Integration analysis of Central and Eastern Europe countries: A Copula approach

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Abstract

In recent years the research conducted in both the academic field and European institution has intensively addressed the topic related to financial and economic integration of the Central and Eastern Europe's (CEE) countries. The importance of this topic occurs from the fact that financial and economic integration have implications for portfolios allocation, trading activities, asset pricing, external competiveness, price's stability and other micro and macro aspects. Three of the nominal convergence's criteria of the Maastricht Treaty are related to the evolution of exchange rate. Therefore dependence among exchange rates of CEE countries represents an important measure of the integration with the Euro Zone's economy. Empirical observation evidenced a very asymmetric dependence in exchange rates between periods of appreciation and depreciation. Both the distinct reactions of Central Banks and the portfolios restructuring due to adverse selection are the main probably causes for the asymmetry in exchange rates dependences. From a technical view point, the capturing of this asymmetric behaviour is very difficult with the standard statistical tools such multivariate distribution or kernels. This fact is owing to some stylized facts of the exchange rates as autocorrelation, heteroskedasticity, leverage effect or fat tails phenomenon.

Keywords: economic integration, copulas, FIGARCH, EVT, monotonicity.

1. Introduction

Many researchers have elaborated different analysis regarding the financial and economic integration, this process having implications for portfolio allocation and asset pricing at micro and macro level and also for external competiveness and price's stability. Both the distinct reactions of Central Banks and the portfolios restructuring due to adverse selection are the main causes for the asymmetry in exchange rates dependences. Each country attends to sustain the level of exports and so is tempted to depreciate the domestic currency. This tendency is strongly correlated with the price stability preference and leads to an appreciation of national currency. The fact that both objectives could be achieved alternatively may produce asymmetric dependence between the analyzed exchange rates.

The countries from the European Union are attending to accede the Euro Zone as soon as the economic convergence is achieved. The economic convergence is referring to the nominal and real convergence. The certain conditions necessary for accession Euro Zone which are explicitly mentioned as nominal criteria are influenced by the national monetary authority (bounds fixed for inflation, interest rates and inflation). On the other hand, many of the empirical results have not confirmed the Purchasing Power Parity (PPP) theory, which means that exchange rates affect in different ways the cost of consumption across countries. If the Uncovered Interest Rate parity is not confirmed we assist to a flight to opportunity of the capital, considering that higher interest rate lead to a capital inflow and to an appreciation of the national currency. Therefore a higher degree of integration is reflected by a higher correlation between behaviour of monetary policies adopted by national central banks and ECB. Considering that the exchange rate is located at the congruence of UIP and PPP, the lines of domestic monetary policies are reflected by the evolution of exchange rates. So the dependence of selected exchange rates from CEE describes the state of integration to Euro Zone's economy. If financial markets aren't integrated, some speculative opportunities across the economies may appear and affect not only the investors' portfolios but also the real economy. Therefore the manner in which the real economy absorbs the external shocks is reflected by the exchange rate. Taking into account the remark mentioned previously and the fact that adjustments of national monetary policies will occur as responses to ECB's policy changes, the dependence between domestic currencies and euro represents an important measure of integration.

A series of far-back observation have been reported the non-normality of distribution in the case of almost economic and financial variables. In this sense Mandelbrot (1963) highlighted for the first time existence of *leptokurtosis* effect, he indicating the fact that large changes tend to be also followed by several large changes of either sign (*volatility clustering* effect). Later in 1976, Black underlined the *leverage* effect as tends of assets prices correlates negatively with volatility movements. Furthermore in 1998 Ramchand and Susmel emphasized the evidence of common volatility tends across markets that could lead to contagion effects.

In this paper we aimed to analyze the use of copulas in financial and economic application, namely to investigate the financial integration of exchange rates. For this purpose we have used a portfolio consisting in four currencies from Central and Eastern Europe. Due to some stylized facts observed in exchange rate series we have filtered the data with a FIGARCH model. To model the univariate distributions of the four exchange rates we engaged a semi-parametric approach. Also we used some methods to analyze the monotonic relation regarding the co-movements of the four exchange rates.

The paper is structured as follows, Section I, presents the methodology used, in the second Section the data and Empirical results are displayed. In the final part we have presented the conclusions of this research and the annexes with the results.

2. Literature Review

The use of copula functions in modelling the economic and financial processes has recorded a fast growth in recent years, even though the first applications of copulas date back to late 70s.

Copula concept was firstly introduced in mathematics by Sklar (1959) who defined a theorem according to which any multivariate joint distribution can be decomposed into a dependence structure and its *n* marginal distributions. 1959 actually refers only to the appearance of this theorem for decomposition of multivariate distributions. Sklar explicitly calls *copula* concept in 1996 as a function that satisfies the theorem formulated by him in 1959. Epistemology of *copula* word comes from Latin and means connection or link. But until 1996, the functions that fulfils the Sklar's theorem from 1959 circulated under different names as: *dependence function* (Deheuvels, 1978), *standard form* (Cook and Jonson, 1981) or uniform representation (Hutchinson and Lai, 1990).

Copula was used for the first time in the joint-life models by of Joe Clayton (1978), studying the bivariate life tables of sons and fathers. Others important contributions to the Clayton's models have been made by Cook and Johnson (1981) and Oakes (1982).

After 2000 a wave of copula applications works in finance came due the growing interest for risk management. Rockinger and Jondeau (2001) used Plackett copula to analyze the dependence among S&P500, Nikkei 225 and some European stock indices. Patton (2002) computed the first conditional copula in order to allow first and second order moments of distribution function to vary over time. Patton (2004) used conditional copulas to analyze the asymmetric distribution between Deutsche Mark and Yen against Dollar. Jondeau and Rockinger (2006) use time-varying Gaussian and Student copula to model the bivariate dependence between countries, while for univariate marginal distributions propose Skewed-t GARCH models.

