A New Measure of Business Cycle Concordance

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Abstract

We develop a new measure of business cycle co-movement that satisfies a number of desirable properties before checking its performance alongside that of already existing measures via a simulation study. Moreover, we investigate the ability of our measure to detect changes in co-movement making use of recently developed methods for discovering structural breaks. We find that our proposed measure of business cycle co-movement is very effective at detecting changes in co-movements and appears to have good size properties. When applied to recent business cycle series from major developed and developing nations, we find that breaks indicated by the impulse indicator saturation approach significantly reflect the structural change in the concordance of business cycles.

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1 Introduction

In this paper we develop a new measure of business cycle co-movement that satisfies a number of desirable properties before assessing its performance alongside that of already existing measures via a simulation study. We also investigate the ability of our measure to detect changes in co-movement making use of recently developed methods for discovering structural breaks, before conducting an analysis of business cycle co-movement.

We find that our proposed measure of business cycle co-movement is very effective at detecting changes in co-movements and at the same time having favourable size properties. When applied to recent business cycle series from developed and developing nations, we find that breaks indicated by the impulse indicator saturation approach significantly reflect the structural change in the concordance of business cycles.

In Section 2 we propose a new measure of business cycle co-movement building on the work of De Haan et al. (2007). We perform a graphical and simulation exercise to demonstrate the properties of this measure and empirically investigating business cycle co-movement based on our proposed measure. In Section 2.1 we firstly discuss the ideal properties of a business cycle co-movement measure before introducing both existing measures of co-movement (Section 2.2) and then our proposed new measure (Section 2.3). We introduce our methodology for detecting structural breaks (Section 3) subsequently and describe (Section 4) and analyse (Section 4.1) a simulation study to establish the properties of our measure, and in particular its performance in detecting changes in co-movements. We then apply our measure in an analysis of business cycle co-movement for a panel of G7 and BRICS economies (Section 5). Section 6 concludes.

2 Measures of Business Cycle Co-Movement

There are multiple steps in moving from the measured statistics on output in an economy to measures of co-movement between the business cycles of countries. Few if any of the steps are without controversy. The extensive literature on locating and defining turning points of business cycles as well as on properties of cycle phases such as duration and amplitude is comprehensively discussed by Harding and Pagan (2006*a*) and advanced on testing synchronisation in Harding and Pagan (2006*b*). Moreover we refer to Pagan and Harding (2005) for a framework for locating the research approaches and traditions in the literature upon what data series the cycle measurement is based on and what characteristics eventually determine a cycle in the respective concept.

We disregard considerations on the accuracy of national statistics and price indices, and also sidestep the (justified) concerns (e.g. Cogley and Nason, 1995) regarding how the resulting measure of real GDP (denoted y_{it}) is detrended to remove a permanent component and retain a transitory one so as to arrive at data for the business cycle (g_{it}) by using filtering techniques such as the commonly applied Hodrick and Prescott (1981) (HP) filter. We do so on the grounds that our methods developed here can be applied invariant to the detrending procedure used to arrive at a business cycle component.

2.1 Ideal Properties of a Co-Movement Measure

An ideal co-movement measure, we propose, has the following properties:

- It is bounded. Many previously proposed measures are unbounded and often take extreme values meaning that standardising them to lie on the unit interval does not yield any more information.

- In the limit (as noise tends to zero) it tends to:

  -1 if cycles are perfectly the opposite of each other, hence $g_i(t) = -g_r(t)$, $\forall t$.

  0 if cycles are completely unrelated to each other, hence $g_i(t) \neq f(g_r(t))$, $\forall t$, or $\mathsf{Cov}\,(g_i, g_r) = 0$.

  1 if cycles move together perfectly, hence $g_i(t) = g_r(t)$, $\forall t$.

The extreme bounds of such a measure are obviously theoretical concepts rather than likely actual outcomes, yet it is useful to know this when appraising the output of a measure, not least to allow comparability between countries.

To test the latter property, we design a Monte Carlo simulation to include these three cases, and we consider the average of it over increasing dataset sizes. This is of importance since we do not have infinitely large datasets at our disposal in applied economics, and it is thus important to know how quickly these properties are attained for various measures (if at all).

2.2 Existing Measures

An elementary measure of co-movement is the sample correlation coefficient, used by Baxter and Kouparitsas (2005). This measure is usually applied to entire samples of data, yet in our case we seek to understand changes in co-movements over time, making the correlation coefficient somewhat less reliable to use.

De Haan et al. (2007) decompose business cycle co-movements into two parts: *synchronicity* and *amplitude*. They propose measuring each separately as follows:

$$\text{Synchronicity (group)}: \qquad \phi(t) = \frac{1}{N}\sum_{i=1}^{N}\frac{g_{it}g_{rt}}{|g_{it}g_{rt}|}, \tag{1}$$

$$\text{Synchronicity (indiv)}: \qquad \phi_i(t) = \frac{g_{it}g_{rt}}{|g_{it}g_{rt}|}, \tag{2}$$

$$\text{Amplitude (group)}: \qquad \gamma(t) = -\frac{\sum_{i=1}^{N}|g_{it} - g_{rt}|}{\sum_{i=1}^{N}|g_{it}|}, \tag{3}$$

$$\text{Amplitude (indiv)}: \qquad \gamma_i(t) = -\frac{|g_{it} - g_{rt}|}{|g_{it}|}. \tag{4}$$

The latter measure for the dimension of amplitude takes zero if the two business cycles are identical to each other (hence $g_i(t) = g_r(t) \forall t$), and becomes more negative the less alike they are. When the object cycle is not the largest in absolute terms, then $\gamma_i(t)$ can be smaller than negative unity.[1] Thus, γ is unbounded, and while De Haan et al. do propose standardising to achieve boundedness, large negative values affect the mean and standard deviation and hence distort the effectiveness of this as a measure of amplitude.

[1] A method to circumvent this difficulty is to alter the denominator to, for example, $\max(|g_i(t)|, |g_r(t)|)$.

2.3 Our Proposed Measure

Our proposed measure is motivated by the desirable properties outlined in Section 2.1, which lead us to the concept of realised covariance (or correlation, since to satisfy our desired properties we scale). As we seek to only measure covariance over short periods of time, the scaling factor of T in standard covariance would distort our inference. We thus propose:

$$\psi_i(t) = 1 - \frac{(g_{it} - g_{rt})^2}{g_{it}^2 + g_{rt}^2}. \tag{5}$$

This measure achieves zero when $\mathsf{Cov}\,(g_i, g_r) = 0$ since $\mathsf{E}\,[g_{it}g_{rt}] = 0$ in that situation, unity when $g_{it} = g_{rt}$, and negative unity when $g_{it} = -g_{rt}$. If noise is assumed (e.g. $g_{it} = g_{rt} + e_t$ where $e_t \sim (0, \sigma_e^2)$), then as the variance of the noise tends to zero, the measure tends to plus or minus unity.

Hence this measure combines information on the direction of co-movement (if two cycles move in opposite directions, which is equivalent to $\Delta g_{it} = -\Delta g_{rt}$, then it is negative), and also the extent of co-movement, since the nearer two cycles are to satisfying $g_{it} = g_{rt}$, the nearer is the measure to unity. Furthermore, the greater the noise (so the greater is σ_e^2), the measure still preserves the sign of co-movement.

An added value of this measure is that combining time periods is elemental given its nature as a covariance measure, and doing so will likely add information. Hence we argue that our measure captures everything that the De Haan et al. (2007) measures capture, but in one single number, which is suitable for detecting structural breaks — of great interest in the post-financial crisis world.

3 Changes in Co-Movements — Structural Change

An important contribution of this paper is to bring to bear recent developments in structural break detection on the problem of detecting changes in co-movements. Identifying changes in relationships between variables through time, referred to as structural breaks, is demanding because by definition such changes are misspecifications of standard linear regression models. Once we have departed from the assumptions underlying such models, we can no longer be sure about statistics calculated on the basis of such models, which creates difficulties in attempting to understand such changes.

Recursive methods can be employed, but as Hendry and Krolzig (2003) point out, there are limits to the informativeness of sub-sample model selection, and with such methods it can be difficult to detect precisely *when* a break happened even if one can be sure of the existence of one. Additionally, techniques that allow time-varying coefficients can be employed. There is a rich literature on detecting structural breaks (see Castle et al. (2010) for example). The general conclusions drawn in that literature point towards the difficulty of detecting structural breaks using standard regression techniques when structural parameters shift (although when seemingly innocuous parameters like the constant change, this can be much more easily detected). Additionally, it often takes a number of time periods before any such structural change can be detected. At least two observations are required since one observation could simply be an outlier, whereas two or more observations would be indicative of a more fundamental change. It is worth noting at this point that using quarterly data over annual data may thus be useful in detecting structural breaks, since it affords more observations over the same time period in which a change may have taken place (Hubrich and Hendry, 2006); Quarterly data would in principle provide additional observations for subsequent econometric analysis but at the same time would potentially induce spurious variability in the cyclical component by giving greater weight to temporary and/or seasonal shocks.

An alternative methodology of detecting structural breaks is that of *dummy saturation*, or *impulse indicator saturation*, as proposed by Hendry et al. (2008). Dummy saturation adds an impulse dummy variable for each observation in a sample; hence for our sample of T observations, we would add T impulse dummy variables to cover each observation. To avoid perfect multicollinearity the dummies would be added in blocks; for example, two blocks of $T/2$ dummies each. A regression is run on each block, and the significant dummies retained. A union model is then formed of all retained dummies, and only the significant dummies from this union model are retained overall. Significant dummy variables over a particular time period are consequently suggestive of a structural change. The regression methodology allows the investigator to augment visual inspection of data with sophisticated regression analysis that takes into account the usual variation expected in data series. Hendry et al. note a number of prominent uses of dummy saturation, and one particularly pertinent is to identifying structural breaks that other methods, such as detecting large residuals, may fail to spot. We therefore employ impulse indicator saturation in the context of changes in co-movements.

We firstly employ it in our simulation study in order to establish its effectiveness in detecting structural breaks before applying it to our actual dataset. We use the resulting dummies as an explanatory variable as we attempt to understand more about changes in business cycle co-movements in Section 5.

4 Simulation Design

Our first aim is to document summary statistics on the behaviour of the various potential measures for business cycle co-movement. We thus vary the relationship between business cycles in a number of ways and calculate the average value and standard deviation of the measure over a number of replications. The simulation study will indicate whether each measure yields the type of output we desire for co-movements, and how accurately it does so.

We consider the following scenarios, and list what we expect to find from a co-movement measure. Recall that we report the simple correlation coefficient ρ, the De Haan et al. measures ϕ (whether cycles are same/opposite side of potential GDP) and γ (how far apart are cycles standardised on subject cycle), and our proposed ψ measure (a covariance-type measure).

1. Business cycles are identical; $g_{it} = g_{rt}$. We would expect to find that $\rho = 1$, $\psi = 1$, $\phi = 1$ and $\gamma = 0$ in this situation. We would also expect standard deviations of these measures to be zero.

2. Business cycles differ only by noise. We generate $u_t \sim \mathsf{N}\left(0, \sigma_u^2\right)$ and hence set $g_{it} = g_{rt} + u_t$. Hence we anticipate that in expectation $\rho = 1 - \sigma_u^2$, $\phi = 1$ and $\gamma = 0$ and $\psi_t \to 1$ as $\sigma_u^2 \to 0$, since $\mathsf{E}(u_t) = 0$ and $\mathsf{Var}(u_t) = \sigma_u^2$. We expect that standard deviations will be somewhat higher, and particularly for ϕ since it varies between -1 and 1.

3. Business cycles are identical but not perfectly synchronised:

   (a) Business cycle of country i is lagged one period behind that of country r: $g_{it} = g_{r,t-1}$.

   (b) Business cycle of country i is lagged two periods behind that of country r: $g_{it} = g_{r,t-2}$.

