 2007	FW	1.011	[0.730]	[0.806]	(1,41.23)	1.000	1.021	1.024	[0.119]	[0.564]	[0.252]
Q1 1970-Q3 2007	Win	1.025	[0.574]	[0.751]	(1,4.10)	1.016	1.029	1.031	[0.249]	[0.829]	[0.503]
Sweden											
Q1 1980-Q2 2012	All	0.998	[0.256]	[0.483]	(1,-29.63)	1.000	-	-	-	-	-
Q1 1980-Q3 2011	FW	0.998	[0.370]	[0.493]	(1,15.51)	1.000	-	-	-	-	-
Q1 1980-Q4 2007	Win	1.021	[0.139]	[0.549]	(1,58.13)	1.018	1.018	1.028	[0.078]	[0.096]	[0.053]
United States											
Q1 1970-Q2 2012	All	0.999	[0.163]	[0.641]	(1,12.46)	1.000	-	-	-	-	-
Q1 1970-Q1 2008	FW	1.000	[0.024]	[0.874]	(1,-84.89)	1.000	-	-	-	-	-
Q1 1970-Q4 2006	Win	1.024	[0.000]	[0.636]	(1,-36.08)	1.016	1.027	1.031	[0.000]	[0.712]	[0.000]

† The method used for selection of the end date of the sample. All: all data available. FW: A Plillips et al. based test for bubbles with an expanding window and fixed start date. Win: A Pillips et al. based test for bubbles with a rolling window with fixed length