A Comparative Analysis of Parametric and Discrete Time Models in Forecasting Portfolio Credit Risk

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Abstract

This paper offers a comparative analysis between the performance of the Weibull and Bernoulli mixture survival models in predicting portfolio credit risk for UK private firms. The intensity rate is measured using a reduced form framework defined in the context of the Basel II Accord. Both intensity models show the yield curve to be a key determinant of the risk of failure. Industry gross operating surplus and mixed income (an innovative predictor), are shown to be important determinants of portfolio credit risk. The correlation between the times to default for firms within the same industry sector is about 36.5%. Based on Shannon's entropy measure, the models can predict firms heading to default almost five years prior to failure. The overall performance of the Weibull model in terms of the conditional information entropy ratio outperforms the Bernoulli mixture model.

1. Introduction

Default probabilities are indispensable for assessing the creditworthiness of firms and identifying financially distressed counterparts. Radical developments in techniques of modelling credit risk have been motivated by the Basel Capital Accord II which emphasised estimating credit risk in the portfolio context. In this regard, the modelling process requires estimating three essential parameters: the default probability for each obligor's financial position over a multi-period time horizon; the default correlations across obligors; and the magnitude of expected financial loss in the event default (Zhou, 2001). Recent literature on credit risk uses duration analysis methods to estimate the default probabilities. A number of techniques which consider modelling dependence across defaults have been developed since 1997. The mixture[1] models have become a standard for the measurement of the correlation between defaulters (See, Carling et al., 2007a and Das et al., 2007, among others).

The literature on modelling credit risk, which is reviewed in Section 2, shows that researchers have given particular attention to the prediction of default intensity rates and to analysing the time to default. They have also concerned with modelling dependence between defaulters. Furthermore, it is not surprising that the existing literature has concentrated on modelling credit portfolio for rated firms using market-based models. The two market models are the Credit Metrics (CM) and CreditRisk+ (CR+). As these models are calibrated market data, it is not straightforward to use them to analyse the loan portfolios of private firms in the context of the Basel II agreement. Only a few academics have considered a direct comparison between the

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[1] A mixture model clusters corporate defaults into sub-homogenous groups based on their exposure to either common or correlated risk factors. The risk factors may be unobservable 'frailties' or contagious. They induce correlated changes in firms' conditional default probabilities.

methods that are used for modelling credit risk portfolio of public listed corporations (Koyluoglu and Hickman, 1998; Gordy, 2000; Fry and McNeil, 2003 and Jarrow, Lando and Turnbull, 1997). To my knowledge, there is no evidence that the literature has compared the performance of the advanced suitable methods of forecasting credit risk portfolio for private firms.

The purpose of this paper, lies in its contribution to a comparison of two advanced credit risk modelling approaches of private firms' loan portfolio. I compare discrete time to parametric survival analysis techniques in multivariate settings. Emphasize is given to modelling dependence across defaulters based on latent unobservable risk factors. The primary goal of the present paper is to evaluate the performance of the Bernoulli and the Weibull survival mixture models in quantifying private firms' portfolio credit risk. A reduced-form framework is developed to estimate conditional default rate.

The instantaneous default rate is affected by two sets of observable time-varying risk drivers; (i) firm-level and macroeconomics factors and (ii) unobservable common risk factors. I test the joint effect of firm-specific factors in determining default risk. Results show that firms are less likely to survive under high financial pressure and intense business risk. Positive values of industry median sales and profitability allow firms to overcome economic turmoil and excessive leverage. Consistent with the literature the results show firm size to be negatively associated with the intensity rate. Moreover, a firm's age works as a proxy for knowledge of technology and competitive environment. A large volume of cumulated information leads to higher survival chances.

The implications of the macroeconomic effects on the instantaneous rate of default are estimated at the aggregated and disaggregated levels. We find that the yield curve and industry gross operating surplus and mixed income are important determinants of default risk. We also find that unobserved common risk factors have significant impact on the conditional default rate. The correlation between firms that share the same industry segment is about 36.5%.

The models performance is assessed using Shannon's entropy measures. This estimates the degree of uncertainty associated with the probability of default triggering. Overall, the entropy measures confirm that both models are informative. The measures identify firms that will be in financial difficulties almost five years prior to failure. The out-of sample results confirm the same results.

The paper is organized as follows. The literature on credit risk management reviewed in Section 2. Section 3 describes the specifications of the econometric models. The models' estimations and results are presented in Section 4. The models' performances are presented in Section 5. Section 6 provides a conclusion.

2. Literature Review

Over the past decade, significant progress has been made with models of credit risk. Recent literature has concentrated on understanding how corporate defaults are correlated. Three main approaches for modelling credit portfolio have been developed: CreditMetrics (CM); CreditRisk+ (CR+); and CreditRiskPortfolio (CPV). All quantify credit risk at the portfolio level and also account for pro-cyclical effects. However, the distributional assumptions and methods of quantification are different.

Koyluoglu and Hickman (1998) analyse the differences and similarities amongst the three models. All share a similar framework for modelling portfolio credit risk. Three main factors dominate the estimation. First, a conditional default rate is calculated for each obligor for the relevant economic conditions. Second, joint-default behaviour, i.e., the conditional distribution of a homogenous sub-portfolio default rate, is estimated. Lastly, the unconditional distribution of portfolio defaults is obtained. The difference between the models is in the distributions used to model dependence across defaults. Gordy (2000) compares the frameworks of the CR+

model with a two-state version of the CM model. He finds the primary sources of discrepancy between the models dependent on the choice of the distribution for systematic risk factors and the functional form for the instantaneous rate probabilities. Together they give the shape of the joint distribution over defaulters in the portfolio. Frey and McNeil (2003) analyse the mechanism used in estimating dependence between defaults of the CR+ and CM models. They conclude that the Bernoulli mixture approach is preferable to the latent variable approach: the maximum likelihood of fitting the Bernoulli mixture model presents a feasible method for obtaining the parameter estimates.

The literature has identified different mechanisms for measuring default dependence among defaulters, namely latent variables, common risk factors and contagion factors. Firstly, the mechanism of the latent variables is used by the CreditMatrics model. The latent variable approach recognizes the dependence between defaulters when the value of a firm's assets falls below the value of its liabilities. Secondly, in the mechanism of the common risk factors, the default dependence is defined via some constant unobserved risk factors that are shared within a group of debtors. Lastly, the contagion systematic factors assume default dependence occurs through close relationships between firms with their partners web (e.g. parents and subsidiaries) (Giesecke and Weber, 2004 and Azizpour et al., 2010).