   We expect in this case that the departures from unity of ρ and ϕ will be more substantial since synchronisation is broken. As cycles last longer than one or two periods, ϕ will not achieve -1 very often other than at the point where cycles cross potential output. We anticipate ψ to get nearer to zero since $\mathsf{Cov}\left(g_{it}, g_{rt}\right)$ must fall. We expect γ will be more pronounced in its departure from zero now $\mathsf{E}(g_{it}) \neq \mathsf{E}(g_{rt})$.

4. Business cycles differ only by noise and are not synchronised. Here we consider again lagging the cycle in country i by:

   (a) one period and

   (b) two periods.

   The added noise ought to increase the departure of ρ and ψ from unity because each case adds σ_u^2 to the departure from synchronisation. In expectation, there ought to be no impact on ϕ or γ.

5. Business cycles are exactly the opposite of each other; $g_{it} = -g_{rt}$. Here we expect that $\rho = \psi = -1$ and furthermore that $\phi = -1$ and $\gamma = -2$ by substituting $g_{it} = -g_{rt}$ into their respective formulas.

6. Business cycles are the opposite of each other with noise added: $g_{it} = -g_{rt} + u_t$ with u_t defined as in case 2. Here, σ_u^2 moves ρ and ψ from -1 and also creates enough noise to move ϕ away from -1. On average, since $\mathsf{E}(u_t) = 0$, γ ought to be unaffected.

7. Business cycles are opposite of each other and not synchronised: We add this setting to see whether indeed the measures are further increased from all previous cases, and we repeat the sub-cases listed in case 3 and referred to in case 4.

   (a) Business cycle of country i is lagged one period behind that of country r: $g_{it} = -g_{r,t-1}$.

   (b) Business cycle of country i is lagged two periods behind that of country r: $g_{it} = -g_{r,t-2}$.

   Because g_{it} and g_{rt} are persistent time series, then $g_{rt} \approx g_{r,t-1} = g_{it}$, it is highly likely that if ρ is calculated over sufficiently long time periods, ρ will remain high even though cycles are not synchronised. Similarly, we suspect that while ϕ will move from -1, it will not move particularly far since the only time when cycles will be either side of potential GDP will be when they are crossing it, which is not particularly frequent for cycles. Since $g_{it} = -g_{r,t-1}$ an extra difference is added to the calculation of γ hence we anticipate it will increase compared to case 5. We anticipate ψ will be nearer to zero.

8. Business cycles are opposite of each other with noise added and not synchronised: We repeat the sub-cases listed in case 3 and referred to in case 4.

   (a) Business cycle of country i is lagged one period behind that of country r: $g_{it} = g_{r,t-1}$.

   (b) Business cycle of country i is lagged two periods behind that of country r: $g_{it} = g_{r,t-2}$.

   In this case we anticipate little impact on the mean-type measures (ϕ and γ) since $\mathsf{E}(u_t) = 0$, whereas the variance-related measures (ρ, ψ) ought to be closer to zero than in cases 7(a) and 7(b) by the size of σ_u^2.

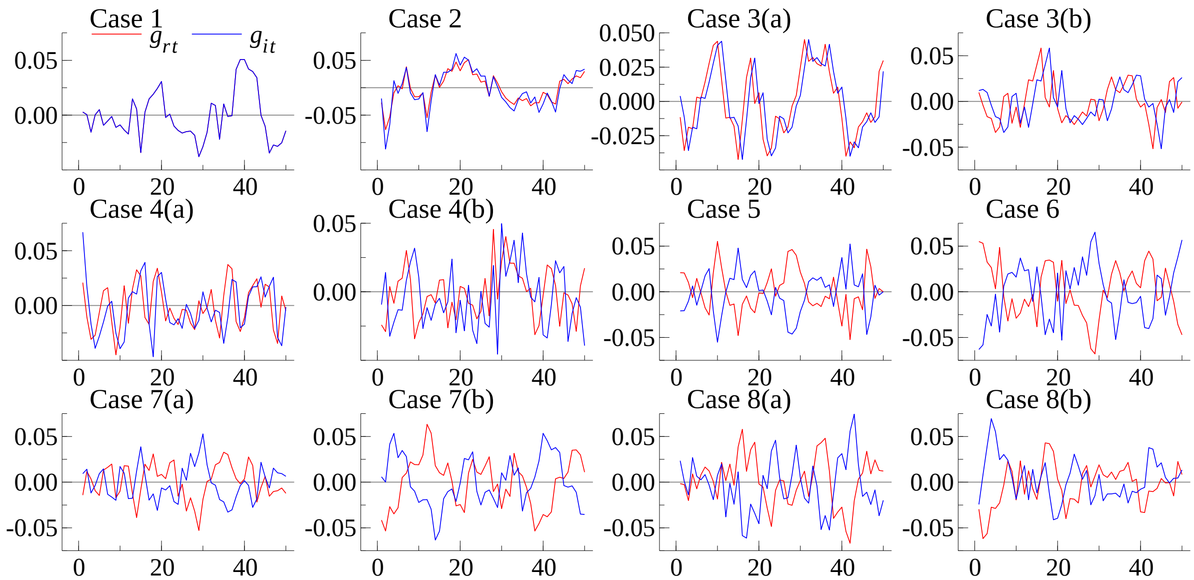

Figure 1: Samples of generated business cycles using trend stationary DGP.

Figure 1 presents one realisation of all of the 12 cases listed above. Our simulation is designed in this manner to ascertain whether any measures achieve the desired properties listed earlier. Notably case 1 should yield positive unity, case 5 negative unity, as the two polar extreme cases, while cases 4(b) and 8(b) should achieve values closest to zero as the cases with the largest departures from co-movement.

Given the central question of this paper, notably the detection or otherwise of changing in co-movements, we thus will analyze whether the measures are able to detect changes in coupling, and how quickly. Naturally, all kinds of variations on changes in coupling could be considered, and hence we must be restrictive. We consider thus only episodes of coupling, decoupling and recoupling, and within that only one specification for each. For coupling, we refer to case 2 above; when we allude to non-coupled economies, we refer to our polar opposite case considered above in case 4, where the cycles are the opposite of each other and lagged twice with noise added.

Our simulation studies cover 50 observations, i.e. 50 years. In order to best mimic potential actual events, we consider for both single changes (to coupled, to decoupled) that they happen 5 and 10 years from the end of the sample. For recoupling we consider that after 30 observations decoupling happens, and then for the final 5 observations recoupling occurs. Thus we investigate three scenarios:

(I) Decoupling, dc: Two cycles move from case 2 (separated only by noise) to case 8(b) (opposite, lagged twice with noise added) 5 and 10 observations from end of sample (where $T = 50$). This scenario is henceforth labeled "dc5" and "dc10" respectively.

(II) Coupling, c: Two cycles move from case 8(b) (opposite, lagged twice with noise added) to case 2 (separated only by noise) 5 and 10 observations from end of sample (where $T = 50$). This scenario is henceforth labeled "c5" and "c10" respectively.

(III) Recoupling, rc: Two cycles are initially coupled for the first 30 observations (case 2) before being decoupled between observations 31 and 45 (case 8(b)), and in the final five observations becoming recoupled again (case 2).

In Figure 2 we plot one of our generated time series for coupling in the final five time periods (scenario (II)). For the first 45 observations we note that the two cycles (solid blue line and solid red line) show no obvious co-movement, but for the final five observations both move together closely. The dotted blue line is the co-moving series from case 2, which for the final five observations becomes scenario (II), while the dashed blue line is the series from case 8(b), where the coupling series (scenario (II)) is for the last five observations.

In each of the cases 1–8(b) we discussed the anticipated impact on our proposed measures, and hence we expect that in scenarios (I)–(III) measures move from behaving akin to case 2 to 8(b) and vice versa, depending on the respective scenario. Hence we should expect, for decoupling, ρ to turn negative and decrease in absolute value, ϕ to similarly change sign and likely fall in absolute value, γ to increase in size, and ψ to also increase, and vice versa for the coupling case.

It might be contented that we propose here rather unrealistic structural breaks on economies — surely they do not shift as dramatically and suddenly as this in reality? However, it is important to know whether candidate measures of co-movement are able to pick up dramatic changes in the ideal case where we know exactly where the change happened. If we are unable to detect changes in this context, then there is little hope that we will be able to detect them in real-world economic data either.

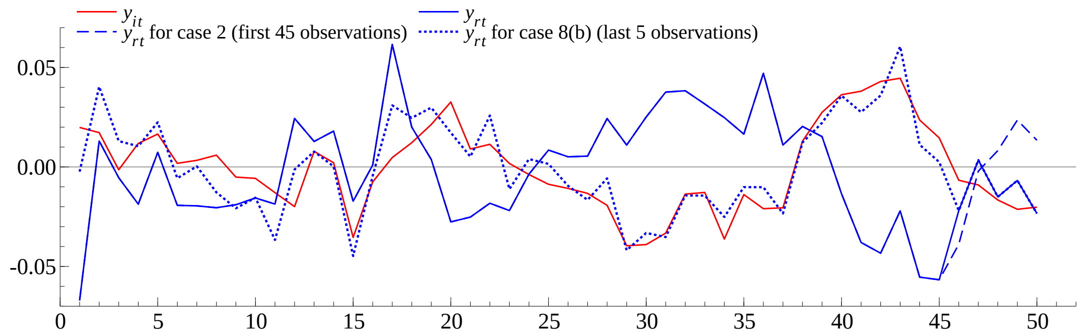

Figure 2: Example of structural break in business cycle movements as generated in simulation.

4.1 Simulation Results

The simulation results are provided in Table 1 (trend stationarity). The findings are displayed for the correlation coefficient ρ and the measures ϕ, γ and ψ calculated each period as well as cumulated over a number of time periods (3, 5 and 10 periods — a panel of the table for each cumulation). The rows of the table refer to the specific cases 1–8(b) listed above. In each cell, the average over the replications (10,000) is reported in large typeface, with the standard deviation reported beneath in small typeface. The bottom part of the table (dc5, ..., rc) refers to the structural change scenarios and will be discussed later in this Section.

The first measure we consider is the sample correlation coefficient, ρ, reported in each panel. As would be expected, for cases 1–4 ρ is positive and is negative for cases 5–8(b). In the first panel ρ is calculated over the entire sample, whereas for the 3, 5 and 10 periods panels, it calculated only over these shorter periods. The difference between the panels for cases 1, 2, 5 and 6 is not particularly large, but for the other cases there is a distinct difference; if ρ is calculated over just 3 periods it is essentially zero in all other cases; this suggests that ρ over short time periods may be very effective at determining a lack of synchronisation, as is the specification in these cases. As mentioned earlier, this is because over longer time horizons the persistence in data mean that $g_{rt} \approx g_{r,t-1} = g_{it}$, but calculating only over 3 or 5 observations makes this approximation much more unstable. The distinction remains but is less stark when ρ is calculated over 5 and to a lesser extent 10 time periods. Over these longer horizons, the lag distinction is clear — the 1-lag specifications (cases 3(a) and 4(a)) have a much higher correlation coefficient than the 2-lag specifications (cases 3(b) and 4(b)) — suggesting again that ρ is able to yield insight on to the relative level of synchronisation.

Considering next the De Haan et al. measures, we note that indeed ϕ_{it} achieves its upper bound and lower bounds in cases 1 and 5, and outside of these cases, where noise and lagged values reduce the probability that the cycles are of opposite signs, steadily moves towards zero. The rate at which ϕ_{it} declines is faster than that of the correlation coefficient ρ for non-cumulated measures but slower for cumulated, reflecting that ϕ_{it} is concerned with only synchronisation not co-movement. The cumulation variant used does not alter ϕ_{it} noticeably.

De Haan et al.'s γ measure of amplitude varies considerably over the twelve cases and, as expected, registers its highest values for cases with greater noise. Notably though, γ appears to pick up decreases in synchronisation as well as ϕ; for example when moving from case 1 to 3(a) and 3(b) only the synchronisation of the cycles is affected yet γ increases. The most notable feature though of γ is the standard deviation. They are remarkably large, reflecting the aforementioned lack of a lower bound for the measure. Even if we rescale the measure to be on the unit interval as De Haan et al. suggest, such large negative values will distort the remaining values that have been rescaled, and hence yields undesirable characteristic of this measure.