3. Methodology

Extreme Value Theory (EVT) represents a domain of the probability theory that deals with the study of extreme events. Such events are characterized by extreme deviations from the normal median of their probability distributions. More exactly, the EVT studies and models the behaviour of distributions in their extreme tails. These rare events are described by a thickening of the tails that determines an excess of the *kurtosis* above the characteristic value for of the Gaussian distribution. Therefore the apparitions of the so-called *fat tails* are also known as the *leptokurtic* distributions. An important remark about the modelling of extreme events is that it is not necessary to make a prior specification or assumption about the shape of the studied distribution. In literature exists two main theories that provided two approaches for applying the EVT theory: Generalized Extreme Value distributions and Generalized Pareto Distribution.

Generalized Pareto Distribution

This approach supposes to set a threshold value such that all the realizations over this limit are considered and also modelled as extreme events. The main idea behind this method called also *peak-over-threshold* is that difference between the realized extreme events and the set threshold are considered as excesses. Therefore peak-over-threshold approach involves the estimating of a conditional distribution of the excesses situated above a given set threshold. For a random vector $X = (X_1, \dots, X_n)$ with a distribution function φ, let consider the threshold V as $V < x_{\varphi}$. Thus φ_v denotes the distribution function of excesses over the threshold V:

$$(1)\varphi_v(x) = \mathrm{P}(X - v \leq x | X > v), x \geq 0.$$

Independently Balkema and de Haan *et al.* (1974) and Pickands *et al.* (1975) provided theorems that demonstrated since the threshold v was estimated and for a sufficiently high v to satisfy $v \to \infty$, the conditional distribution φ_v can be fit using *Generalized Pareto Distribution* (GPD). Therefore we will define the following relationship regarding the fitting of conditional distribution function φ_v using GPD:

$$(2)\ \varphi_v(x) \cong G_{\xi,\gamma,\beta}(x), v \to \infty, x \geq 0,$$

where

$$(3)G_{\xi,\sigma,\beta}(x) = \begin{cases} 1 - \left(1 + \xi \dfrac{x - \beta}{\gamma}\right)^{-\frac{1}{\xi}}, for\ \xi \neq 0 \\ 1 - e^{-\frac{(x-\beta)}{\gamma}}, \qquad for\ \xi = 0 \end{cases}$$

and

$$(4)\ x \in \begin{cases} [\beta, \infty], for\ \xi \geq 0 \\ \left[\beta, \beta - \dfrac{\gamma}{\xi}\right], for\ \xi < 0 \end{cases}$$

In the above relations, the parameter γ denotes the scale parameter, while β represents the location parameter. An important observation is that in the case when $\beta = 0$ and $\gamma = 1$, then the relations (8) and (9) constitute a standard GPD.

The relationship between GEV and GDP approaches can be expressed as following:

$$(5)\ G_{\xi,}(x) = 1 + \log \Psi_{\xi}(x), for\ \log \Psi_{\xi}(x) > -1.$$

Copula models

In probability field, a joint distribution can be decomposed in a dependence structure that represent a copula and into marginal distributions related to the number of random variables. So, the copulas describe the dependence between two or more random variables, with different marginal distributions. The main advantage of using copulas is that this procedure allows the modelling of both parametric and non-parametric marginal distributions into a joint risk distribution. More exactly the dependence structure of these joint risk distributions are characterized more in detail by copulas than the information provided by a simple correlation matrix. Mathematically speaking, in order to notations used by Nelsen (1999), the notion of copula can be described as following:

Definition. A function $\boldsymbol{C}$ **:** $[0,1]^n \to [0,1]$ is a copula with *n* dimensions only if it follows the properties:

i) $\forall\, u \in [0,1]$, $C\,(1, \dots, 1, u, 1, \dots, 1) = u$;
ii) $\forall\, u_i \in [0,1]$, $C\,(u_1, \dots, u_n) = u$ if at least one of the u_i's equals zero;
iii) C is *n*-increasing and grounded, therefore the *C*- volume of every box is positive only if its vertices are ranging in $[0,1]^n$.

Also there have to be mentioned that if a function fulfils the property *i*) then respective function is grounded. The name of "copula" attributed to the function $\boldsymbol{C}$ results from the following theorem.

Sklar's Theorem (1959). *If F is a n-dimensional joint distribution function with the continuous marginal distributions* $F_1, \dots, F_n$, *then there exist a unique n-copula C* $[0,1]^n \to [0,1]$, *such that*:

$$(6)\ F(x_1, \dots, x_n) = C\big(F_1(x_1), \dots, F_n(x_n)\big),$$

for every $x_1, \dots, x_n \in \bar{R}$. A very important remark about Sklar's theorem is that C is unique only if the $F_1, \dots, F_n$ are continuous. In conclusion, the theorem mentioned above shows that any joint distribution can be dimensioned in a copula and into marginal distribution functions. In 1996, Sklar defined copula like "a function that links a multidimensional distribution to its one dimensional margins".

Inversely, if there are known the density functions for the *n*-dimensional joint distribution and marginal distributions, then the copula is given by the following formula:

$$(7)\ C(u_1, \dots, u_n) = F\left(F^{-1}{}_1(u_1), \dots, F^{-1}{}_n(x_n)\right),$$

as Nelsen (1999) mentioned that above relation hold only if the $F_1, \dots, F_n$ are continuous. Also Nelsen (1999) shown that for a bidimensional distribution function, the two margins F_1 and F_2 are given by $F_1(x_1) = F(x_1, +\infty)$, respectively $F_2(x_2) = F(+\infty, x_2)$.