Giesecke (2004) examines the structural model of correlated default where firms are subject to cyclical correlation and contagion processes. The result indicates that disclosure matters a lot in default prediction. It increases transparency and reduces the likelihood of contagion effects due to the incomplete information of investors. Giesecke concludes that his model outperforms estimation, by the CreditMetrics model of correlated credit risk. This is attributed to its ability to accommodate information-based contagion effects. Giesecke and Weber (2004) employ Bernoulli's model mixture approach to studying credit loss. Giesecke and Weber's model incorporates both cyclical correlations and contagion effects. They conclude that macro-economic fluctuations are the main source of loss risk. The strength of the additional contagion-induced loss variability and the probability of large losses depend on the complexity of the business partner network, *i.e.* the degree of correlation between firms. Zhou (2001) evaluates default correlations across multiple defaults based on the CreditMatrics model. The default correlations are small in the short term and increase with time. He argues that the business cycle cannot explain this phenomenon. Similarly, Frey et al. (2001) estimate credit portfolio losses in the context of the latent variable mechanism. They find that individual default probabilities and asset correlations are insufficient to determine the distributions of portfolio losses.

Carling et al. (2007a) estimate the creditworthiness of Swedish firms' credit lines in two international retail banks. They developed a reduced form framework to identify credit risk drivers. The findings underline the importance of macroeconomic variables in explaining default risk in parallel with firm level information. These authors assert that macroeconomic factors: yield curve, output gap and households' expectations can capture the absolute level of default risk, while firm-specific effects can only rank firms according to their level of risk. Their model accommodates the duration dependency that permits the monitoring of a firm's credit worthiness. They argue that the inclusion of systematic risk factors, such as indicators of default correlation, will not fully capture credit losses in the event of an economic downturn. They extend the scope of their previous work to allow for dependencies between defaulters through both common risk factors and industry specific disturbances. They show that intra-industry correlations of defaults matter in estimating portfolio credit risk, and neglecting them will lead to an underestimation of losses in the event of default (Carling et al., 2007b).

Duffie et al. (2009) provide evidence for the magnitude effect of unobserved risk factors on default probability, relative to the information provided by an observable attribute vector for predicting individual firm defaults in the US portfolio of corporate debt. Their model tests reveal that overlooking the frailty effect results in an underestimation of the probability of extreme positive or negative events in the portfolios of corporate credits. The impact of unobserved frailty on default intensities increases proportional annual volatility by roughly 40%. Das et al. (2007) argued that models which capture the magnitude effects of uncertainty regarding common factors, after controlling for firms observable factors, are important determinates for estimating dependence across defaulters.

The literature in this area is new. The researchers concentrate on modelling portfolio credit risk for public-listed companies. A few researchers have estimated the portfolio credit risk for private firms. Therefore, the main objective of this paper is to compare and to examine the performance of the survival parametric and the discrete time mixture models in forecasting the credit risk portfolio of UK private firms.

3. Econometrics Models

This section presents econometrics models that are used to predict the life time of UK private firms. Duration analysis is the most appropriate approach. Its distinct features are twofold. First, the dependent variable is time. A duration model sequentially records a firm's financial status s from its entry in the experiment to the time at which point either the default event occurs or the firm is right censored i.e. where the lifetime of a firm exceeds the analysis the experimental period or the firm is lost to for unknown reasons. Second, the hazard function estimates the lifetime of a firm that is at the risk of default. This function computes the probability that a firm will default within a short interval subject to it survive at the beginning of the period. The model uses time varying predictors to describe the dynamic behaviour of a firm's creditworthiness.

The aim of this section is to compare the continuous- time mixture model with the discrete-time mixture model. Section 3.1 outlines the fundamental definitions and assumptions of the duration models. Section 3.2 gives the distributions of the durations of each technique. Mixture models specifications are given in section 3.3. Section 3.4 defines models of risk factors.

3.1 Definitions and assumptions

Default event: consider an economy with a cohort of private firms. A firm i, $i \in 1,\ldots,n$ can be in two states. State 0 corresponds to the non-default occasion. State 1 corresponds to the default occurrence. Let T_i be a continuous random variable representing time-to- the default for each firm i. It is assumed that T_i is infinite if the default does not occur. All firms facing credit risk in the sample survive at $T_i = 0$. Let t be the realized duration for each firm i where a default indicator variable D_i takes value 1 if the event is experienced and value 0 if the duration is right censored [*i.e.* representation to firm i state at time t_i]. $T_i = t$ means that the default happens at time t , the condition $T_i > t-1$ means that firm i did not default before t . The probability distribution of duration time T_i can be given by either the cumulative distribution function $F(t)$ in the case of continuous time, or the probability density mass function $f(t)$ in the case of discrete time. For firm i continuity time is a minimum of one year. The choice of one year is suitable for many reasons. The default risk or court petition request action can be taken within a year. Private firms provide yearly financial statements. This choice also allows controlling for the exit time of censored firms. It is assumed that firms can only exit or be censored at the end of each year. Moreover, since the presence of censoring data has significant implications for estimating the likelihood function, it is assumed that the censoring time C is random and independent of firms' default time. It is also assumed that the probability distribution of the surviving time of censored firms at time C is identical to that of firms that are not censored and survive at least to c after allowing for explanatory variables.

3.2. Distributions of durations

We address the characteristics of the hazard and survival functions of the continuous and discrete survival time models. In continuous time, the event of default is assumed to occur at any point in time and that time is recorded in finite time units. The lifetime of a firm is a realization of a random variable that is drawn from a specific distribution and a homogenous population. In this study, the Weibull distribution is considered.