Our proposed measure, ψ, performs as expected. For cases 1 (perfect co-movement) and 5 (cycles are opposites) it is 1 and -1 respectively, while for other cases it moves towards zero. The movement away from ± 1 is more marked than for ρ, but the patterns are similar, as would be expected. For the 3, 5 and 10 period measures with ψ, we sum the measure rather than taking the average, as can be inferred from cases 1 and 5. Consequently the values of ψ for the respective columns are approximately proportional.

The simulation results where structural change is specified are presented in the bottom part of the table. The "dc" lines represent the decoupling scenarios detailed in Section 4, where the structural change is induced for economies that were moving together and cease to do so either five ("dc5") or ten ("dc10") observations from the end of the sample. The "c" represent coupling and "rc" signifies scenarios of recoupling.

In all structural change scenarios we report the co-movement measures for the phases before and after the specified structural changes (i.e. for "dc5", 45 periods when the cycles were moving together ("$\mathrm{dc5}_{[\mathrm{pre}]}$") and the 5 periods where they do not move together ("$\mathrm{dc5}_{[\mathrm{post}]}$")). This is clearly a rather optimistic assessment, but here our question is whether in the best case scenario we can detect changes in co-movement using particular measures.

We find that for all measures bar the correlation coefficient, the structural change is picked up. For decoupling all measures apart from the correlation coefficient, ρ, fall in size as would be expected since the co-movement moves from case 2 to case 8(b), while in the coupling scenario the opposite happens. For the recoupling scenario, the middle period ("$\mathrm{rc}_{[\mathrm{mid}]}$"middle row) is where the cycles are decoupled and hence here the measures are lower before increasing for the final line, the recoupled period ("$\mathrm{rc}_{[\mathrm{post}]}$"). This is expected since the correlation coefficient is calculated over the entire sample. It is clear that ϕ and ψ offer a greater probability of detecting structural change since the standard errors for γ are of such size that it is hard to imagine any change being statistically significant, and ϕ only measures synchronisation and not amplitude.

	1 Period				3 Periods				5 Periods				10 Periods			
Case	ρ	ϕ	γ	ψ	ρ	ϕ	γ	ψ	ρ	ϕ	γ	ψ	ρ	ϕ	γ	ψ
1	1.00 0.00	1.00 0.00	0.00 0.00	1.00 0.00	1.00 0.00	1.00 0.00	0.00 0.00	3.00 0.00	1.00 0.00	1.00 0.00	0.00 0.00	5.00 0.00	1.00 0.00	1.00 0.00	0.00 0.00	10.00 0.00
2	0.91 0.00	0.74 0.14	−2.43 11837.50	0.65 0.08	0.75 0.09	0.74 0.06	−2.40 3846.83	1.97 0.32	0.83 0.04	0.74 0.04	−2.41 2285.91	3.28 0.62	0.88 0.01	0.74 0.02	−2.42 924.29	6.58 1.56
3(a)	0.57 0.00	0.41 0.11	−5.57 405256.11	0.33 0.07	−0.00 0.05	0.41 0.08	−5.60 137221.72	1.01 0.38	0.19 0.04	0.42 0.06	−5.60 79368.74	1.69 0.83	0.41 0.03	0.42 0.04	−5.56 33900.05	3.40 2.26
3(b)	0.28 0.00	0.20 0.06	−6.59 781941.82	0.16 0.06	−0.11 0.07	0.21 0.07	−6.66 264852.39	0.48 0.39	−0.07 0.05	0.21 0.06	−6.74 157540.95	0.82 0.89	0.10 0.04	0.21 0.04	−6.78 74194.44	1.66 2.37
4(a)	0.52 0.00	0.37 0.11	−7.86 10538716.38	0.30 0.07	0.01 0.05	0.38 0.08	−8.01 3569927.81	0.91 0.38	0.17 0.05	0.38 0.06	−8.08 2124144.87	1.52 0.82	0.37 0.04	0.38 0.04	−7.45 776640.46	3.05 2.28
4(b)	0.26 0.00	0.18 0.06	−21.68 257483204.14	0.14 0.06	−0.10 0.06	0.19 0.07	−21.71 87173323.70	0.44 0.38	−0.05 0.05	0.19 0.06	−22.08 51835548.50	0.75 0.85	0.10 0.04	0.19 0.04	−23.97 24580290.44	1.51 2.28
5	−1.00 0.00	−1.00 0.00	−2.00 0.00	−1.00 0.00	−1.00 0.00	−1.00 0.00	−2.00 0.00	−3.00 0.00	−1.00 0.00	−1.00 0.00	−2.00 0.00	−5.00 0.00	−1.00 0.00	−1.00 0.00	−2.00 0.00	−10.00 0.00
6	−0.91 0.00	−0.73 0.14	−4.11 105092.07	−0.65 0.08	−0.75 0.09	−0.74 0.06	−3.96 25599.05	−1.96 0.33	−0.83 0.04	−0.74 0.04	−3.94 14856.92	−3.27 0.64	−0.88 0.01	−0.74 0.02	−3.77 4199.82	−6.56 1.56
7(a)	−0.57 0.00	−0.41 0.11	−6.23 405309.61	−0.33 0.07	0.00 0.05	−0.41 0.08	−6.26 137255.64	−1.01 0.38	−0.19 0.04	−0.42 0.06	−6.27 79404.27	−1.69 0.83	−0.41 0.03	−0.42 0.04	−6.23 33925.81	−3.40 2.26
7(b)	−0.28 0.00	−0.20 0.06	−6.90 782035.64	−0.16 0.06	0.11 0.07	−0.21 0.07	−6.98 264884.16	−0.48 0.39	0.07 0.05	−0.21 0.06	−7.07 157570.72	−0.82 0.89	−0.10 0.04	−0.21 0.04	−7.12 74218.16	−1.66 2.37
8(a)	−0.52 0.00	−0.37 0.11	−8.00 10363251.69	−0.30 0.06	−0.01 0.05	−0.37 0.07	−6.42 1223390.98	−0.90 0.37	−0.18 0.05	−0.37 0.06	−6.06 459565.66	−1.51 0.81	−0.37 0.04	−0.38 0.04	−5.73 128965.45	−3.06 2.21
8(b)	−0.25 0.00	−0.18 0.06	−7.31 115023.92	−0.14 0.06	0.10 0.06	−0.19 0.07	−7.25 36029.49	−0.44 0.38	0.05 0.05	−0.19 0.06	−7.18 20501.02	−0.74 0.86	−0.10 0.04	−0.19 0.04	−7.07 9019.45	−1.51 2.32
Scenario																
(I)																
$dc5_{[pre]}$	0.84 0.00	0.75 0.15	−2.39 12467.25	0.66 0.09	0.75 0.09	0.75 0.06	−2.34 3231.16	1.98 0.34	0.83 0.04	0.75 0.04	−2.35 1925.82	3.30 0.65	0.88 0.01	0.75 0.02	−2.38 863.52	6.60 1.60
$dc5_{[post]}$	0.84 0.00	−0.07 0.25	−7.99 453091.13	−0.05 0.18	0.36 0.23	0.25 0.18	−6.75 76188.32	0.67 0.98	0.55 0.11	0.42 0.08	−5.43 27203.68	1.82 1.29	0.72 0.03	0.58 0.02	−3.98 6818.48	5.12 1.32
$dc10_{[pre]}$	0.73 0.00	0.75 0.16	−2.32 10622.77	0.66 0.09	0.75 0.10	0.75 0.07	−2.31 3585.02	1.98 0.36	0.83 0.04	0.75 0.04	−2.32 1935.95	3.30 0.68	0.88 0.01	0.75 0.02	−2.34 744.65	6.61 1.61
$dc10_{[post]}$	0.74 0.00	−0.12 0.14	−7.00 268209.47	−0.10 0.12	0.21 0.15	0.04 0.14	−6.20 55091.39	0.13 0.83	0.27 0.12	0.12 0.12	−5.53 20496.40	0.58 2.02	0.46 0.06	0.33 0.06	−4.27 5141.76	3.04 3.87
(II)																
$c5_{[pre]}$	−0.18 0.00	−0.19 0.07	−7.23 111921.30	−0.15 0.06	0.10 0.07	−0.20 0.08	−7.10 36855.09	−0.46 0.40	0.05 0.05	−0.20 0.06	−7.12 21745.12	−0.77 0.90	−0.10 0.04	−0.20 0.04	−7.10 9635.64	−1.55 2.38
$c5_{[post]}$	−0.11 0.00	0.72 0.39	−2.79 29565.97	0.62 0.22	0.38 0.23	0.37 0.18	−4.64 25048.06	0.96 1.08	0.23 0.11	0.18 0.09	−5.13 5915.95	0.84 1.50	0.06 0.04	−0.00 0.02	−5.54 1680.18	0.11 1.51
$c10_{[pre]}$	−0.07 0.00	−0.20 0.08	−7.39 123531.96	−0.15 0.06	0.10 0.07	−0.20 0.08	−7.36 41250.08	−0.47 0.42	0.05 0.05	−0.20 0.07	−7.28 24146.19	−0.78 0.94	−0.10 0.04	−0.20 0.04	−7.20 10138.26	−1.56 2.41
$c10_{[post]}$	−0.05 0.00	0.73 0.30	−2.85 37542.91	0.64 0.17	0.57 0.19	0.55 0.15	−3.39 10502.89	1.45 0.84	0.54 0.15	0.46 0.14	−4.00 6230.23	2.03 2.21	0.31 0.07	0.23 0.06	−5.03 3770.50	2.11 4.30
(III)																
$rc_{[pre]}$	0.59 0.00	0.75 0.18	−2.27 8995.43	0.66 0.11	0.75 0.12	0.75 0.07	−2.25 3008.57	1.99 0.39	0.83 0.04	0.75 0.05	−2.23 1463.52	3.32 0.73	0.88 0.01	0.75 0.02	−2.28 522.79	6.64 1.56
$rc_{[mid]}$	0.57 0.00	−0.19 0.12	−7.09 224609.85	−0.14 0.10	0.18 0.12	−0.07 0.13	−6.39 62409.48	−0.12 0.72	0.20 0.10	−0.00 0.12	−6.26 31153.76	0.08 1.90	0.28 0.09	0.15 0.09	−5.66 7846.42	1.51 6.27
$rc_{[post]}$	0.53 0.00	0.72 0.39	−2.79 29565.97	0.62 0.22	0.38 0.23	0.37 0.18	−4.64 25048.06	0.96 1.08	0.23 0.11	0.18 0.09	−5.13 5915.95	0.84 1.50	0.06 0.04	−0.00 0.02	−5.54 1680.18	0.11 1.51

Table 1: Simulation results for trend stationary data.

As noted above, however, this is a rather optimistic picture of detecting structural breaks in economic data series. We implicitly assume knowledge of the breakpoint that one is not privy to in reality and must instead attempt to discover it. Employing dummy saturation, as was discussed in Section 3, we check whether an observation is an outlier or may be part of a structural break.

It is important to assess via simulation that in our stylised cases and scenarios of co-movement, dummy saturation on some or all of our measures will enable us to detect structural breaks. Thus we run dummy saturation on each generated series of our simulation and consider how often the dummy for each observation is retained. If that observation does not belong to a period in which we specify a structural break, we expect that most of the time that dummy variable is not retained; if we carry out dummy saturation at the 5% level we would expect that 5% of the time a dummy is retained when no structural break has been specified for that observation. This is referred to as the size of the procedure. On the other hand, if a structural break has occurred for a particular observation, we expect that the number of occasions in our simulations that a dummy is retained is substantially higher than 5%. This alternative aspect of our testing is referred to as the power: Does dummy saturation using our measures of co-movement have the *power* to detect changes in co-movement?

We represent this information graphically, starting in Figure 3. This plots the three measures (we do not plot ρ since it does not change throughout the sample) in separate panels. The size cases are cases 1-8(b), while the power cases are the "dc", "c" and "rc" scenarios. Considering size first, in cases 1-8(b) (1 and 5 are not reported since there is no variation in the data series for these cases by definition), all measures have consistent size in the region of 5%, although for ϕ and ψ this is temporarily somewhat lower.