Other powerful property registered by all the copulas is referring to theirs invariance:

Invariance Theorem. *Let define n continuous random variables $Y_1, \dots, Y_n$ that have a C copula. So, if $g_1(Y_1), \dots, g_n(Y_n)$ are increasing functions on the range of $Y_1, \dots, Y_n$, then the random variables $X_1 = g_1(Y_1)$, …, $X_n = g_n(Y_n)$ have also the same copula C.*

More exactly, the above theorem underlines one of the most important advantages of the modelling using copulas, namely that the dependence structure is insensitive to the monotonically changes of random variables.

In accordance with the Lipschitz's condition of continuity on $[0,1] \times [0,1]$, we will define the following property of copulas:

Theorem. *Let consider an n-copula C. Then for all $u_1, \dots, u_n \in [0,1]$ and all $v_1, \dots, v_n \in [0,1]$:*

$$(8)\ |C(v_1, \dots, v_n) - C(u_1, \dots, u_n)| \le |v_1 - u_1| + \dots + |v_n - u_n|.$$

The above relation is given by the property that copulas are *n*-increasing. Roughly speaking, the theorem states that every copula ***C*** is uniformly continuous on its domain.

Other important property of these dependence structures refers to the partial derivatives of a copula with respect to its variables:

Theorem. *Given a n-dimensional copula C, for every* ϵ [0,1], *the partial derivative* $\partial C/\partial v$ *exists for every* $v \epsilon$ [0,1], *such that:*

$$(9)\ 0 \leq \frac{\partial C}{\partial v}(u, v) \leq 1\ .$$

Also it have to been mentioned that the analogous is true for $/\partial v$. Additionally the functions $u \rightarrow C_v(u) = \partial C(u, v)/\partial v$, respectively $v \rightarrow C_u(v) = \partial C(u, v)/\partial u$ are defined and non-decreasing almost everywhere on [0,1].

The most important examples of elliptical copulas are the Gaussian and Student copulas. In fact, from technical viewpoint, these two copulas are very close to each other. Furthermore the two copulas become closer and closer in their tail only when the number of freedom degrees of Student copula increases.

Gaussian **(Normal) Copula**

According to the notations used by Yannick Malevergne and Didier Sornette (2005), a *Gaussian n*-copula ***C*** can be defined as following:

$$(10) C_{\rho,n}^{Gauss}(u_1, \ldots, u_n) = \varphi_{\rho,n}(\varphi^{-1}(u_1), \ldots, \varphi^{-1}(u_n))\ ,$$

where φ denotes the standard *Gaussian* distribution, $\varphi_{\rho,n}$ is the *n*-dimensional Normal distribution with correlation matrix ρ. The *Gaussian* copulas are derivate from the multivariate Gaussian distributions.

So the density function of the Normal copula is given by:

$$(11) c_{\rho,n}^{Gauss}(u_1, \ldots, u_n) = \frac{\partial C_{\rho,n}(u_1, \ldots, u_2)}{\partial u_1 \ldots \partial u_2}.$$

Noticing with $y^t(u) = (\varphi^{-1}(u_1), \ldots, \varphi^{-1}(u_n))$, then it will result**:**

$$(12) c_{\rho,n}^{Gauss}(u_1, \ldots, u_n) = \frac{1}{\sqrt{det\ \rho}} exp(-\frac{1}{2} y^t(u)(\rho^{-1} - I)y(t).$$

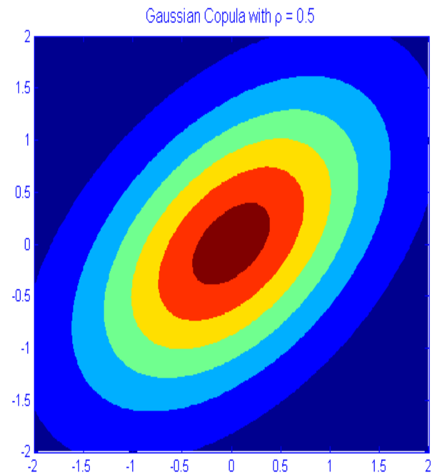

Figure1 :Contour Plots of Gaussian Copula

Archimedean Copulas

Unlike the *meta-elliptical* copulas, Archimedean copulas are not derived from the multivariate distributions through the use of Sklar's theorem. In addition the Archimedean copulas can be defined as the closed-form solutions. A copula belongs to Archimedean family if it fulfils the properties:

Definition. *Given* φ *as a continuous function from* $[0,1]$ *onto* $[0,\infty]$, *strictly decreasing and convex, such that* $\varphi(1) = 0$ *and* $\varphi^{[-1]}$*is a pseudo-inverse of* φ*:*

$$(13)\varphi^{[-1]}(t) = \begin{cases} \varphi^{[-1]}(t), if\ 0 \leq t \leq \varphi(0) \\ 0, \quad if\ t \geq \varphi(0) \end{cases},$$

then the function

$$(14)\ C(u,v) = \varphi^{[-1]}(\varphi(u) + \varphi(v))$$

is an Archimedean copula with generator φ .

A strict condition for ***C*** to be an Archimedean copula is that :

$(-1)^k \frac{d^k \varphi^{[-1]}(t)}{dt^k} \geq 0$, as $\forall\ k = 0,1,\ldots,n$, or more exactly if $\varphi^{[-1]}$ is monotonic.

Thus we can generalize the relation for ***n***-Archimedean copulas:

$$(15)C_n(u_1,\ldots,u_2) = \varphi^{[-1]}\big(\varphi(u_1) + \ \ldots\ + \varphi(u_n)\big).$$

The main idea behind Archimedean copulas is that the dependence structure among n variables is represented by a function of a single variable, which is the generator φ.

From the large Archimedean family of copulas, we will mention the most known of these ones:

Clayton Copula

Joe Clayton (1978) has used for the first time the concept of copula in the joint-life models, studying the bivariate life tables of sons and fathers. Others important contributions to the Clayton's models were developed by Cook and Johnson (1981) and Oakes (1982). A Clayton copula can be defined as following:

$$(16) C_\theta^{Clayton}(u, v) = \max(\left[u^{-\theta} + v^{-\theta} - 1\right]^{-\frac{1}{\theta}}, 0), \theta \in [-1, \infty),$$

having the role of a limit copula, with the generator $\varphi(t) = \frac{1}{\theta}(t^{-\theta} - 1)$, whose *Laplace* transformation is a *Gamma* distribution.