The advantages of choosing the Weibull distribution stem from the fact that it provides estimates of both the proportional hazard and the accelerated time to failure models. It also assumes that the shape of the hazard changes overtime. The Weibull distribution is a more suitable alternative than the Cox proportional hazard and the exponential distribution in modelling the event of default. The Cox model makes no assumption about the shape of the hazard function. Although the exponential distribution can estimate both the hazard rate and the time to default, it assumes the hazard function remained constant overtime. In the Weibull models, the distribution has two parameters denoted as α and γ. The instant rate function $\lambda(t)$ given by Equation (1) is time dependent. The default probability at time t monotonically increases if $\alpha > 1$, monotonically decreases if $\alpha < 1$, and is constant if $\alpha = 1$ (exponential case). γ scales the base line hazard function multiplicatively through a vector X which incorporates a number of risk drivers i.e. $\gamma \equiv \exp\{-\beta' X\}$.

$$\lambda(t) = \gamma\alpha t^{\alpha-1} \tag{1}$$

The probability that a company is randomly selected from the UK population of private firms and will have a survival time less than or equal to some stated time t and be insolvent, is given by the cumulative distribution function (2).

$$F(t) = 1 - \exp(-\gamma t^{\alpha}) \tag{2}$$

The probability of firm i surviving past some specified time t and remaining solvent is obtained by (3) where $t \geq 0$:

$$S(t) = \exp(-\gamma t^{\alpha}) \tag{3}$$

The density function (4) is another way to describe the T distribution. It gives the instantaneous rate that is the probability that firm i defaults or is financially distressed subject to survival up until time $t-1$.

$$f(t) = \gamma\alpha t^{\alpha-1} \exp(-\gamma t^{\alpha}) \tag{1}$$

The integrated hazard function, $\Lambda(t)$, (5) contributes to the estimation of the likelihood function. First, the contribution of the default group in the likelihood function equals $\lambda(t)\exp\{-\Lambda(t)\}$. Second, the contribution of the censored observation in the likelihood function equals $\exp\{-\Lambda(t)\}$. Thus, the likelihood function is given by (6) (Hougaard, 2000).

$$\Lambda(t) = \gamma t^{\alpha} \tag{2}$$

$$L(\lambda) = \lambda(t)^{D} \exp\{-\Lambda(1)\} \tag{3}$$

The discrete time model summarizes the data in intervals, although the event of default usually occurs at any instant in a year. This way of collecting the data permits modelling the event of default using discrete time approach (Allison, 1982). The response variable is dichotomous following the Bernoulli distribution. It takes on a value of one if the event occurs and zero otherwise. Essentially, the model is defined in terms of the conditional probability of the default occurring. The complementary log-log (Clog-log afterwards) distribution is utilized as a link function. The Clog-log function directly estimates proportional hazards.

For the discrete time model time takes only positive integer values $(t = 1, 2, 3, ...)$ in which $t_1 < t_2 < ... < t_j$ interval. The observed lifetime of firm i is defined as a random integer variable T_i. Firms are assumed to be independent. Firm i continues up to time t_j and then either exists or is censored. As mentioned above, it is assumed that the time of censoring is independent of the hazard rate. The probability density mass function and the survival function are given by (7) and (8) respectively

$$f(t_j) = P(T_i = t_j) \tag{7}$$

$$S(t) = \mathrm{P}(T_i \geq t_j) = \sum_{j:t_j \geq t} f(t_j) \tag{8}$$

The discrete hazard function is a proportion of the relationship between the probability density function and the survival function which is given by (9). This relationship defines the distribution of T_i.

$$\lambda(t_j) = \mathrm{P}(T_i = t_j | T \geq t_j) = \frac{f(t_j)}{S(t_j)} \tag{9}$$

3.3 Survival mixture models specifications

A reduced form framework of doubly stochastic process is developed to capture the correlation between default probabilities of firms in the same industry sector. This framework incorporates three essential sources that are assumed to trigger the default event. These are broken down into the following components:

a. A time varying vector capturing idiosyncratic risk $U_i(t)$ contains firm-specific covariates that are observed for firm i from entering at time t to exit time T. They are assumed to be unique and predetermined to the individual firm i and do not affect the other firms. These covariates include a set of the firm's financial ratios (See Section 3.4).
b. A time-varying vector $M(t)$ capturing systematic risk at the aggregated and disaggregated levels that describes the state of the economy effects on the firms and is observed at all times. It incorporates two macroeconomic risk factors. All firms are assumed to respond to systematic risk in the same way at the aggregated level. The impact of some systematic risk factors is assumed to vary across industries. Macroeconomic indicators are assumed to be strictly exogenous and therefore unaffected by the default event.
c. A vector of unobserved shared frailty Y_h that models industry sectors intergroup correlation. For each industry sector h, $h \in 1, ..., K$ where K is the number of industry groups and h refers to a specific sector. Y_h is assumed to have mixture distribution, and as a result it controls for the unobserved risk factors that are not captured by the above mentioned vectors. Unobserved common risk factors are

assumed to induce the dependence between common fallings across firms in the same industry sector (Cleves et al., 2010).

For notational purposes, $X(t)$ is defined as the vector of firm-specific and macroeconomic covariates and Θ as unknown parameters. First we consider the generalized Weibull hazard-based model conditional on frailty effect Y_h that is defined by (10). The parameter Y_h is a random positive quantity which is assumed to have mean 1 and finite variance θ. Any industry group that have $Y_h > 1$ is said to be frail for the responses that are left unexplained by the observed covariates in the model and will have an increased risk to failure and vice versa. Finally, we assume that the shared frailty follows the inverse Gaussian function (Gutierrez, 2002).

$$\lambda_{ih}(t|Y_h) = Y_{ih}(t) \tag{4}$$

The multivariate Weibull mixture model conditional on the frailty effects is given by (11). The term $\Gamma(I)$ refers to the model error. This error follows the extreme value distribution of type I error. Equations (11) define the specifications of the Weibull hazard mixture model conditional on frailty effects.

$$\lambda_{ih}(t|\mathrm{Y}_h,\mathrm{X(t)},\Theta)=Y_{ih}\lambda_0 \exp(\beta U_i(t) + \mathrm{A}M(t))\alpha t^{\alpha-1} + \Gamma(1) \tag{5}$$

The generalized Bernoulli mixture hazard function in (12) accommodates both systematic and unsystematic effects as well as the shared frailty across firms of the same industry segment. The term ζ_h is the measure of dependence across firms in the same industry. It follows the normal distribution and is assumed to be independent across the industry sectors. The term ε_{ihj} is the residual error effects of firm i in industry sector h at time j. The residuals of Bernoulli mixture mode follow Gumbel distribution with a mean of about 0.577 and a variance of $\pi^2/6$ (Rabe-Hesketh and Skrondal, 2008).

$$\lambda_{ih}(t|,\mathrm{X(t}_j),\Theta)=\beta_0 \ln(t_j) + \beta U_{ih}(t_j) + \mathrm{A}M(t_j) + \zeta_h + \varepsilon_{ihj} \tag{12}$$

Eventually, both the hazard models in (11) and (12) share two important features which would facilitate comparison between the models' results. First of all, the residuals of the Weibull and Clog-log models have a standard extreme value of type-1, Gumbel distributions. Secondly, the exponential of ζ_h is the measure of frailty of the discrete time model which is equivalent to the measure of the frailty Y_h in the continuous model when Y_h follows the inverse Gaussian distribution. Finally, to facilitate the discrete time models' compassion with the Weibull models in Section 4, we replaced the time dummies with the logarithm of time to characterise the baseline hazard function in the model (12).