Turning to power, the structural breaks all happen towards the end of the sample and in all three panels this can be observed, as some of the series do jump up for the last 5 or 10 observations. The size of that jump up though is important, and can be learnt from the vertical axis. For γ, the jump up is only to just over 10%, whereas for ϕ and ψ it is to usually above 30%, indicating that for the latter measures, there is a greater probability of a change in co-movement being detected. For γ approximately 10% of dummies are rejected in the event of a structural break yielding a low probability of actually detecting a break having occurred. Thus, γ has lower power. The power of ϕ and ψ is also not particularly high, with still less than a 50/50 chance of detecting a break using these measures. Furthermore, the coupling cases do not yield any evidence that they would be detected in this manner.

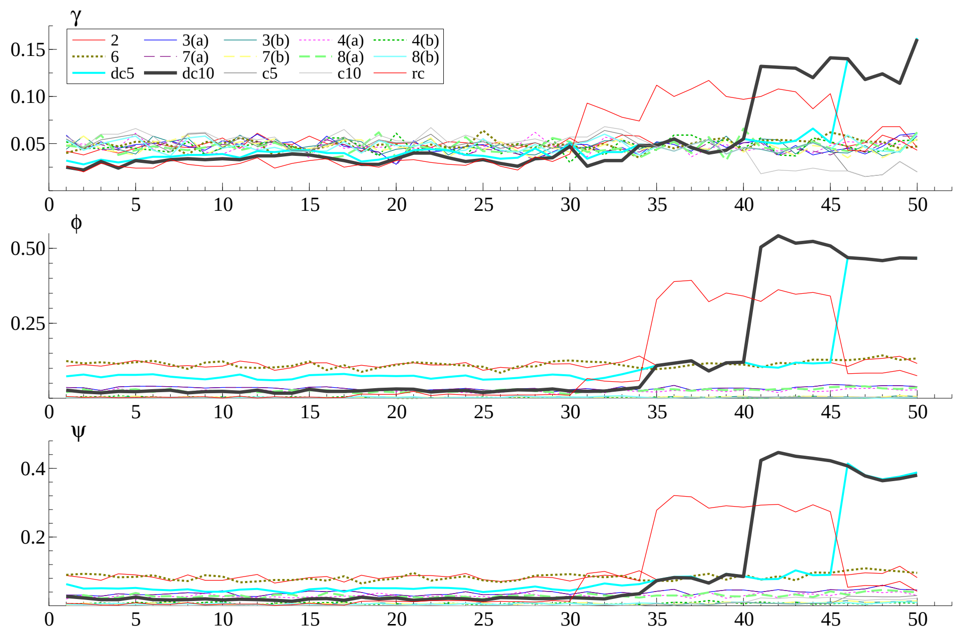

Figure 3: Retention frequencies of dummy variables using dummy saturation for measures γ, ϕ and ψ.

In Figure 4 we combine information over 2, 3 and 5 periods in order to determine whether this beneficial in detecting a break. Since structural breaks, by definition last for a number of time periods, and hence if information for more than one of these time periods is used, the probability of their detection is increased. Figure 4 confirms that hypothesis. Even with two periods of information, coupling episodes are much more likely to be detected, and the chance of detecting decoupling jumps to more than a 50% probability. This pattern continues if three or five periods of information is used, and when five periods is used the chance of detecting breaks is over 50% for all cases (including coupling).[2] However, there is a trade-off, since for instance with real-life data, if a break was suspected of happening after 2008, then on annual data there is now only just five years of information to use — combining over longer time horizons naturally has a cost in terms of observations lost. Nonetheless, from the bottom right panel of Figure 4 there is a distinct pattern in the rejection frequencies the moment a break happens — they start to increase dramatically, suggesting that some information could be used before the five years have passed to help determine whether a break might have happened.

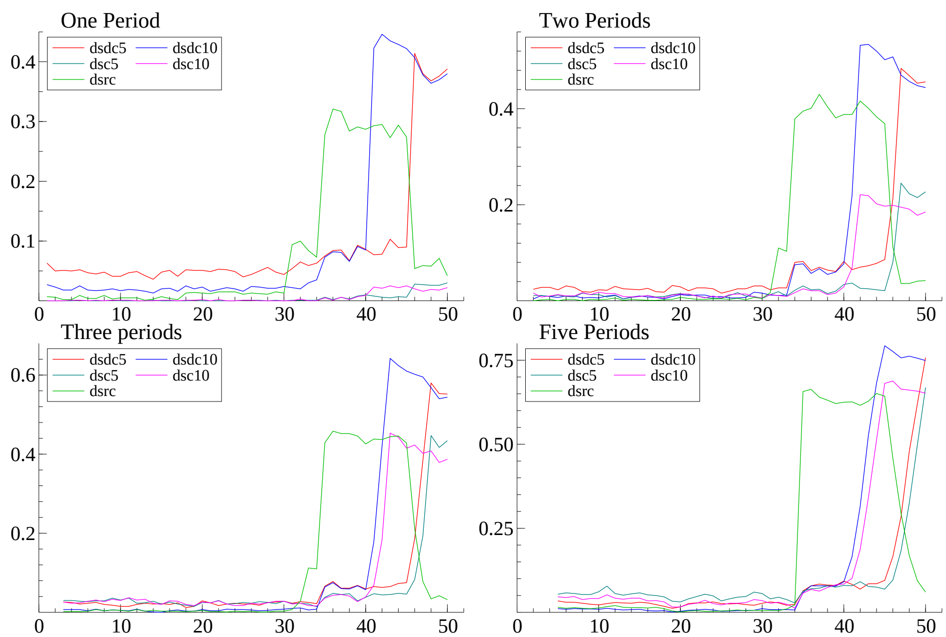

Figure 4: Retention frequencies of dummy variables using dummy saturation for ψ with different numbers of time observations.

We conclude from this section that our ψ measure of co-movement is able to perform as well as either of the individual measures proposed by De Haan et al., and furthermore when information is combined over two or three time periods, it seems highly probable that structural breaks would be detected in co-movements. This exemplifies the utility of

[2] This pattern is also apparent for the γ and ϕ measures, although not as distinct for the coupling cases, and also the size more generally increases with γ and ϕ as we increase the number of periods over which we calculate. See Figures 8 and 7 in the Appendix.

the impulse indicator saturation method, and also confirms the usefulness of our proposed measure of business cycle co-movements.

5 Data Analysis

Having considered the properties of each measure of co-movement, and the likelihood of detecting structural breaks using dummy saturation, in this section we put both sets of results into practice by using our ψ measure of co-movement alongside impulse indicator saturation to understand more about potential changes in co-movement.

We run a regression model containing the indicator variables selected by dummy saturation and consider whether including trade intensity, a conventional explanatory variable for business cycle co-movement, reduces the significance of the dummy variables. If the dummy variables remain significant, there is evidence for structural changes during the period we consider.

The dataset comprises the G7 economies (Canada, France, Germany, Italy, Japan, United Kingdom and USA) as well as the BRICS countries (Brasil, Russia, India, China and South Africa) on an annual basis ranging from 1980 to 2011.[3] In extracting the cyclical component of the real GDP series in USD at market rates we account for the widely discussed sensitivity of estimation results relying on data based on high-pass and band-pass filtering techniques and therefore apply methods suggested by Baxter and King (1999) (BK hereafter), Butterworth (1930) (BW hereafter), Christiano and Fitzgerald (2003) (CF hereafter) and the widely used Hodrick and Prescott filter (Hodrick and Prescott (1997)) (the HP-filter hereafter). The HP-filter integrates the smoothing parameter λ which is varied following Ravn and Uhlig (2002)[4]. The cross-correlations of the cyclical components extracted by the various detrending methods are reported in Tables 3-15, indicating overall high correlations

[3]GDP data (series codes: 99B..ZF..., 99B.CZF..., 99B.CZW...) exchange rates of national currencies in USD (series code: ..AE.ZF...) and GDP deflator series (series code: 99BIRZF...) are collected from the International Financial Statistics database of the IMF. Trade data were obtained from the IMF Direction of Trade Statistics (series codes: 70..DZF111, 71..DZF111).

[4]Due to space limitations we restrict ourselves to reporting the estimation results for cyclical components derived with HP-filtering ($\lambda = 4$). Applying the other afore mentioned filtering techniques and their various configurations yield comparable results that are available upon request.

for the data we analyse, irrespective of the applied filtering technique. The calculation of the proposed concordance measure based on the particular transitory components will therefore yield highly correlated quantities for the data under consideration.

The general regression model

$$\Psi_{it} = \alpha_0 + \beta_1 \Psi_{i,t-k} + \gamma \Delta_{i,t} + \beta_2 X_{it} + \beta_3 X_{i,t-k} + u_{it} \quad (6)$$

examines the concordance of the business cycles in reference to the US cycle that is measured as proposed in (5). The matrix $\Psi_{i,t-k}$ contains the (k) lagged values of the concordance measure. $\Delta_{i,t}$ is a matrix of the impulse indicator series that account for structural breaks in the business cycle synchronicity of the respective country vis à vis the US cycle. The matrix X_{it} comprises bilateral trade intensity as a major determinant of business cycle synchronicity (see Baxter and Kouparitsas (2005) among others).[5]

Figure 5 illustrates the calculated three periods moving average of the $\psi_{(3)}$ measure for business cycle concordance, exemplified for the United Kingdom, France, Italy and Germany.[6] The graphical illustration indicates an intensified co-movement of the UK business cycle component with the US cycle since 2004 . While the co-movement of the US and UK cycle series can be described with a lagging behind of the UK cycle until the mid 1990s, the characteristic of the interrelation of their time patters changes during the remaining half of the century. Figure 6, which displays the impulse indicator series in the lower part of the graphs, confirms this notion by indicating a significant structural break of the co-movement pattern for the US and UK cycle in 1995.[7]

The regression results (Table 5) with heteroskedasticity and autocorrelation robust standard errors highlight the importance of the impulse indicator saturation (IIS $\psi_{(3)}$) in detecting breaks when assessing business cycle synchronicity. The dummy indicators significantly detect changes in the concordance of the business cycle components with respect to the US

[5]Trade intensity is proxied by the mean of aggregate bilateral exports and imports vis-à-vis the US for each period.

[6]The illustrations for the remaining countries are shown in Figures 9 and 10 of the Appendix.

[7]The illustrations of the cyclical components and structural breaks as indicated by dummy saturation for the remaining countries are shown in Figures 11 and 12 in the Appendix.

cycle for the underlying sample. The evidence for trade intensity vis-à-vis US as a determinant for business cycle synchronicity is mixed for the panel of economies we analyse. It has to be noted here though, in lieu of considering a larger set of business cycle determinants yielding more robust results for the importance of trade in determining business cycle synchronicity our main goal here is to illustrate the utility of the proposed measure as well as the effectiveness of the impulse indicator approach in detecting structural changes.