Thus the density function of the Clayton copula is:

$$(17) c_\theta^{Clayton}(u, v) = (1 + \theta)[uv]^{-\theta-1}(u^{-\theta} + v^{-\theta} - 1)^{-2-\frac{1}{\theta}}.$$

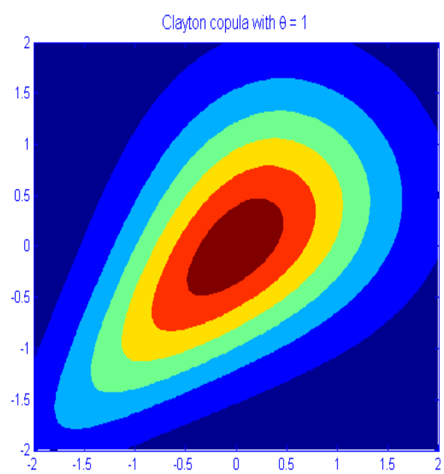

Figure 2:Contour Plots for Clayton Copula

Symmetrised Joe-Clayton (SJC)-Copula

Starting from ‘BB7’ copula of Joe (1997) or Joe-Clayton as it is also known in literature, Patton (2004) introduced the Symmetrised Joe-Clayton(SJC) copula:

(18) $C^{SJC}(u,v|\tau^U,\tau^L) = 0.5 * (C^{SJC}(u,v|\tau^U,\tau^L) + C^{SJC}(1-u,1-v|\tau^U,\tau^L) + u + v - 1).$

Unlike originally ‘BB7’, the Symetrised Joe-Clayton copula may take into account for completely presence or absence of asymmetry in the tail dependence. In fact the SJC copula represents a special case of the Joe-Clayton when $\tau^U = \tau^L$. Empirical facts indicate SJC copula as a more interesting choice to model the dependence in economic and financial processes.

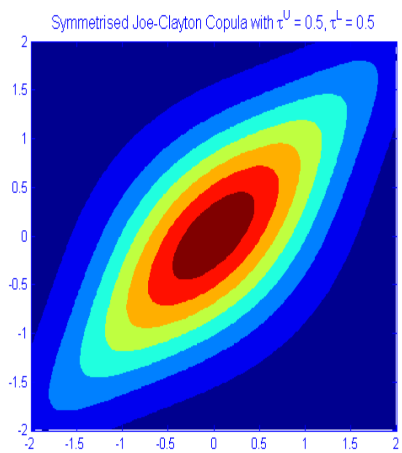

Figure 3: Contour Plots for SJC Copula

<u>Canonical Copula-Vine</u>

From Bayes Law is well-known the fact that a multivariate joint distribution can be decomposed using iterative conditioning as following:

$$(19)\quad f(y_1,\dots,y_n) = f(y_1) * f(y_2|y_1) * f(y_3|y_2,y_1) * \dots * f(y_{n-2}|y_1,\dots,y_{n-1}) * f(y_n|y_1,\dots,y_{n-1})$$

Thus we can decompose the first conditional density in terms of copula:

$$(20) f(y_2|y_1) = c_{12}\big(F_1(y_1),F_2(y_2)\big)f_2(y_2)$$

Further we can continue with the second conditional density as:

$$(21)\ f\big(y_3|y_2,y_1\big) = c_{23|1}\left(F_{2|1}(y_2|y_1),F_{3|1}(y_3|y_1)\right)f(y_3|y_1)$$

where

$$(22) f(y_3|y_1) = c_{13}\big(F_1(y_1), F_3(y_3)\big) f_3(y_3).$$

This model of conditional copula are called *Canonical Vine Copula* and was introduced by Bedford and Cooke (2002) and in financial application were firstly used by Aas (2007) and Berg and Aas (2007). The notations called here are according those used by Aas (2007). A general form of Canonical Vine Copula can be defined as:

$$(23)\ c(y_1, \dots, y_n) = \prod_{j=1}^{n-1}\prod_{i=1}^{n-j} c_{j,j+i|1,\dots,j-1}\left(F\big(y_j|y_1, \dots, y_{j-1}\big), F\big(y_{j+i}|y_1, \dots, y_{j-1}\big)\right).$$

Copula-GARCH Model

A major criticism of the copula models in the favour of multivariate GARCH model was that former suppose a static measure of dependence. Even though the separately modelling of the marginal distribution and dependence structure provides a higher degree of robustness over time of the copula parameters, the empirical findings proved that the high frequency data records a continuously switching of the regimes. Thus in 2001, Patton took the first initiative to extend the copula function to conditional case, in order to account the impact of the past information on the state of copula parameters. He introduced for the first time the concept of time varying dependence which does nothing to incorporate the heteroschedasticity in dynamic copula modelling. So to extend the Sklar's theorem to conditional cumulative distribution functions, Patton has defined the following σ-algebra:

$$(24)\sigma - \text{algebra} = \sigma\{\mathrm{y_{1t-1}}, \mathrm{y_{2t-1}}, \dots, \mathrm{y_{nt-1}}, \mathrm{y_{1t-2}}, \mathrm{y_{2t-2}}, \mathrm{y_{nt-2}}, \dots\}$$

for $t = 1, \dots, T$. In fact the above equation tell us that σ-algebra is generated by all the past information up to time t. Therefore the Sklar's theorem cam be expressed as:

$$(24) F(y_{1t}, \dots, y_{2t}|\sigma - \text{algebra}) = C_t(F_{1t}(y_{1t}|\sigma_t - \text{algebra}), \dots, F_{nt}(y_{nt}|\sigma_t - \text{algebra})|\sigma_t - \text{algebra})$$

More exactly the main idea behind the equation (24) and (25) is that in modelling of the marginal distributions, the conditional mean follows an autoregressive process, while the conditional variance is modelled as a GARCH(1,1) process.