3.4. Models of risk factors

We define possible risk drivers for triggering a default event. An attribute vector accommodates the effects of both the idiosyncratic and systematic risk factors on a firm's default intensity rate. For Idiosyncratic risk factors, we use six financial ratios that appear in the literature as commonly significant predictors for the credit risk. Table 1 shows the candidate variables and the expected relationship between each financial ratio and the intensity rate.

Table 1: The Idiosyncratic Risk Factors

Risk factors	Description	Transformation	Expected sign
$TLTA$	Total debt / total assets.	Logarithmic	+
$NITA$	Net income after interest and taxes/ total assets.	Logarithmic = $\ln(NITA-(\min(NITA+1)$	-
VOL	The standard deviation of the net income for two consecutive years before the estimation year/ total assets.	None	+
$SIZE$	The logarithm of the total assets/ the nominal GDP index year 2000	Logarithmic	-
AGE	The difference between the financial statement year and the foundation year of a firm.	Logarithmic	-
$SSIC3$	The industry median sales. [sales – 3 digit SIC industry median sales/ sales]/ total assets	None	-

With regard to systematic factors, Basel II has indicated that business cycle, especially are dominant in triggering default. Significant macroeconomic predictors include GDP growth, inflation rate, yield curve, market indices and the exchange rate. A number of studies have measured such factors based on an aggregate data. For example Carling et al. (2007a) find output gap, yield curve and the households' expectations are important factors in predicting the survival time to default. Consistent with the literature, we consider the impact of the macroeconomic indicators on the survival time to default of UK private firms. We use the yearly nominal spot interest rate, LIBOR which is a macroeconomic indicator of monitory policy. It captures the up-turning point of a business cycle before it inverts to the recession. In an expansion phase, credit is injected. The new credit increases liquidity resulting in a lowering of the short term interest rate thereby an increase in the supply of investment funds and vies versa in the recession phase. Therefore, a positive association between nominal yield curve and the credit risk is expected in the expansion time and vice versa in the recession. LIBOR is measured as the difference in rates $\mathrm{Year}_t\text{-}\mathrm{Year}_{t-1}$.

The use of macroeconomic predictors at the disaggregate level in the credit risk literature is infrequent. Perhaps, this is so because the long time series databases for disaggregated macroeconomic covariates are not available in many countries. Another possible reason is that the literature has concentrated its focus on portfolio credit risk using the market-base model, such as the Merton model in which aggregated macroeconomic factors can be easily included and are globally available across countries. Both the Credit Risk+ and CreditRiskPortfolio models estimate portfolio credit risk based upon sectorial analysis.

We use the industry gross operating surplus and mixed income, GOSMI as another proxy for macroeconomic conditions at the disaggregate levels of the industry sectors. The operating surplus and mixed income of a government budget indicates that the government revenues (inflow) are greater than its expenses (outflows). Financial deficits increase during the recession leading the government to increasing the costs of its services in order to compensate for its deficits.

Consequently, the industries' profits are affected but not all industry sectors are equivalently impaired. One would expect to find $GOSMI$ differ across industry sectors markedly and hence the default rate. To our knowledge no research has considered this macroeconomic indicator in measuring a portfolio credit risk. In order to estimate the impact of $GOSMI$ on credit risk portfolio, firms are sorted into 34 industry sectors using Fama and French industry codes and then scaled the $GOSMI$ by the yearly nominal GDP.

4. Estimation

4.1. Data structure and analysis

Data are obtained from the Financial Analysis Made Easy, FAME database which is supplied by Bureau Van Dijk. The database provides financial and income statements data for private companies in the UK. It also contains information corresponding to bankruptcy filing details. The UK Standard Industrial Classification of Economic Activities 2003 code, UK SIC is also provided. The research population is defined in the framework of the Basel II Committee definition for small and medium corporations as the set of private companies with a maximum turnover of €50,000,000 (approximately £42992261.4)[2] in their last financial statement. (BIS3, 2004). The dataset is an unbalanced panel and covers the period from 2001 to 2008. It is unbalanced because the defaulted firms can exit at any year. In the sample, there are 599 corporations that experienced either financial distress (a court petition) or were legally closured (liquidation). The complete time path of such corporations is measured from their entry to their default state with the exception of 50 corporations that were left censored. The sample also includes 5607 corporations that are right censored. The exit time for these corporations is at the end of the experiment. All data are lagged one year in order to be sure that the defaulted corporations are observed at the beginning of the year in which default occurs. Another reason is to avoid the possible endogenous relationship between the leverage ratio and the instantaneous rate (Gujarati, 2003). One of the covariates, earnings volatility, is measured over two consecutive years before the estimation year. Consequently the estimation starts at year 2003. Data of corporations surviving in year 2009 are used for ex-ante prediction. For extreme outliers, the firm risk factors are winsorized 1% form the upper and lower percentiles of the sample. The yield curve data are collected from the Bank of England online database. The industry gross operating surplus and mixed income data is obtained from the Economic and Social Data Service (ESDS) database. ESDS supports both national and international aggregate (disaggregate) macroeconomic data.

Table 2: Data structure and frequency of failure event

Time series distribution of survival data structure for the sample of UK private firms at risk failure. The onset starts from year 2001 to exit year 2008. The data are presented in uniform intervals. The survivor and cumulative hazard probabilities and hazard rate are non-parametrically estimated.

Interval	Beg.	Total	No. Default	Survival	Cum. Failure	Hazard	Std Error
1	2	32609	50	0.9983	0.0017	0.0017	0.0002
2	3	27019	74	0.9953	0.0047	0.0031	0.0004

[2] The average exchange rate was 1.163 by European Central Bank in 2008.