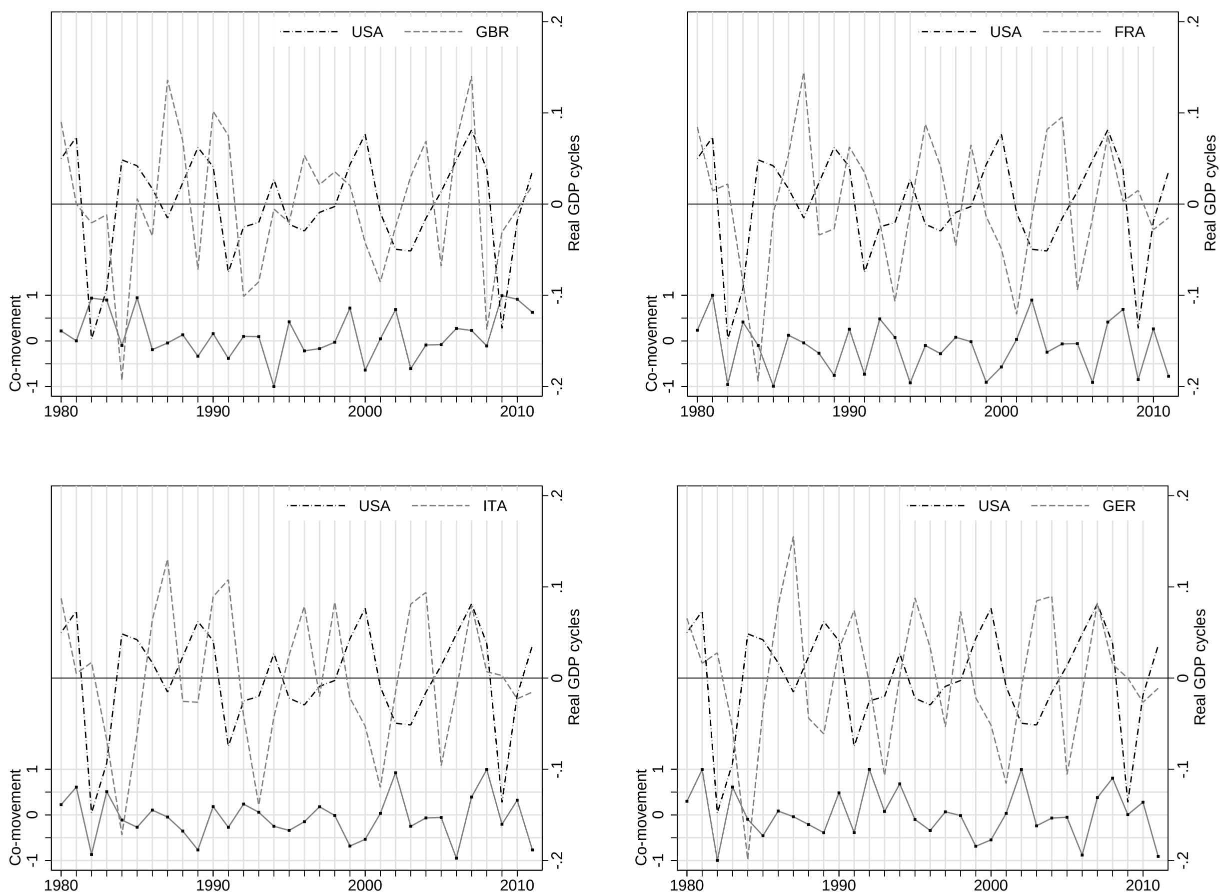

Figure 5: Real business cycles; co-movement as indicated by $\psi_{(3)}$, calculated in reference to the US cycle.

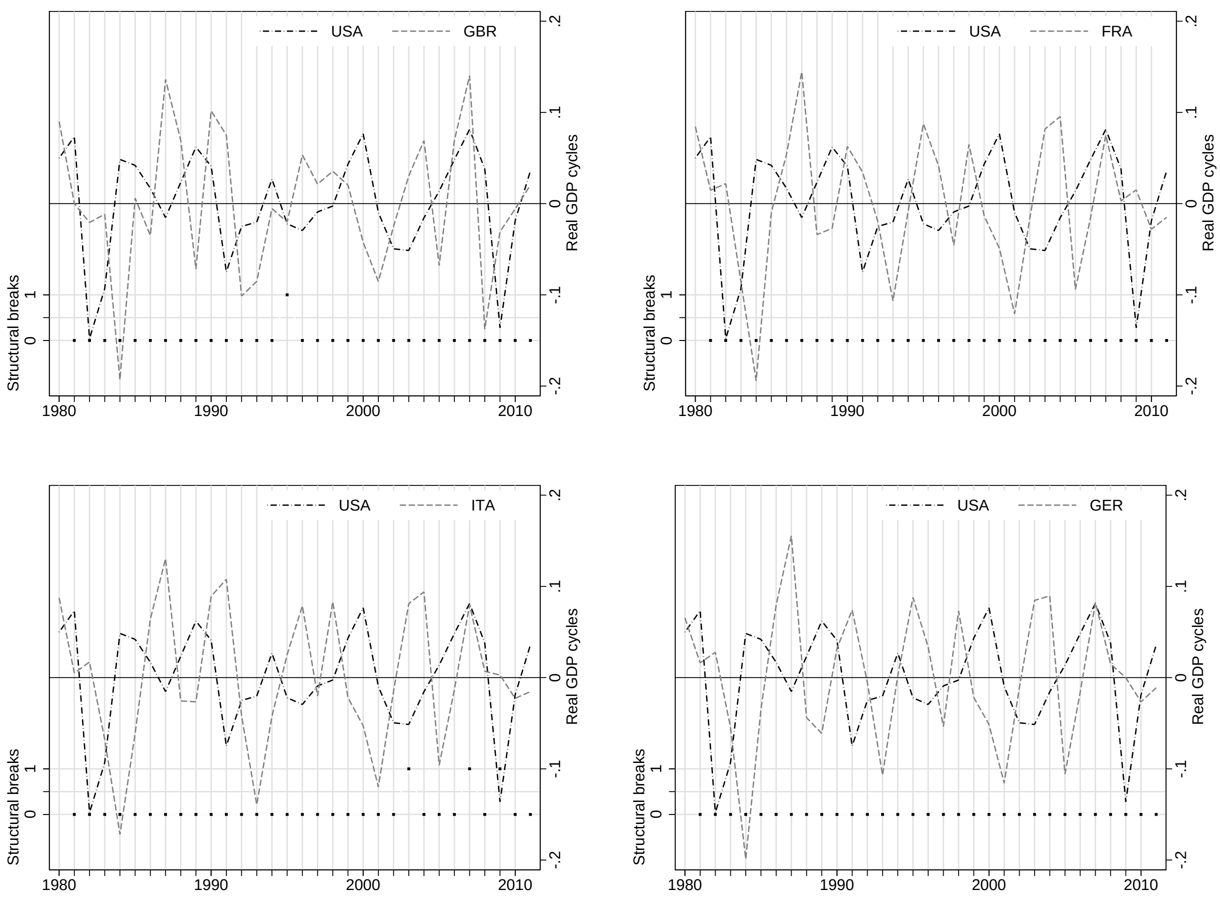

Figure 6: Real business cycles; structural breaks of the co-movement calculated in reference to the US cycle.

	POLS	FE	First difference GMM
$\psi_{(3)}$(-1)	0.821534***	0.782069***	0.602259***
		0.821534	
	(0.056875)	(0.031924)	(0.042659)
		[0.091885]	
$\psi_{(3)}$(-2)	-0.315109***	−0.344908***	-0.435283***
		−0.315109	
	(0.056343)	(0.060908)	(0.069585)
		[0.090768]	
IIS $\psi_{(3)}$	-0.071884*	−0.141535***	-0.166163***
		−0.071884	
	(0.040036)	(0.044171)	(0.047669)
		[0.055366]	
Trade intensity vis-à-vis US	-0.257543	−0.242650	-0.454834
		−0.257543	
	(0.182494)	(0.239122)	(0.277411)
		[0.207863]	
Trade intensity vis-à-vis US(-1)	0.463385**	0.474934**	0.406052
		0.463385	
	(0.230198)	(0.285399)	(0.284340)
		[0.268337]	
Trade intensity vis-à-vis US(-2)	-0.126002	−0.183189	-0.174288
		−0.126002	
	(0.159674)	(0.123613)	(0.169219)
		[0.199534]	
Intercept	-0.010785	−0.001299	
		−0.010785	
	(0.016955)	(0.020919)	
		[0.022947]	

Significance levels : $*$: 10% $**$: 5% $***$: 1%

Table 2: Business cycle synchronicity ($\psi_{(3)}$) - Impulse indicator saturation and trade intensity estimates.

Notes: Robust Huber/White/sandwich errors in parentheses (see Rogers (1994) and Williams (2000)) when not indicated otherwise. The Breusch-Pagan LM test indicates no evidence of significant differences across units. Pesaran's test of cross-sectional independence (see Pesaran (2004)) indicates residual correlation across entities (test statistic=3.633; P-value=0.0003). Panel corrected standard errors and coefficients are therefore reported for the FE estimates in square brackets and small type face respectively. The annotations of the significance levels are based on the robust standard errors.

GMM regression results are presented for one-step difference GMM estimation with Arellano-Bond robust variance/covariance standard errors. *IIS* $\psi_{(3)}$ is used as standard instrument for the first difference equation. $\psi_{(3)}$ and *Trade intensity vis-à-vis US* are employed as GMM-type instruments with lags(1/3) (Roodman (2006)).

6 Concluding Remarks

The main contribution of this paper is a new metric of business cycle co-movement that satisfies a number of desirable properties. Its performance alongside that of already existing measures is assessed through a simulation study. We also investigate the ability of our measure to detect changes in co-movement making use of recently developed methods for discovering structural breaks. We find that our proposed measure of business cycle co-movement is highly effective at detecting changes in co-movements and appears to also have desirable size properties. When applied to recent business cycle series from developed and developing nations, we find that breaks indicated by the impulse indicator saturation approach significantly reflect the structural change in the concordance of business cycles. Augmenting our dataset with conventional variables of business cycle determinants and combining them with the impulse indicator approach leaves scope for further research and assessment of changes in business cycle synchronicity against the backdrop of structural changes in the state of the economies such as the global financial crisis.

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A Appendix

A.1 Additional Material

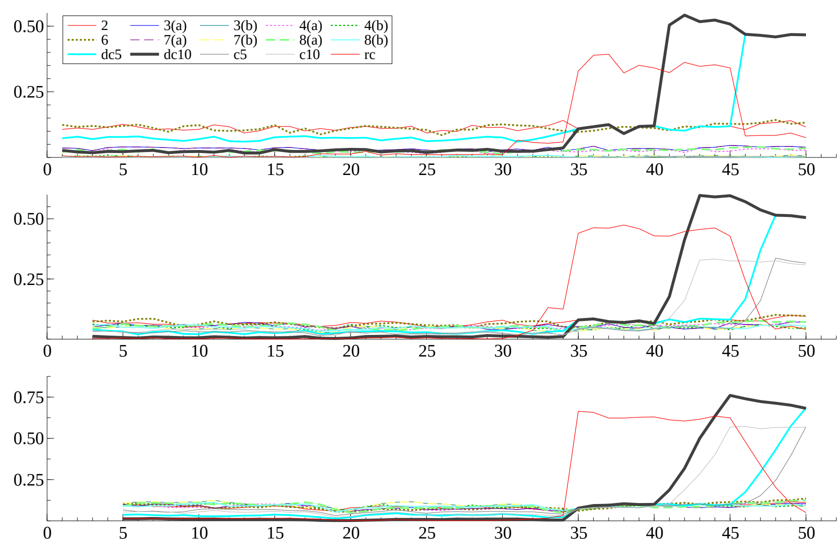

Figure 7: Dummy saturation on ϕ measure as number of periods used increases.

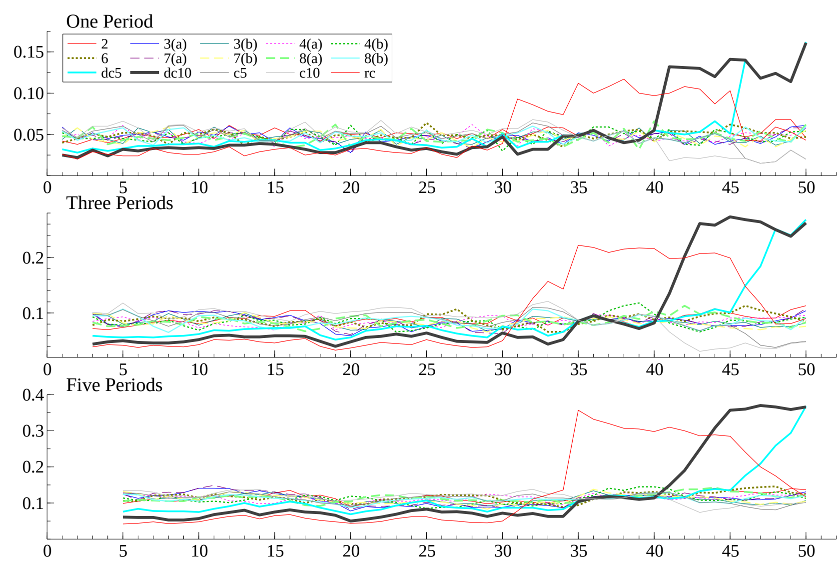

Figure 8: Dummy saturation on γ measure as number of periods used increases.