Further I will define the time-varying equations for Gumbel and SJC copulas which I will use later to model the dependence between exchange rates over the analyzed period. A general form of the conditional dependence can be expressed as:

$$(26)\rho_t = \bar{\Lambda}\left(\omega + \beta\rho_{t-1} + \alpha\frac{1}{m}\sum_{j=1}^{m}\emptyset^{-1}(u_{t-j})\emptyset^{-1}(v_{t-j})\right),$$

where $\bar{\Lambda} \equiv \frac{1-e^{-x}}{1+e^{-x}}$ is the modified logistic transformation that holds the dependence parameter ρ_t in the interval (-1,1). The right hand of above equation contains an autoregressive term $\beta\rho_{t-1}$, a forcing variabile and m denotes the window length. Equation (60) was designed for modelling dynamic Elliptical copulas.

For non-Elliptical copulas Patton proposed the following general form to model the evolution of the dependence parameter:

$$(27)\theta_t = \Lambda\left(\omega + \beta\theta_{t-1} + \alpha\frac{1}{m}\sum_{j=1}^{m}|u_{t-j} - v_{t-j}|\right),$$

where Λ is an appropriate transformation function designed to keep the dependence parameter in its domain. This transformation function can take different forms as: $\frac{1}{1+e^{-x}}$ for tail dependence, e^x for Clayton copula or $e^x + 1$. For SJC copula Patton proposed the following dynamic equations:

$$(28)\tau_t^U = \Lambda\left(\omega_U + \beta_U\tau_{t-1}^U + \alpha_U\frac{1}{m}\sum_{j=1}^{m}|u_{t-j} - v_{t-j}|\right)$$

$$(29)\tau_t^L = \Lambda\left(\omega_L + \beta_L\tau_{t-1}^L + \alpha_L\frac{1}{m}\sum_{j=1}^{m}|u_{t-j} - v_{t-j}|\right),$$

where τ_t^U and τ_t^L represents the upper, respectively lower tail dependence and $|u_{t-j} - v_{t-j}|$ denotes the mean absolute difference over the past observations. Thus the window length can be seen as a switching parameter of the forcing variable..A very important remark is that the Patton's model for conditional dependence supposes the time-varying of parameters according to defined dynamic equation, while the functional form of copula remains constant over horizon. Instead Rodriguez (2003) proposed a Markov switching regime for the functional form of copula.

In the fields of economics, finance or actuarial risks it exists a lot of approaches used to estimate the parameters of copulas. Broadly speaking we can divide such techniques of copulas' estimations in three main categories: nonparametric, semi-parametric and parametric. In literature exists different parametric methods used for the estimation of the copula's parameters.

<u>Canonical Maximum Likelihood (CML) Estimation</u>

The CML approach supposes an estimation of the copula's parameters, without any assumption about the parametric form of marginal distributions. Thus CML technique uses nonparametric approaches such the *kernel* estimation for the modelling of marginal distributions. Maashal and Zeevi *et al.*(2002) proposed an estimation algorithm based on crossing of the following two steps:

i) Firstly using the empirical marginal distribution, a given dataset $(y_1^t, \dots, y_n^t)$ with $t = 1, \dots, T$ is transformed into uniform variates $(\hat{u}_1^t, \dots, \hat{u}_n^t) = \left(F_1(y_{1t}), \dots, F_n(y_{nt})\right)$;

ii) Secondly it is estimated the vector of copula's parameters θ as:

$$(30)\,\hat{\theta}_{CML} = \arg\max\ L(\theta), \text{ where}$$

$$(31)\ L(\theta) = \sum_{t=1}^{T} \ln c(\hat{u}_1^t, \dots, \hat{u}_n^t).$$

Therefore the main advantage of the CML approach is the easily of its utilization from the numerical viewpoint.

<u>Fractional Integrated GARCH model</u>

To mimic the behavior of the correlogram of the observed volatility, Baillie, Bollerslev, and Mikkelsen (1996) (hereafter denoted BBM) introduce the Fractionally Integrated GARCH (FIGARCH) model by .The conditional variance of the FIGARCH (*p*,*d*,*q*) is given by:

$$(32)\quad \sigma_t^2 = \omega[1-\beta(L)]^{-1} + \{1-[1-\beta(L)]^{-1}\phi(L)(1-L)^d\}\varepsilon_t^2 \text{ ,with } 0 \le d \le 1.$$

If d = 0, the FIGARCH (0, d, 1) process reduces to a GARCH (0,1) process and if d = 1, the

FIGARCH process becomes an integrated GARCH process. Baillie, Bollerslev and Mikkelsen[3] (1996) note that the effects of a shock on the conditional variance of FIGARCH (p, d, q) processes decrease at an hyperbolic rate when $0<d<1$.Thus, by analogy with ARFIMA processes, the long-term dynamics of the volatility is taken into account by the fractional integration parameter *d*, and the short-term dynamics is modeled through the traditional GARCH parameters.

Invertible processes have a degree of integration between -0.5 and 0.5 and the effect of shocks decays at the slow rate to zero .When d is zero the process is stationary also known as short-memory and the effect of shocks decays geometrically. For d =1 the process has a unit root and when d is positive but smaller than 0.5 the process implies long memory and for $-0.5<d<0$ the process exhibits negative dependence between distant observations, so called anti-persistence.