[3] BIS (2004): *International Convergence of Capital Measurement and Capital Standards: A Revised Framework*. Basel Committee on Banking Supervision, Bank for International Settlements, June 2004.

3	4	21465	85	0.9908	0.0092	0.0045	0.0005
4	5	15961	113	0.9824	0.0176	0.0085	0.0008
5	6	10517	171	0.9612	0.0388	0.0218	0.0017
6	7	5173	106	0.9226	0.0774	0.041	0.004

4.2. Duration analysis

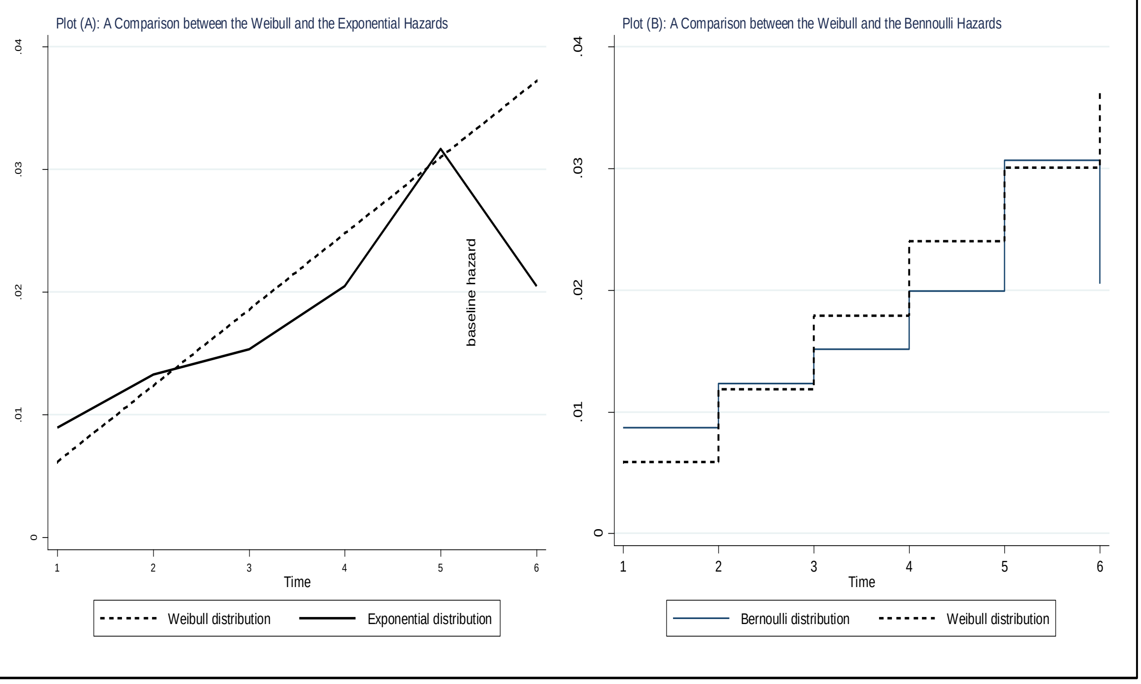
In this section, we provide a comparison between the estimated parametric and discrete time survival mixture models' performance in quantifying credit risk of UK private firms' portfolio. These multivariate mixture models are developed using the same reduced form framework (See Section 3) and database. Initially, the characteristics of the estimated hazard functions are outlined in order to describe the dynamic behaviour of the risk of default. This is followed by assessments of the joint effects of both idiosyncratic and systematic risk factors on estimating the conditional default probabilities. We then consider the role of common risk factors in modelling dependent defaults.

4.2.1 The hazard function

The hazard function outlines the main characteristics of the dynamic behaviour of the risk of default. It estimates the probability that firm i will fail in the current period conditional on not having failed in the previous period. The graphical representation of the hazard function provides insight into the overall impact of the effect of macroeconomic condition on UK private firms' propensity to survive. Figure 1 compares the Weibull hazard function to the Exponential hazard and the Benrnoulli time hazard models. Plot A of Figure 1 shows the characteristics of the instantaneous rate of the Weibull and the Exponential models. The Weibull model shows that the default rate is not constant but has steep upward trend. It estimates the accelerated failure time parameter α equal to 2 suggesting that the default rate increases at an increasing rate. Again, this is a violation of the proportional hazard assumption. A visual inspection of the Exponential model confirms that the likelihood of default varies over time. In Plot B, the

estimated hazard by the Bernoulli model shows a higher probability of failure in the first and the second intervals than the Weibull model. In contrast, from the third interval, the Weibull model shows the hazard to be increased rapidly. Both the Exponential and Bernoulli models show that the hazard rate declines at the last interval. However, the

Figure 1: Estimated Hazard Function for the Weibull vs. the Exponetial and the Bernoulli Models



Weibull model shows the opposite. These differences in the estimation of the instantaneous rate could come out of the differences amongst the distributions moment generating functions. So far, the models described the threat of default as an increasing function of time. These visual inspections indicate that the recent recession has significant implications on the dynamic behaviour of UK private firms.

4.2.2 The mixture models

We focus now on estimating the joint effect of the idiosyncratic and the systematic risk drivers as well as the common risk factors on the conditional default rate. Two functional forms are tested. The first aims to estimate the joint effect of firm-specific risk factors. The second examines whether a hybrid framework, that incorporates both firm-specific and macroeconomic risk drivers can measure the likelihood of default more accurately. Finally, we test whether dependence between defaults stem from a set of common risk factors.

Table 3 provides the estimated coefficients and standard errors for four alternative duration models. Models (1) and (2) are parametrically estimated while Models (3) and (4) follow a discrete time approach. To make statistical inferences about the models' performance, we rely on the Akaike Information Criterion (AIC), the Bayesian [4] information criterion (BIC) and the log likelihood measure (LL).

In Models (1) and (3), all covariates enter the models with the expected sign and are highly significant. A likelihood ratio test is undertaken on the null hypothesis that all coefficients are jointly not different from zero. The estimate of LR tests for the Weibull model, Model (1), is 606.56 and for the Bernoulli model, Model (3) is 1005.24. From these results, the null hypothesis is rejected. Thus we infer that the observable firm-specific risk factors pick up the relevant internal information regarding a firm's performance, financial pressure, growth opportunities, and market experience.