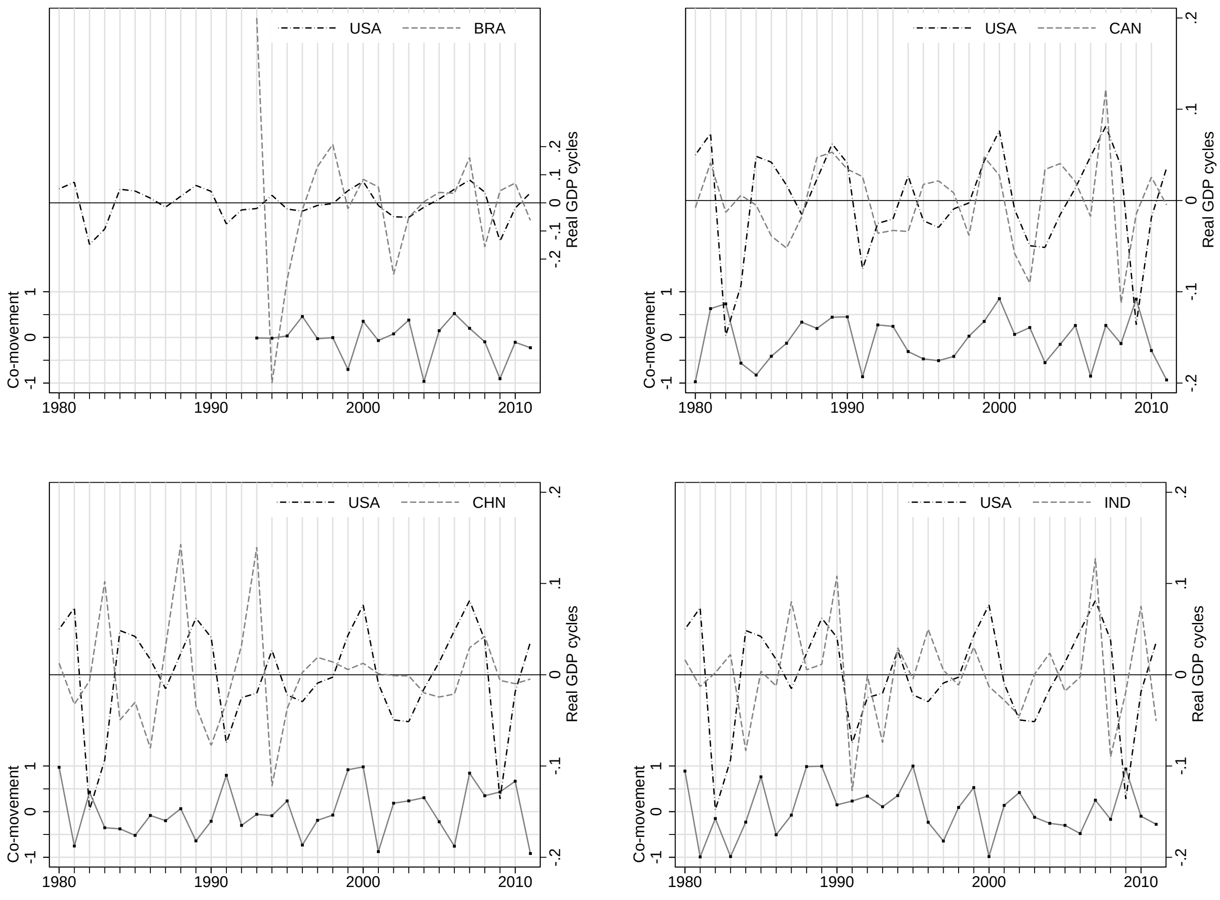

Figure 9: Real business cycles; co-movement as indicated by $\psi_{(3)}$, calculated in reference to the US cycle.

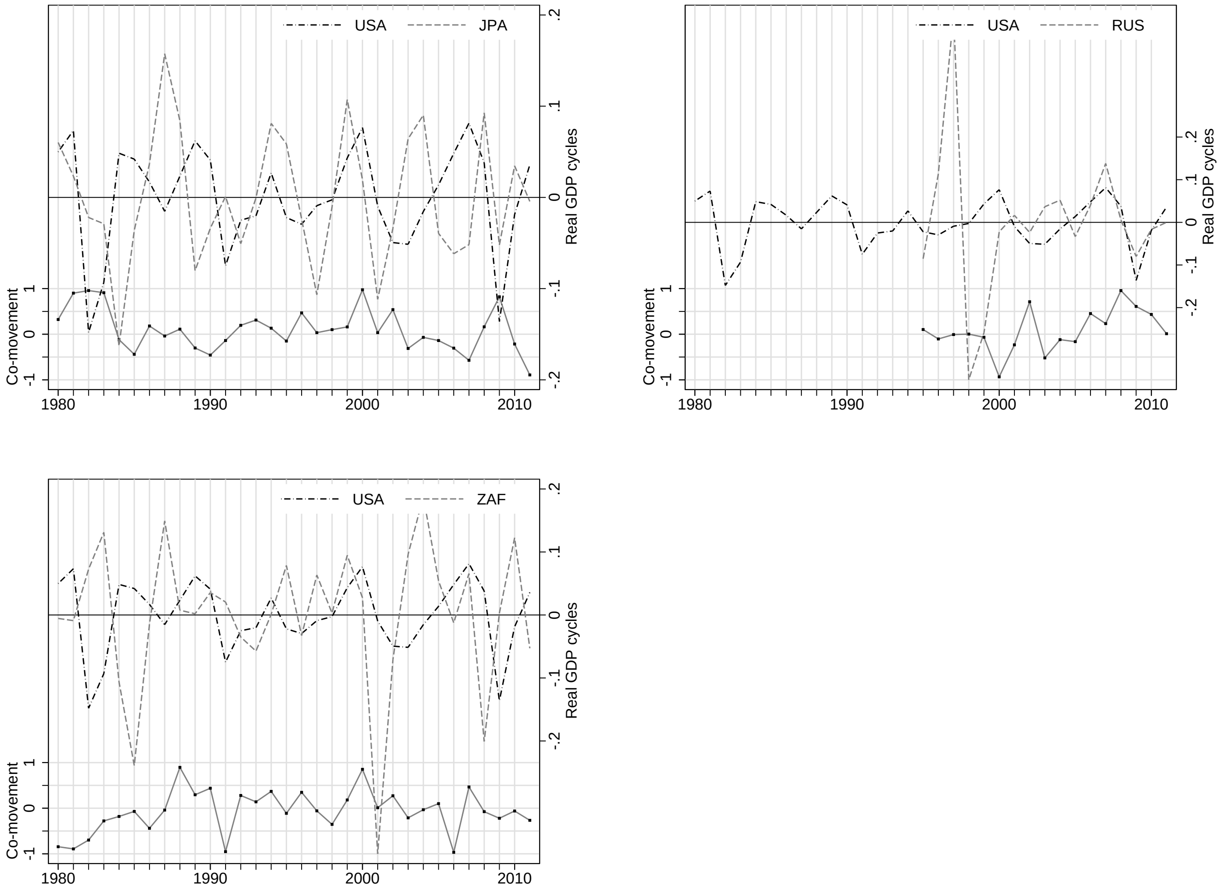

Figure 10: Real business cycles; co-movement as indicated by $\psi_{(3)}$, calculated in reference to the US cycle.

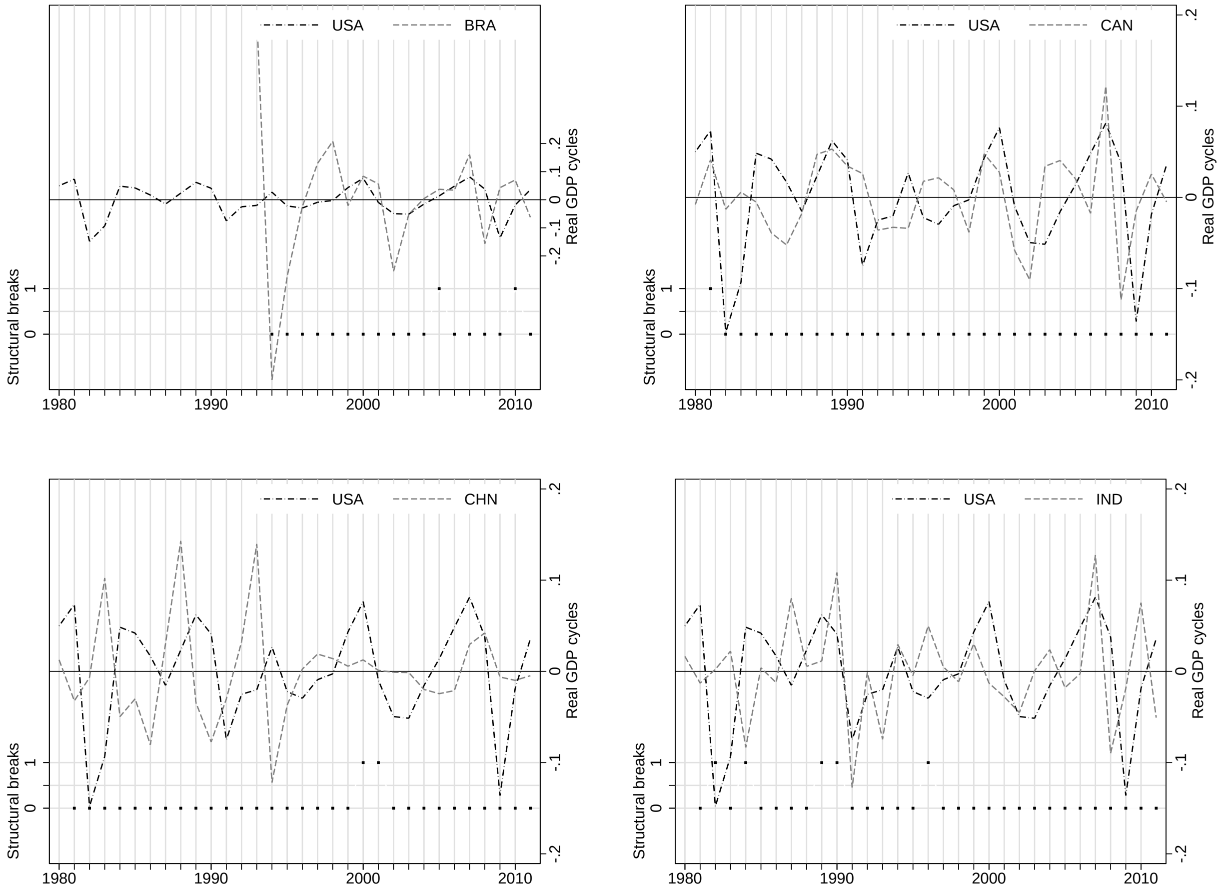

Figure 11: Real business cycles; structural breaks of the co-movement calculated in reference to the US cycle.