4. Data and Empirical results

In this paper we have used daily closing rates of four exchange rates, namely the Euro against Czech Koruna (CZK), Hungarian Forint (HUF), Polish Zloty (PLN) and Romanian New Leu (RON). The analyzed data sample is provided by Bloomberg, ranging between February 1999 and February 2010. From a macro view, the four countries present a high degree of homogeneity due to several facts as all of them are intending to attend to Euro Zone in the near future and all have adopted inflation target regime to conduct the monetary policy. Also have to mention here that a very important point required by inflation target regime is the assurance by local monetary institutions to a high flexibility of exchange rates. Instead the four countries are using different types of exchange rate regimes:

Exchange Rate Regimes	
Czech Republic	Classical administrated floating
Hungary	Target zones against Euro
Poland	Independent floating
Romania	Managed floating

[3] Baillie, R.T., T. Bollerslev and H.-O. Mikkelsen, 1996, Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity, Journal of Econometrics, 74, 3-30.

For this reason and due to different changes in the social, political and economic medium of each analyzed country, the four exchange rates have recorded very distinct evolutions in the selected period. While the Czech Koruna has registered the most important appreciation against Euro, Romanian New Leu has situated at the opposite pole. In the same timeframe the other two exchange rates, EUR/HUF and EUR/PLN exhibited a smooth tendency of decreasing to the beginning of financial crisis in September 2008, when all the four currencies from Central and Eastern have recorded significant depreciations against Euro.

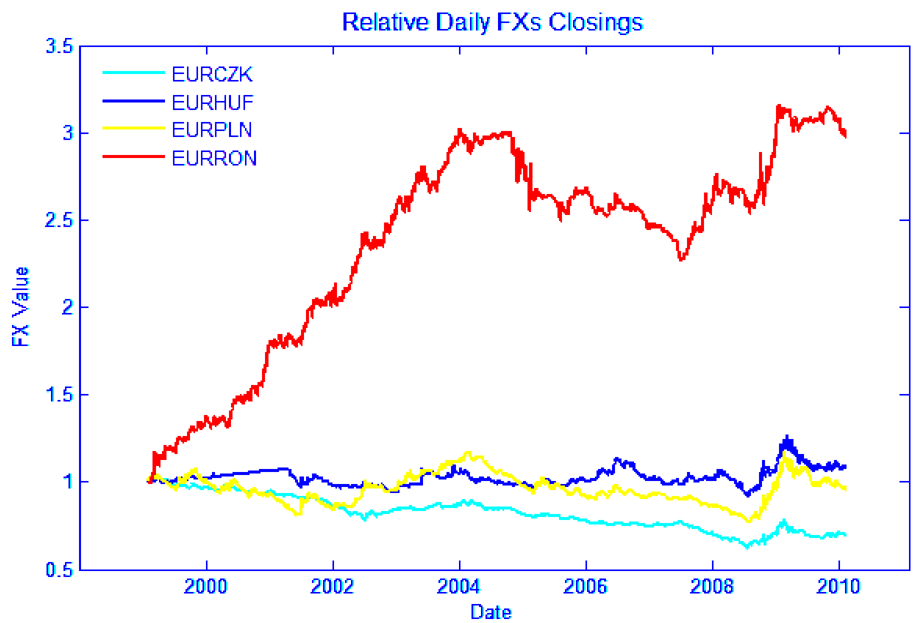

Figure 4: Relative Daily FX Data.

Thus the various patterns of the four exchange rates are explained by both different structural reforms adopted by CEE economies and distinct policies used to stabilize the nominal exchange rates and domestic inflation. In order to analyze the dependence among the evolution of the analyzed exchange rates we used the returns of these rate computed as: $R_{t+1} \approx \ln\left(\frac{S_{t+1}}{S_t}\right)$, where R_{t+1} denotes the return and S_t is daily exchange rate.

Looking at descriptive statistics (Annexes, Table 1) we can observe quite distinct features of the exchange rate returns, as the above plot indicated. Thus the EUR/RON recorded much larger extreme values compared with those registered by the other analyzed currencies. In addition, the standard deviation of EUR/RON was the highest one among the four exchange rates, while the EUR/CZK recorded the smallest one, but the beginning of the financial crisis pushed the Czech Koruna and Hungarian Forint to historical minima and maxima. On the other hand, the all analyzed series posted positive skewness and excess kurtosis (the value of kurtosis is higher than 3).

As we expected, the exchange rates exhibit some typical stylized facts as the excess kurtosis, heteroskedasticity, volatility clustering and autocorrelation. To verify the existence of unit roots in the returns series we have computed the ADF and KPSS tests (Annexes, Table 2). The null hypothesis of unit roots for ADF test was rejected in all the cases for three confidence levels. Instead the null hypothesis of stationary series was also rejected for KPSS test which indicates the existence of both the microstructures noises and a fractional order of integration. In order to compensate for autocorrelation and heteroskedasticity and also taking into account for fractional order of integration we engaged an AR (p) × FIGARCH (p,d,q) model (Annexes, Table 3). But to test if the residuals are *i.i.d.*[4] we used a Ljung-Box test. The results showed that the null of no serial correlation was accepted for all the four exchange rates (Annexes, Table 4).

But as Embrechts (1997) underlined that before applying some statistical methods, the used data must be well studied. Thus we have computed the Mean Excess Function (MEF):

$$(33)\mathrm{MEF}(t) = \frac{\sum_{i=1}^{n}(Y_i - t)}{\sum_{i=1}^{n} 1_{\{Y_i > t\}}}$$

where t denotes the threshold, $1_{\{Y_i < t\}}$ is an indicator function that accounts for values higher than respective threshold. Ascending ordered sample values are successively chosen as thresholds and it is calculated the average of excesses over the threshold. Because the threshold was chosen successively on an increasing order, the MEF should have a negative slope converging to zero. If the empirical MEF is a positively slope straight line above 0, there is an indication of extreme values and need to use EVT theory. All four currencies have shown signs of excess kurtosis, with EUR/RON recorded the highest extremes, while the EUR/CZK posted the lower ones as the previous analyzes have indicated (Annexes, Figure 1).