Table 3: Estimated Instantaneous Default Rate of the UK Private Firms at Risk during 2001-2008 Using Observable Risk Drivers Covariates

Models M(1) and M(2) are parametrically estimated while Models M(3) and M(4) are discretely measured. Models (1) and (3) consider idiosyncratic risk factors. Models (3) and (4) estimate total observed risk drivers including idiosyncratic and systematic risks. The duration models presented in the hazard metric and the standard errors are given in parentheses. All variables are lagged one year. Information Criterion (AIC) & (BIC) and log likelihood (LL) are given. α is the Weibull models' ancillary parameters. The significance of covariates level is* p < 0.05, ** p < 0.01, *** p < 0.001.

	The Weibull Models		**The Discrete-time Models**	
Covariates	M(1)	M(2)	M(3)	M(4)
$TLTA_{it-1}$	12.27^{***}	11.81^{***}	13.06^{***}	12.49^{***}
	(2.97)	(2.83)	(3.29)	(3.11)
$NITA_{it-1}$	0.0804^{***}	0.0780^{***}	0.0854^{***}	0.0762^{***}
	(0.03)	(0.03)	(0.04)	(0.03)
VOL_{it-1}	4.011^{**}	3.952^{**}	2.861^{*}	2.756^{*}
	(1.75)	(1.73)	(1.41)	(1.37)
$SIZE_{it-1}$	0.646^{***}	0.657^{***}	0.638^{***}	0.633^{***}
	(0.03)	(0.03)	(0.02)	(0.02)
AGE_{it-1}	0.482^{***}	0.482^{***}	0.492^{***}	0.481^{***}
	(0.04)	(0.04)	(0.04)	(0.04)
$SSIC3$	0.917^{***}	0.913^{***}	0.914^{***}	0.914^{***}
	(0.01)	(0.01)	(0.01)	(0.01)
$GOSMI_{ht}$		0.496		0.624
		(0.20)		(0.25)

[4] The BIC is estimated in STATA software by the following equation $BIC_k = -2\ln N\hat{L}(M_k) + df_k \ln N$ where K the number of regressors , $\hat{L}(M_k)$ the likelihood of the model. According to STATA smaller value of BIC is better.

$LIBOR_t$		0.0465*** (0.02)		0.375* (0.15)
$Ln-time$			4.896*** (0.59)	5.970*** (0.88)
AIC	3283.35	3230.16	4977.66	4973.17
BIC	3350.5	3314.1	5036.4	5048.7
LL	-1633.7	-1605.1	-2481.8	-2477.6
$Ch2$	955.2	1012.4	7150.5	7088.8
α	2.498	2.849		

The literature has found that macroeconomic predictors have a measurable impact on the event of default (e.g. Gieseck and Weber, 2004; Das et al., 2007 and Carling et al. 2007). We explore the potential role of the macroeconomic covariates on the likelihood of default. In Models (2) and (4) of Table 3, the likelihood of default is not only determined by idiosyncratic risk factors, but also by the state of economy. The yield curve appears to be an important indicator of portfolio credit risk. Both models show a negative association between the yield curve and the likelihood of default. These results indicate an expectation of negative economic growth, therefore an increase in the default rate. This result is consistent with Carling et al. (2007a). The operating surplus and mixed income is significant at 95% confidence interval in the discrete time model and appears significant at 90% in the parametric model. This result is not conclusive. Finally, I compare the models regarding their information content using the measures of fit AIC, BIC, and LL. The results in Table 3 show that the AIC and BIC of Model (1) exceeds Model (2) by 53 and 36 points respectively. Model (2) provides a more accurate estimation of a firm's credit quality than Model (1). Model (4) gives similar results. These results indicate that the effects of macroeconomic conditions should be considered when predicting portfolio credit risk.

In duration analysis, the mixture models are also known as frailty models. Shared frailty is the third dimension in modelling portfolio credit risk. Common risk factors that describe random variability across industry segments and their effect on joint defaults of many obligors are considered. Recent literature on portfolio credit risk has shown that ignoring unobservable risk factors will lead to underestimation of the conditional default probabilities. For example, Carling et al. (2007b) and Duffie et al. (2009) among others found that the common risk factors are important determinates in modelling of dependence between default events.

In order to examine the extent to which uncertainty surrounding the values of common risk factors influences the conditional default probabilities estimation of firms sharing the same frailties, we consider the impact of unobservable risk factors that cause dependence between default events of firms within the same industry sector. After accounting for unobserved industry heterogeneity, robustness checks are undertaken in order to explore the significant role of the observable macroeconomic risk factors. We estimate three nested alternative shared frailty models for each survival mixture approach. Table 4 reports the parameter estimates, shared frailty output and information criteria tests for the Weibull and discrete time mixture models. The Weibull models are W1-W3, and the discrete time models are Models C1-C3. Models (2) and (4) in Table 3 are used as a benchmark. The signs and statistical significance of the coefficients of the observable covariates are similar to those of Models (2) and (4). The exception is the coefficient on the GOSMI, which is significant at 90% in Model (2), it becomes highly significant after considering the frailty effect. The mixture models show that the common risk factors have a significant contribution in triggering the default event. The estimated frailty variance, θ, which measures the dependence between default events of firms in the same industry sector, are significant in all of the models. The Chi-square values of the test of the null hypothesis that the joint default events between firms in the same industry sector is zero are highly significant in all of the model types.

After allowing for the effects of industry frailty, a comparison of the parameters is carried out. There are a decline in the estimated coefficients and an increase in standard deviation of the models relative to those of Models (2) and (4). For instance, in Model W3, a firm propensity to default increases by almost 11 times, as the leverage increases by 1%. In contrast, Model (2) estimates the hazard to be almost 12 times. The reason of this discrepancy is that the mixture models relaxe the assumption of the independence between the survival times of the firms in the same industry sector which results in a reduction in the expected misspecification errors and gives more accurate parameter estimates. This highlights the significant contribution of the unobserved risk factors on estimating portfolio credit risk. It also indicates that Models (2) and (4) are misspecified.

To analyse the relative importance of macroeconomic risk drivers and shared frailty as an explanation for a default event, we test different model specifications. For the Weibull mixture model, results of the sensitivity tests verify that the joint effect of the GOSMI and LIBOR covariates have a significant impact on scaling conditional default probabilities. Similarly, both the LL and AIC measures show that Model W3 gives the best fit. On the other hand, BIC criterion favours of Model W2. This indicates that LIBOR factor is a strong predictor when it comes to accommodating macroeconomic conditions, and conditioning on the frailty effects. Parallel results were found for the Bernoulli mixture approach.