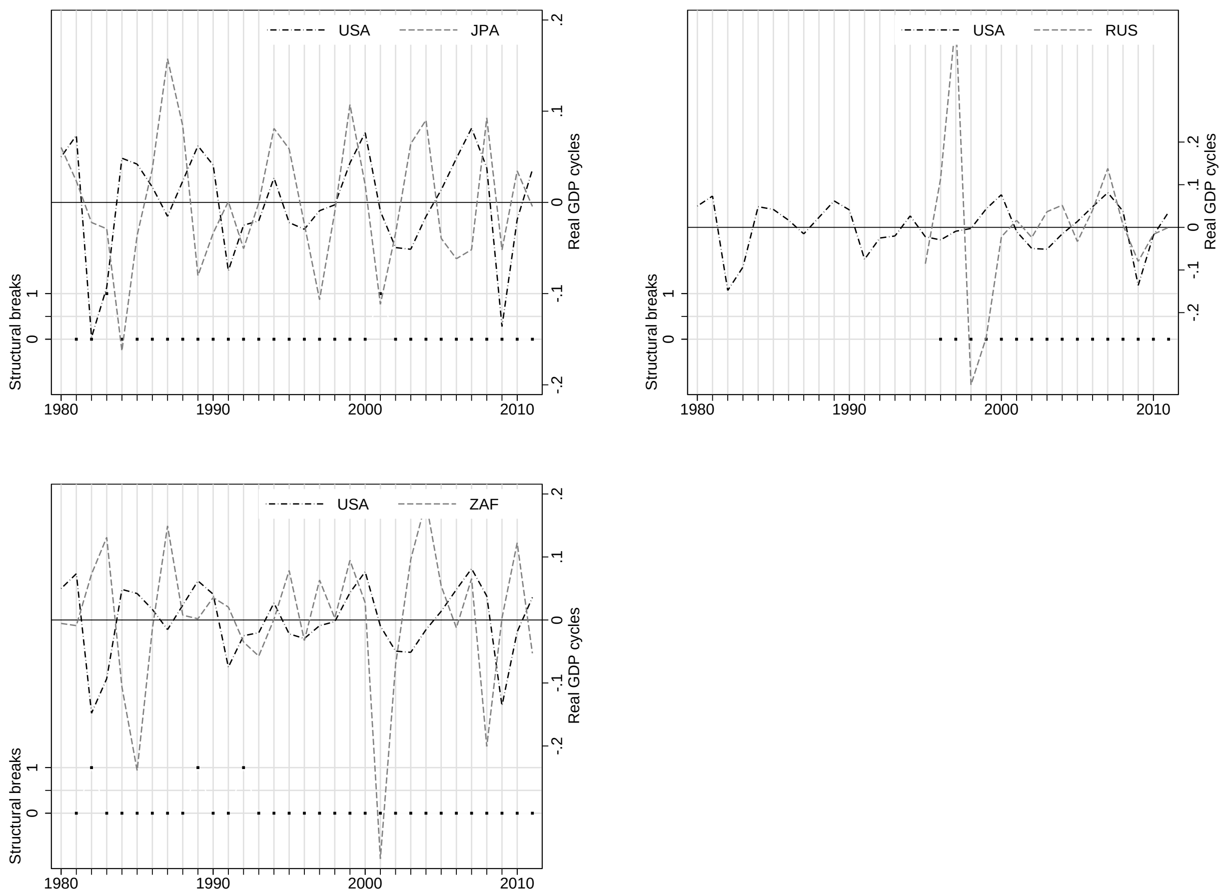

Figure 12: Real business cycles; structural breaks of the co-movement calculated in reference to the US cycle.

A.2 Cross-Correlations of Cyclical Components

Variables	HP($\lambda = 4$)	HP($\lambda = 6.25$)	HP($\lambda = 10$)	HP($\lambda = 100$)	HP($\lambda = 1600$)	BK($q = 1$)	BK($q = 2$)	CF	BW (n=2)	BW (n=6)
HP($\lambda = 4$)	1.000									
Nb. Obs.										
HP($\lambda = 6.25$)	0.998	1.000								
Nb. Obs.	19									
HP($\lambda = 10$)	0.993	0.998	1.000							
Nb. Obs.	19	19								
HP($\lambda = 100$)	0.891	0.907	0.924	1.000						
Nb. Obs.	19	19	19							
HP($\lambda = 1600$)	0.603	0.623	0.647	0.877	1.000					
Nb. Obs.	19	19	19	19						
BK($q = 1$)	0.908	0.885	0.865	0.720	0.227	1.000				
Nb. Obs.	17	17	17	17	17					
BK($q = 2$)	0.971	0.959	0.947	0.831	0.437	0.789	1.000			
Nb. Obs.	15	15	15	15	15	15				
CF	0.985	0.984	0.978	0.870	0.577	0.850	0.908	1.000		
Nb. Obs.	19	19	19	19	19	17	15			
BW (n=2)	0.997	0.991	0.982	0.867	0.579	0.925	0.973	0.984	1.000	
Nb. Obs.	19	19	19	19	19	17	15	19		
BW (n=6)	0.518	0.485	0.458	0.366	0.238	0.774	0.447	0.473	0.555	1.000
Nb. Obs.	19	19	19	19	19	17	15	19	19	

Table 3: Cross-correlations of real GDP cyclical components - Brasil

Variables	HP($\lambda = 4$)	HP($\lambda = 6.25$)	HP($\lambda = 10$)	HP($\lambda = 100$)	HP($\lambda = 1600$)	BK($q = 1$)	BK($q = 2$)	CF	BW (n=2)	BW (n=6)
HP($\lambda = 4$)	1.000									
Nb. Obs.										
HP($\lambda = 6.25$)	0.996	1.000								
Nb. Obs.	32									
HP($\lambda = 10$)	0.983	0.995	1.000							
Nb. Obs.	32	32								
HP($\lambda = 100$)	0.838	0.878	0.916	1.000						
Nb. Obs.	32	32	32							
HP($\lambda = 1600$)	0.673	0.713	0.755	0.911	1.000					
Nb. Obs.	32	32	32	32						
BK($q = 1$)	0.911	0.881	0.849	0.686	0.559	1.000				
Nb. Obs.	30	30	30	30	30					
BK($q = 2$)	0.987	0.977	0.962	0.838	0.684	0.926	1.000			
Nb. Obs.	28	28	28	28	28	28				
CF	0.949	0.926	0.891	0.675	0.531	0.899	0.947	1.000		
Nb. Obs.	32	32	32	32	32	30	28			
BW (n=2)	0.995	0.983	0.960	0.790	0.626	0.930	0.989	0.965	1.000	
Nb. Obs.	32	32	32	32	32	30	28	32		
BW (n=6)	0.921	0.892	0.855	0.651	0.503	0.915	0.947	0.937	0.944	1.000
Nb. Obs.	32	32	32	32	32	30	28	32	32	

Table 4: Cross-correlations of real GDP cyclical components - Canada

Variables	HP($\lambda = 4$)	HP($\lambda = 6.25$)	HP($\lambda = 10$)	HP($\lambda = 100$)	HP($\lambda = 1600$)	BK($q = 1$)	BK($q = 2$)	CF	BW (n=2)	BW (n=6)
HP($\lambda = 4$)	1.000									
Nb. Obs.										
HP($\lambda = 6.25$)	0.999	1.000								
Nb. Obs.	32									
HP($\lambda = 10$)	0.997	0.999	1.000							
Nb. Obs.	32	32								
HP($\lambda = 100$)	0.960	0.967	0.974	1.000						
Nb. Obs.	32	32	32							
HP($\lambda = 1600$)	0.429	0.437	0.447	0.617	1.000					
Nb. Obs.	32	32	32	32						
BK($q = 1$)	0.926	0.916	0.906	0.862	0.419	1.000				
Nb. Obs.	30	30	30	30	30					
BK($q = 2$)	0.997	0.994	0.990	0.965	0.581	0.930	1.000			
Nb. Obs.	28	28	28	28	28	28				
CF	0.967	0.959	0.950	0.901	0.411	0.907	0.978	1.000		
Nb. Obs.	32	32	32	32	32	30	28			
BW (n=2)	0.999	0.997	0.992	0.950	0.420	0.928	0.998	0.975	1.000	
Nb. Obs.	32	32	32	32	32	30	28	32		
BW (n=6)	0.986	0.980	0.972	0.920	0.402	0.914	0.987	0.990	0.991	1.000
Nb. Obs.	32	32	32	32	32	30	28	32	32	

Table 5: Cross-correlations of real GDP cyclical components - People's Republic of China

Variables	HP($\lambda = 4$)	HP($\lambda = 6.25$)	HP($\lambda = 10$)	HP($\lambda = 100$)	HP($\lambda = 1600$)	BK($q = 1$)	BK($q = 2$)	CF	BW (n=2)	BW (n=6)
HP($\lambda = 4$)	1.000									
Nb. Obs.										
HP($\lambda = 6.25$)	0.994	1.000								
Nb. Obs.	32									
HP($\lambda = 10$)	0.975	0.994	1.000							
Nb. Obs.	32	32								
HP($\lambda = 100$)	0.820	0.872	0.918	1.000						
Nb. Obs.	32	32	32							
HP($\lambda = 1600$)	0.757	0.814	0.867	0.991	1.000					
Nb. Obs.	32	32	32	32						
BK($q = 1$)	0.910	0.882	0.848	0.681	0.617	1.000				
Nb. Obs.	30	30	30	30	30					
BK($q = 2$)	0.989	0.983	0.970	0.848	0.784	0.931	1.000			
Nb. Obs.	28	28	28	28	28	28				
CF	0.931	0.903	0.864	0.671	0.607	0.880	0.927	1.000		
Nb. Obs.	32	32	32	32	32	30	28			
BW (n=2)	0.990	0.969	0.935	0.744	0.677	0.927	0.984	0.948	1.000	
Nb. Obs.	32	32	32	32	32	30	28	32		
BW (n=6)	0.875	0.820	0.757	0.514	0.449	0.911	0.905	0.911	0.930	1.000
Nb. Obs.	32	32	32	32	32	30	28	32	32	

Table 6: Cross-correlations of real GDP cyclical components - France

Variables	HP($\lambda = 4$)	HP($\lambda = 6.25$)	HP($\lambda = 10$)	HP($\lambda = 100$)	HP($\lambda = 1600$)	BK($q = 1$)	BK($q = 2$)	CF	BW (n=2)	BW (n=6)
HP($\lambda = 4$)	1.000									
Nb. Obs.										
HP($\lambda = 6.25$)	0.994	1.000								
Nb. Obs.	32									
HP($\lambda = 10$)	0.977	0.994	1.000							
Nb. Obs.	32	32								
HP($\lambda = 100$)	0.858	0.904	0.941	1.000						
Nb. Obs.	32	32	32							
HP($\lambda = 1600$)	0.794	0.843	0.885	0.982	1.000					
Nb. Obs.	32	32	32	32						
BK($q = 1$)	0.908	0.870	0.827	0.678	0.629	1.000				
Nb. Obs.	30	30	30	30	30					
BK($q = 2$)	0.989	0.976	0.957	0.861	0.827	0.934	1.000			
Nb. Obs.	28	28	28	28	28	28				
CF	0.922	0.883	0.839	0.671	0.616	0.942	0.943	1.000		
Nb. Obs.	32	32	32	32	32	30	28			
BW (n=2)	0.991	0.971	0.940	0.791	0.725	0.936	0.990	0.951	1.000	
Nb. Obs.	32	32	32	32	32	30	28	32		
BW (n=6)	0.855	0.799	0.739	0.543	0.483	0.936	0.910	0.930	0.908	1.000
Nb. Obs.	32	32	32	32	32	30	28	32	32	

Table 7: Cross-correlations of real GDP cyclical components - United Kingdom

Variables	HP($\lambda = 4$)	HP($\lambda = 6.25$)	HP($\lambda = 10$)	HP($\lambda = 100$)	HP($\lambda = 1600$)	BK($q = 1$)	BK($q = 2$)	CF	BW (n=2)	BW (n=6)
HP($\lambda = 4$)	1.000									
Nb. Obs.										
HP($\lambda = 6.25$)	0.995	1.000								
Nb. Obs.	32									
HP($\lambda = 10$)	0.978	0.994	1.000							
Nb. Obs.	32	32								
HP($\lambda = 100$)	0.809	0.858	0.906	1.000						
Nb. Obs.	32	32	32							
HP($\lambda = 1600$)	0.699	0.751	0.805	0.957	1.000					
Nb. Obs.	32	32	32	32						
BK($q = 1$)	0.928	0.907	0.880	0.706	0.600	1.000				
Nb. Obs.	30	30	30	30	30					
BK($q = 2$)	0.991	0.987	0.976	0.840	0.725	0.943	1.000			
Nb. Obs.	28	28	28	28	28	28				
CF	0.953	0.929	0.892	0.674	0.547	0.902	0.946	1.000		
Nb. Obs.	32	32	32	32	32	30	28			
BW (n=2)	0.993	0.975	0.945	0.742	0.631	0.939	0.987	0.969	1.000	
Nb. Obs.	32	32	32	32	32	30	28	32		
BW (n=6)	0.915	0.873	0.819	0.566	0.463	0.925	0.932	0.952	0.954	1.000
Nb. Obs.	32	32	32	32	32	30	28	32	32	