Thus taking into account these reasons, the appliance to Extreme Value Theory is an appropriate choice. Even the Student distribution of innovations capture a high degree of the leptokurtosis effect, the

[4] *independent and identically distributed.*

unimodal distribution as T or Gaussian are not designed to provide a good fit in the tails. The main reason for this effect is that tails are low density areas and the unimodal distributions are an appropriate choice to fit in the areas where data are most concentrated, namely in mode[5]. Also the exchange rates contain many microstructure noises which Student or Gaussian distributions cannot capture.

A very important concern in modelling the tails of distribution using GPD approach is to chose an appropriate threshold over which are considered the excesses, because various methods for estimating parameters of distributions are very sensitive to the choice of threshold. Embrechts (1997) has suggested the usage of Hill estimator for threshold determination. Hill (1997) proposed the following estimator:

(34) $\hat{\gamma} = \frac{1}{k-1}\sum_{i=1}^{k-1} \ln Y_{i,N} - \ln Y_{k,N}$

for $k \geq 2$. In the above relation k are the upper ordered exceedances, N is the sample size and $\xi = \frac{1}{\gamma}$ is the tail index. Computing the Hill-plot for γ as Embrechts (1997) suggested a threshold will be selected from the plot where γ is fairly stable.Therefore we have ordered the highest, respectively lowest 500 *i.i.d.* observations for each currencies and inferenced the Hill estimator for lower and upper tails (Annexes, Figure 2). The lowest thresholds was recorded by EUR/RON for the right tail and EUR/CZK for the left one, both accounting for about 10% of whole sample data over the most stable area of Hill estimator. Thus for the previously mentioned reason and in accordance to literature about this topic we set a 10 % threshold for each tail as the data that exceeds the thresholds are considered extreme events. To do this we used a semi-parametric approach to fit the residuals' distribution, namely for tails we applied the Generalized Pareto Distribution (GPD) approach, while the interior of distribution was fitted by a Gaussian kernel (Annexes, Figure 3).

In order to analyze the dependence among the four exchange rates, firstly we have engaged two Elliptical copulas to estimate the correlation among currencies, namely Student, respectively Gaussian copulas. The estimated parameters with both Elliptical copulas showed some positive correlation among the four currencies (Annexes, Table 5). Correlations estimated with Student copula are higher than those fitted with Gaussian copula, due to the fact that T copula takes into account for fat tails. Another approach to compute the correlation matrix is to first estimate the rank correlation matrix[6]. Then, given

[5] See Embrechts (1997) for more details.

[6] Zeevi and Mashal (2002).

the previous estimation we can use a robust sine transformation to obtain a correlation matrix. We can observe that correlation matrix obtained from rank correlation provides higher coefficients of correlation than those estimated with Gaussian copula because the former approach accounts for tail dependence.

From the estimated correlation matrix we can observe that in all three methods the highest correlation is recorded between EUR/PLN and EUR/HUF, while the lowest one is registered between EUR/CZK and EUR/RON. Another important remark is that each currency is most correlated with the EUR/PLN and at least with the EUR/RON. However the correlation coefficients are smaller than 0.5 that means a low dependence in the evolution of the four currencies. But the empirical events revealed a high dependence among the four exchange rates on depreciation side as the episode from October 2008 showed, when all these currencies have sharply decreased against European currency. This fact brings the discussion about the existence of both asymmetric dependence and leverage effect as stylized facts and also about the contagion of shocks among these countries. This is a very interesting result taking into account that Poland is the largest country by population and the biggest economy from CEE zone.

Thus a shock of Poland’s economy on other countries in the region would have the greatest impact on the analyzed exchange rates. In order to account for this kind of scenario we engaged an analysis based on conditioned dependence among the four exchange rates when EUR/PLN plays a pivotal role.

Thus we selected the EUR/PLN as pivot in the Canonical Vine Copula model using SJC copula to capture the bivariate dependence among the decomposed pairs. The estimated parameters of bivariate conditioned correlation resulted from Canonical Vine Copula showed that highest dependence was recorded in the upper tail between EUR/CZK and EUR/HUF conditioned by EUR/PLN, while the dependence between EUR/CZK and EUR/RON conditioned by EUR/PLN is much lower (Annexes, Table 6). This fact indicates that among the four currencies it exist quite various degrees of shocks absorption which means that there are recorded some jumps in the financial and economic integration process. Also from estimated results it can be observed that conditioned dependences in upper tail are almost twice than those from lower tail. The previous remark underlines the existence of both the asymmetric dependence among the four currencies and the leverage effect.

Also have to mention that an explanation for much tighter dependency between EURCZK, EUR / HUF and EUR / PLN as compared with EUR/RON is that Czech Republic, Poland and Hungary have adopted in periods very close to each other inflation targeting regime. All four countries are primarily aimed to accede to ERM II, but firstly they have to satisfy the nominal convergence criteria in order to provide a high stability of exchange rate. Different reactions of Central Banks to changes in prices or interest rates, lead to asymmetric dependences among the analyzed exchange rates' evolution. The other fundamental explanation of asymmetric dependence effect is that all four countries are subject to the same problem: the increasing flows of FDIs from last period leads to appreciations of local currencies against Euro, but in the same time this effect coincides with a loss of external competitiveness.

For this reasons we have to take into account the existence of asymmetric dependence because this is one of the main concern in portfolio and risk management or hedging activities. Due to their rigidity in capture the asymmetric dependence, static methods that account for correlation among the four exchange rates could provide incomplete information about the really relations among them. Thus we proposed to analyse the bivariate varying and constant dependence between EUR/PLN and each of other three currencies. As we have seen from the estimation of dependence parameters with Elliptical copulas each currency records the highest correlation with EUR/PLN, thus supporting the proposal we have done. Another important reason for this choice is that Poland is the greatest country by population and largest economy from CEE. So that a shock from Poland' economy posts a high probability of having a significant impact on other economies from CEE.