The framework of the two unrestricted models has important features. Its main advantage over prior work is in defining the role of the industry gross operating surplus and mixed income in forecasting credit risk portfolio. This disaggregate macroeconomic risk factor is an important new indicator of portfolio credit risk. Furthermore, consistent with Carling et al. (2007a) the yield curve is an important determinant of portfolio credit risk. This factor makes the model forward looking, directly reflecting the impact of macroeconomic conditions on a portfolio credit risk of private firms. Finally, after controlling for the observable risk drivers that predict conditional default probabilities, the two models provide strong evidence that UK private firms are exposed to a common latent risk factor driving default correlation for firms in the same industry sector. This finding is consistent with Carling et al. (2007 b) who found the unobserved industry risk factors to be important for predicting Swedish portfolio credit risk. The estimated interclass correlation (roh) for two firms in the same industry category is 36.5%. The implication is a high degree of dependence upon the survival time for firms in the same industry segment and default event.

5. The performance of the Models

In this section, we assess the predictive performance of Models W3 and C3 in forecasting private firms' portfolio credit risk for UK private firms. We use the information entropy, Shannon's entropy to compare the relative performance of the two models. The information entropy measures the uncertainty of an event occurrence represented by a probability distribution. It is used here to measure the degree of uncertainty that associated with the probability of time to default trigger. The assessment involves two stages. In the first stage, we estimate Shannon's entropy indicators in order to determine the incremental information that is added by the models for each year prior to failure and to identify a firm's financial status. In the second stage, we compare the performance of both models. The conditional information entropy ratio (CIER) is used in comparing the model performance (Zavgren, 1985 and Keenan and Sobehart, 1999).

5.1. Shannon's entropy:

The information entropy estimates the required amount of information which assists a decision maker to predict the uncertainty about the event occurrence. This amount of information is measured as the logarithm of the probability that an outcome occurs (default /non-default). The information entropy notation is as follows. Let (p) be the probability of a default trigger and $(1-p)$ be the probability of a censoring state. The probability of a default trigger is assumed not to be equally likely to happen as the probability of a censoring occurrence. Thus the entropy will be informative. If the probability is 50% the entropy will be non-informative. This is because both probabilities are the same for a decision maker. Finally, let the entropy quantity be $H[\wp]=H(p_1, p_2 ... p_n)$, introduced by Shannon's formula (14). Shannon's Entropy is the sum of the logarithm of the probability of default times the probability of the event occurrence. The key properties of the entropy function are being additive and permitting conditional probability estimation (Zavgren, 1985 and Keenan and Sobehart, 1999).

$$H[\wp]=H(p_1, p_2 ... p_n)=-\sum_{k=1}^{n} p_k \log p_k \qquad (14)$$

5.2. Conditional information entropy ratio (CIER):

The conditional information entropy ratio compares the degree of uncertainty of an event occurrence in two cases. The first case is estimating the unconditional model, the null model, in which there is no knowledge about the credit risk drivers. In other words, the model has no prediction in this form. The result is a degree of uncertainty about the default trigger. The second case lies in estimating the conditional model after adding an attribute vector that incorporates more information about the credit risk drivers. We calculate the conditional information entropy ratio using function (15).

$$CIER = 1-\left[\frac{H_0-(H_i|U_{it})}{H0}\right] \qquad (15)$$

Where: H_0 is the entropy value of the null model, H_i the conditional entropy and U_{it} is the knowledge that is added by the model. The rule of thumb is that, the higher the CIER, the greater the predictive power of the model (Keenan and Sobehart, 1999).

First, we estimate Shannon's entropy for each year prior to failure for both the default and non-default groups separately. The models' performances are tracked one year ahead: year 2009, for the non-default firms. Table 5 reports the output of average entropy predictions for the Wiebull Model W3 and the Bernoulli Model C3.

Table 4: Estimated Duration Models with Shared Frailty Effect

The instantaneous rates of the risk of bankruptcy are presented and the standard errors are given in parenthetical. Information Criterion (AIC) and (BIC) and log likelihood (LL) are given The Weibull accelerated failure time parameter is (α). θ is shared frailty parameter of the dispersion across industry categories. The correlation across the joint defaults of many obligors in the same industry is (ρ). The significance level of covariates is * p < 0.05, ** p < 0.01, *** p < 0.001.

	The Weibull models			The Discrete-time models		
	W1	W2	W3	C1	C2	C3
$TLTA_{it-1}$	11.60^{***}	10.60^{***}	10.95^{***}	11.56^{***}	10.69^{***}	11.18^{***}
	(2.87)	(2.60)	(2.69)	(2.72)	(2.49)	(2.63)
$NITA_{it-1}$	0.0484^{***}	0.0483^{***}	0.0455^{***}	0.0458^{***}	0.0430^{***}	0.0421^{***}
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
VOL_{it-1}	3.947^{**}	4.023^{**}	3.912^{**}	2.640^{*}	2.600^{*}	2.592^{*}
	(1.75)	(1.78)	(1.74)	(1.17)	(1.15)	(1.15)
$SIZE_{it-1}$	0.696^{***}	0.678^{***}	0.701^{***}	0.682^{***}	0.655^{***}	0.676^{***}
	(0.03)	(0.03)	(0.03)	(0.03)	(0.02)	(0.03)
AGE_{it-1}	0.511^{***}	0.513^{***}	0.513^{***}	0.514^{***}	0.505^{***}	0.508^{***}
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
$SSIC3$	0.903^{***}	0.911^{***}	0.903^{***}	0.901^{***}	0.911^{***}	0.903^{***}
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$GOSMI_{ht}$	0.230^{**}		0.145^{**}	0.227^{**}		0.201^{**}
	(0.13)		(0.09)	(0.13)		(0.11)
$LIBOR_{t}$		0.0485^{***}	0.0412^{***}		0.390^{*}	0.357^{**}
		(0.02)	(0.02)		(0.15)	(0.14)
$Ln-time$				4.879^{***}	5.822^{***}	6.043^{***}
				(0.57)	(0.81)	(0.85)
AIC	3258.3666	3212.2324	3201.4581	4941.4934	4942.9558	4935.7999
BIC	3342.3	3296.1	3293.8	5017.0	5018.5	5019.7
LL	-1619.2	-1596.1	-1589.7	-2461.7	-2462.5	-2457.9
$Ch2$	883.0	929.1	941.9	1111.4	1116.3	1108.0
α	2.496	2.835	2.863			
θ	0.829	0.460	0.943	0.971	0.822	0.972
ρ				0.365	0.291	0.365

Table 5 Average Shannon's Entropy for Discrete and Parametric Mixture Models

The table presents the average predictive values of the Shannon's entropies in bits of the mixture survival models. The entropies' predictive values were calculated based on the estimation sample in yearly bases. Panels A and B report the information entropies of the default group of Models W3 and C3 respectively. Similarly, Panel C reports the average information entropies for the active firms.