Table 8: Cross-correlations of real GDP cyclical components - Germany

Variables	HP($\lambda = 4$)	HP($\lambda = 6.25$)	HP($\lambda = 10$)	HP($\lambda = 100$)	HP($\lambda = 1600$)	BK($q = 1$)	BK($q = 2$)	CF	BW (n=2)	BW (n=6)
HP($\lambda = 4$)	1.000									
Nb. Obs.										
HP($\lambda = 6.25$)	0.996	1.000								
Nb. Obs.	32									
HP($\lambda = 10$)	0.984	0.996	1.000							
Nb. Obs.	32	32								
HP($\lambda = 100$)	0.845	0.879	0.910	1.000						
Nb. Obs.	32	32	32							
HP($\lambda = 1600$)	0.471	0.498	0.528	0.740	1.000					
Nb. Obs.	32	32	32	32						
BK($q = 1$)	0.940	0.912	0.879	0.711	0.441	1.000				
Nb. Obs.	30	30	30	30	30					
BK($q = 2$)	0.987	0.978	0.962	0.841	0.525	0.942	1.000			
Nb. Obs.	28	28	28	28	28	28				
CF	0.930	0.897	0.857	0.646	0.372	0.948	0.942	1.000		
Nb. Obs.	32	32	32	32	32	30	28			
BW (n=2)	0.995	0.983	0.963	0.803	0.439	0.958	0.989	0.957	1.000	
Nb. Obs.	32	32	32	32	32	30	28	32		
BW (n=6)	0.946	0.916	0.879	0.686	0.364	0.961	0.956	0.979	0.971	1.000
Nb. Obs.	32	32	32	32	32	30	28	32	32	

Table 9: Cross-correlations of real GDP cyclical components - India

Variables	HP($\lambda = 4$)	HP($\lambda = 6.25$)	HP($\lambda = 10$)	HP($\lambda = 100$)	HP($\lambda = 1600$)	BK($q = 1$)	BK($q = 2$)	CF	BW (n=2)	BW (n=6)
HP($\lambda = 4$)	1.000									
Nb. Obs.										
HP($\lambda = 6.25$)	0.994	1.000								
Nb. Obs.	32									
HP($\lambda = 10$)	0.975	0.994	1.000							
Nb. Obs.	32	32								
HP($\lambda = 100$)	0.835	0.885	0.928	1.000						
Nb. Obs.	32	32	32							
HP($\lambda = 1600$)	0.765	0.819	0.868	0.979	1.000					
Nb. Obs.	32	32	32	32						
BK($q = 1$)	0.912	0.883	0.848	0.690	0.620	1.000				
Nb. Obs.	30	30	30	30	30					
BK($q = 2$)	0.989	0.981	0.966	0.848	0.776	0.936	1.000			
Nb. Obs.	28	28	28	28	28	28				
CF	0.914	0.877	0.831	0.635	0.563	0.881	0.913	1.000		
Nb. Obs.	32	32	32	32	32	30	28			
BW (n=2)	0.990	0.968	0.934	0.758	0.686	0.931	0.984	0.943	1.000	
Nb. Obs.	32	32	32	32	32	30	28	32		
BW (n=6)	0.869	0.814	0.750	0.521	0.453	0.893	0.891	0.935	0.924	1.000
Nb. Obs.	32	32	32	32	32	30	28	32	32	

Table 10: Cross-correlations of real GDP cyclical components - Italy

Variables	HP($\lambda = 4$)	HP($\lambda = 6.25$)	HP($\lambda = 10$)	HP($\lambda = 100$)	HP($\lambda = 1600$)	BK($q = 1$)	BK($q = 2$)	CF	BW (n=2)	BW (n=6)
HP($\lambda = 4$)	1.000									
Nb. Obs.										
HP($\lambda = 6.25$)	0.997	1.000								
Nb. Obs.	32									
HP($\lambda = 10$)	0.987	0.997	1.000							
Nb. Obs.	32	32								
HP($\lambda = 100$)	0.874	0.903	0.929	1.000						
Nb. Obs.	32	32	32							
HP($\lambda = 1600$)	0.623	0.651	0.680	0.857	1.000					
Nb. Obs.	32	32	32	32						
BK($q = 1$)	0.908	0.889	0.868	0.763	0.560	1.000				
Nb. Obs.	30	30	30	30	30					
BK($q = 2$)	0.989	0.983	0.974	0.891	0.683	0.932	1.000			
Nb. Obs.	28	28	28	28	28	28				
CF	0.978	0.967	0.951	0.822	0.544	0.883	0.966	1.000		
Nb. Obs.	32	32	32	32	32	30	28			
BW (n=2)	0.995	0.983	0.966	0.832	0.585	0.920	0.989	0.983	1.000	
Nb. Obs.	32	32	32	32	32	30	28	32		
BW (n=6)	0.926	0.895	0.860	0.690	0.473	0.900	0.946	0.953	0.957	1.000
Nb. Obs.	32	32	32	32	32	30	28	32	32	

Table 11: Cross-correlations of real GDP cyclical components - Japan

Variables	HP($\lambda = 4$)	HP($\lambda = 6.25$)	HP($\lambda = 10$)	HP($\lambda = 100$)	HP($\lambda = 1600$)	BK($q = 1$)	BK($q = 2$)	CF	BW (n=2)	BW (n=6)
HP($\lambda = 4$)	1.000									
Nb. Obs.										
HP($\lambda = 6.25$)	0.997	1.000								
Nb. Obs.	32									
HP($\lambda = 10$)	0.988	0.997	1.000							
Nb. Obs.	32	32								
HP($\lambda = 100$)	0.898	0.925	0.949	1.000						
Nb. Obs.	32	32	32							
HP($\lambda = 1600$)	0.802	0.832	0.860	0.962	1.000					
Nb. Obs.	32	32	32	32						
BK($q = 1$)	0.919	0.893	0.861	0.684	0.489	1.000				
Nb. Obs.	30	30	30	30	30					
BK($q = 2$)	0.911	0.911	0.894	0.726	0.524	0.139	1.000			
Nb. Obs.	28	28	28	28	28	28				
CF	0.971	0.959	0.941	0.829	0.743	0.859	0.673	1.000		
Nb. Obs.	32	32	32	32	32	30	28			
BW (n=2)	0.995	0.984	0.968	0.857	0.760	0.937	0.859	0.978	1.000	
Nb. Obs.	32	32	32	32	32	30	28	32		
BW (n=6)	0.497	0.453	0.415	0.317	0.274	0.752	-0.657	0.438	0.547	1.000
Nb. Obs.	32	32	32	32	32	30	28	32	32	

Table 12: Cross-correlations of median real GDP cyclical components

Variables	HP($\lambda = 4$)	HP($\lambda = 6.25$)	HP($\lambda = 10$)	HP($\lambda = 100$)	HP($\lambda = 1600$)	BK($q = 1$)	BK($q = 2$)	CF	BW (n=2)	BW (n=6)
HP($\lambda = 4$)	1.000									
Nb. Obs.										
HP($\lambda = 6.25$)	0.995	1.000								
Nb. Obs.	17									
HP($\lambda = 10$)	0.975	0.992	1.000							
Nb. Obs.	17	17								
HP($\lambda = 100$)	0.741	0.800	0.862	1.000						
Nb. Obs.	17	17	17							
HP($\lambda = 1600$)	0.564	0.622	0.690	0.939	1.000					
Nb. Obs.	17	17	17	17						
BK($q = 1$)	0.933	0.910	0.876	0.676	0.555	1.000				
Nb. Obs.	15	15	15	15	15					
BK($q = 2$)	0.994	0.992	0.981	0.859	0.768	0.963	1.000			
Nb. Obs.	13	13	13	13	13	13				
CF	0.925	0.908	0.877	0.646	0.519	0.895	0.932	1.000		
Nb. Obs.	17	17	17	17	17	15	13			
BW (n=2)	0.995	0.979	0.947	0.675	0.499	0.946	0.989	0.933	1.000	
Nb. Obs.	17	17	17	17	17	15	13	17		
BW (n=6)	0.840	0.802	0.750	0.460	0.320	0.936	0.921	0.887	0.867	1.000
Nb. Obs.	17	17	17	17	17	15	13	17	17	

Table 13: Cross-correlations of real GDP cyclical components - Russia

Variables	HP($\lambda = 4$)	HP($\lambda = 6.25$)	HP($\lambda = 10$)	HP($\lambda = 100$)	HP($\lambda = 1600$)	BK($q = 1$)	BK($q = 2$)	CF	BW (n=2)	BW (n=6)
HP($\lambda = 4$)	1.000									
Nb. Obs.										
HP($\lambda = 6.25$)	0.996	1.000								
Nb. Obs.	32									
HP($\lambda = 10$)	0.985	0.996	1.000							
Nb. Obs.	32	32								
HP($\lambda = 100$)	0.831	0.865	0.898	1.000						
Nb. Obs.	32	32	32							
HP($\lambda = 1600$)	0.597	0.631	0.669	0.887	1.000					
Nb. Obs.	32	32	32	32						
BK($q = 1$)	0.926	0.905	0.879	0.729	0.574	1.000				
Nb. Obs.	30	30	30	30	30					
BK($q = 2$)	0.990	0.983	0.972	0.880	0.785	0.955	1.000			
Nb. Obs.	28	28	28	28	28	28				
CF	0.906	0.874	0.834	0.632	0.448	0.873	0.904	1.000		
Nb. Obs.	32	32	32	32	32	30	28			
BW (n=2)	0.994	0.981	0.961	0.783	0.553	0.941	0.988	0.939	1.000	
Nb. Obs.	32	32	32	32	32	30	28	32		
BW (n=6)	0.811	0.771	0.728	0.533	0.362	0.896	0.902	0.886	0.853	1.000
Nb. Obs.	32	32	32	32	32	30	28	32	32	

Table 14: Cross-correlations of real GDP cyclical components - USA

Variables	HP($\lambda = 4$)	HP($\lambda = 6.25$)	HP($\lambda = 10$)	HP($\lambda = 100$)	HP($\lambda = 1600$)	BK($q = 1$)	BK($q = 2$)	CF	BW (n=2)	BW (n=6)
HP($\lambda = 4$)	1.000									
Nb. Obs.										
HP($\lambda = 6.25$)	0.997	1.000								
Nb. Obs.	32									
HP($\lambda = 10$)	0.987	0.997	1.000							
Nb. Obs.	32	32								
HP($\lambda = 100$)	0.866	0.897	0.928	1.000						
Nb. Obs.	32	32	32							
HP($\lambda = 1600$)	0.696	0.728	0.762	0.892	1.000					
Nb. Obs.	32	32	32	32						
BK($q = 1$)	0.913	0.889	0.864	0.733	0.629	1.000				
Nb. Obs.	30	30	30	30	30					
BK($q = 2$)	0.989	0.980	0.968	0.868	0.759	0.939	1.000			
Nb. Obs.	28	28	28	28	28	28				
CF	0.965	0.952	0.931	0.772	0.644	0.898	0.963	1.000		
Nb. Obs.	32	32	32	32	32	30	28			
BW (n=2)	0.997	0.987	0.971	0.828	0.659	0.928	0.991	0.973	1.000	
Nb. Obs.	32	32	32	32	32	30	28	32		
BW (n=6)	0.951	0.931	0.905	0.734	0.574	0.923	0.970	0.954	0.967	1.000
Nb. Obs.	32	32	32	32	32	30	28	32	32	

Table 15: Cross-correlations of real GDP cyclical components - South Africa