Panel A: Parametric Mixture Model (W3)

Years Prior to Failure (W3: Weibull)							
Failure Year	Y_{03}	Y_{04}	Y_{05}	Y_{06}	Y_{07}	Y_{08}	Overall
Y_{07-08}	0.185	0.198	0.171	0.057	0.067	0.018	0.136
Y_{06-07}	0.216	0.185	0.061	0.083	0.023		0.114
Y_{05-06}	0.178	0.076	0.085	0.026			0.091
Y_{04-05}	0.116	0.124	0.007				0.082
Y_{03-04}	0.093	0.025					0.059

Panel B:Discrete Mixture Model (C3)

Years Prior to Failure (C3: Bernoulli)							
Failure Year	Y_{03}	Y_{04}	Y_{05}	Y_{06}	Y_{07}	Y_{08}	Overall
Y_{07-08}	0.277	0.238	0.186	0.125	0.080	0.030	0.181
Y_{06-07}	0.268	0.220	0.143	0.091	0.041		0.153
Y_{05-06}	0.190	0.136	0.096	0.040			0.116
Y_{04-05}	0.116	0.071	0.025				0.071
Y_{03-04}	0.106	0.041					0.073

Panel C: Models W3 & C3

Years Prior to Active (Weibull: W3)							
Active	Y_{03}	Y_{04}	Y_{05}	Y_{06}	Y_{07}	Y_{08}	Overall
Y_{03-08}	0.057	0.071	0.060	0.016	0.026	0.006	0.039
Years Prior to Active (Bernoulli: C3)							
Y_{03-08}	0.12	0.095	0.068	0.041	0.025	0.009	0.06

Panels A and B of Table 5 show the incremental information flow that is added by Models W3 and C3 respectively, as failure is approached. The expected values of entropies show that the degree of certainty over the default occurrence increases. For example, the entropies estimated by both models show that the predictive survival time for defaulted firms in year 2008 declines from 0.277 bits in year 2003 to 0.030 bits in year 2008. This result indicates that the models are informative and able to detect firms that face financial difficulties almost five years prior to failure. This result is consistent with Zavgren (1985) who estimated the probability of bankruptcy on cross-sectional bases for five years using the logistic regression technique. Panel C shows the expected values of the entropies for the active firms. However, Model W3 shows that the entropy values fluctuate from one year to another. The degrees of uncertainty of these firms to default are significantly lower than those of the failed ones. Model C3 shows a steady decrease in uncertainty in the survival time of the active firms. The result of the tracking sample shows similar behaviour. These findings reveal that Models W3 and C3 can predict the firms ahead to default early.

Table 6 Measuring Models' Performance by Conditional Entropy Ratio

The table presents the yearly conditional entropy ratios of Models W3 and C3.The predictive performance of the models estimated for one year ahead by tracking the active firms' performance in year 2009.

Year	Y_{03}	Y_{04}	Y_{05}	Y_{06}	Y_{07}	Y_{08}	Average
Weibull	0.488	0.629	0.288	0.762	0.720	0.476	0.561
Bernoulli	.0277	0.399	0.438	0.562	0.653	0.688	0.503
Tracking sample results for active firms' performance in Year 2009							
Model	Avg. Entropy	CIER Ratio					
Weibull	0.370	0.132					
Bernoulli	0.254	0.0608					

Table 6 reports the CEIR ratio for each year for Models W3 and C3. The results show that the models are informative and hold high predictive power. The Weibull model's predictive power is higher than that of the Bernoulli model in all years except for 2005 and 2008. In contrast the Bernoulli model shows consistently increasing predictive power in all years. Finally, the Weibull mixture model's overall average of CIER is 56%. This is higher than the CIER Bernoulli mixture model which is 50%. The tracking sample shows a similar pattern. The result shows that the parametric mixture survival model predicts the event of default more accurately than the discrete time mixture model.

6. Conclusion

Modelling portfolio credit risk is of critical importance for financial institutions and banking regulatory and supervisory authorities. In that respect Credit Metrics and CreditRisk+ models have been widely used to forecast portfolio credit risk for public firms. Researchers have focused on modelling dependence across defaulters. These approaches are not suitable for private firms. The contribution of this paper lies in the evaluation of two new credit risk portfolio modelling techniques and their suitability for forecasting private firms' portfolio credit risk. These techniques are the Weibull and Bernoulli mixture models.

Applying a new dataset of UK private firms, to each of the models, estimates are obtained of the instantaneous default rate within an identical framework. This framework incorporates: (i) a vector of idiosyncratic time varying risk factors; (ii) a vector of aggregated and disaggregated macroeconomic covariates; and (iii) a vector of unobserved risk factors that captures the default dependence across firms within the same industry sector. A high value of leverage and high volatilities in cash flows are shown to increase the instant rate of default. In contrast, profitability and industry median sales are negatively associated with the intensity rate. Furthermore, a firm's size and age are important to forecasts of portfolio credit risk. The hazard models also show the yield curve to have a significant impact in determining the hazard rate. As a novel feature, the industry gross operating surplus and mixed income as a predictor is identified as an important determinant of portfolio credit risk. Finally, the role of the unobserved shared frailty is a significant factor in measuring dependence across firms within the same industry sector.

The performance of the survival mixture models is compared using Shannon's entropy measure. The analyses show both models to be informative. They are able to predict the portfolio credit risk almost five years ahead. The tracking sample of the survival firms shows similar results. In terms of the conditional information entropy ratio, the Weibull mixture model gives more accurate in predicting portfolio credit risk than the Bernoulli model.

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