We used a static Symmetrised Joe-Clayton copula for each of the three pairs of exchange rates to estimate the dependence in the lower and upper tails. The dependence in the lower tail denotes the co-movements in tendency of appreciation against Euro that is owing to the convergences process in aiming to accede to ERM II. On the other hand the dependence in upper tail that presents co-movements of the four currencies in depreciation against European currency is mainly explained by the trading activity. Overall the co-movements of four analyzed exchange rates reveal the economic and financial integration among the four countries. The results from static SJC copula showed a very right asymmetric dependence among the dependence in tail (Annexes, Table 7). But the static measure of correlation provided by SJC copula does no capture a full image of the real dependence in for the whole period of analyzed sample. For this purpose we engaged a dynamic model to describe a law motion of dependence for each of the three pairs. The view of dynamic correlation resulted from a SJC Copula-GARCH mode provided very interesting results about the dependence in the three pairs of exchange rates. Thus we can observe a very slight difference between the dependence of lower and upper tails in the case of EUR/PLN-EUR/CZK pair (Annexes, Figure 4). The *p-value* of 0.8386 and of 0.7718 for β^{Lower}, respectively for ω^{Lower}, while α^{Lower} is significant different from zero indicates that the dependence in the appreciation against Euro reveals a bit different degree of integration between the economies of Czech Republic and Poland (Annexes, Table 8). Instead the dependence between EUR/PLN and EUR/HUF was very right asymmetric in the analyzed period, all the three dynamic parameters for the upper tails' dependence being significant different from zero while in lower tail only α^{Lower} is significant (Annexes, Figure 5, Table 8). The dependence between EUR/PLN and EUR/RON was very noisy in the upper tail, only the α^{Upper} being significant different from zero, which could be explained by the fact that slightly correlation in depreciation against Euro was due to the trading and portfolio activities from October 2008 when all the four exchange rates have recorded high upward movements (Annexes, Figure 6, Table 8).

Further analysis was made in order to capture if these four exchange rates are moving monotonically and if the trend is available for the whole portfolio.

Suppose $\mu = (\mu_{0,}\mu_1,..\mu_N)'$ and defining the associated returns differentials as $\Delta_i = \mu_1 - \mu_{i-1}$, the test has the null hypothesis:

$$(35)\ \begin{cases} H_0: \Delta > 0 \\ H_1: \Delta \leq 0 \end{cases}$$

We analyzed different tests and the obtained results of the monotonicity of average returns across series are presented below:

Top-Bottom	t-stat	t p-value	MR p-value	MR_all p-value	*UP* p-value	*DOWN* p-value	Wolak p-value	Bonferroni p-value
0.0005	3.9828	0.0000	0.1980	0.1830	0.0010	0.8290	0.8163	1.0000

Table 1: Tests of monotonicity for returns.

The first column reports the spread in the estimated expected return between the top and bottom ranked series. The column 2 reports the *t-statistic* for this spread (using Newey-West heteroskedasticity and autocorrelation consistent standard errors), while column 3 shows the associated *p-value*. Columns 4 and

5 present the *p-values* from the monotonic relationship (MR) test applied to the decile portfolios, based either on the minimal set of portfolio comparisons, or on all possible comparisons (MRall). Columns 6 and 7 show *p-values* associated with the *"Up"* and *"Down"* tests that consider signed deviations from a flat pattern. The last two columns report the *p-values* from tests based on *Wolak's* (1989) test and a *Bonferroni* bound.

The obtained results showed that the tests indicates the same monotonically increasing of the series. For the first test, MR, the null hypothesis of monotonically increasing trend is accepted with the *p-value* of 0.1830, taking into account all the combinations among series. Results of the *UP test* rejected the null hypothesis of flat patterns of the series, while for *Down test* we accepted the null hypothesis. Both the Wolak[7] and Bonferroni tests accepted the null hypothesis meaning that the ascending trend for the four series has been once again proven.

5. CONCLUSIONS

The exchange rates exhibit some specific stylised facts as autocorrelation, heteroskedasticity, fat tails or leverage effect. Also the exchange rate can be seen as both a macro variable and an asset. For this purpose the exchange rate could be used to analyze the economic and financial integration.

In order to analyze the integration among the four selected economies from Central and Eastern Europe we engaged copula function and models to account for dependence in the evolution of exchange rates. Both the acceptance of the null hypothesis for Bonferroni and Wolak tests and the reject of the null hypothesis of *UP* test concluded that the movements of the series are monotonically uptrend. The ADF and KPSS unit root tests indicated the existence of microstructure noise that means a fractional order of integration for the returns of exchange rates. The used AR-FIGARCH model compensated for autocorrelation and heteroskedasticity in the return series as the results of Ljung-Box test didn't reject the null hypothesis. The computed Mean Excess Function indicated the need of using Extreme Value Theory to capture the behaviour in tails of the returns. Correlation matrix resulted from the application of both Elliptical copulas showed very different dependences among the four series. In addition, the Canonical Vine Copula revealed asymmetric conditional dependence between tails. The asymmetric dependence effect could also be explained by the fact that all four countries are subject to the same

[7] In 1989, Wolak proposed a test that entertains weak monotonicity under the null hypothesis.

problem: the increasing flows of FDIs from last years leads to appreciations of local currencies against Euro, but in the same time this effect coincides with a loss of external competitiveness.

The obtained results from the SJC Copula-GARCH models indicated that the four economies encountered quite different degrees of integration. This fact underlines that the four countries are positioned on different levels of convergence, mentioning that each of them is attending to accede ERM II. We highlight here that our results show the higher degree of economic and financial integration is recorded between Poland and Czech Republic

Different stages of integration lead to an increase of heterogeneity in the European Union. The heterogeneity slows the harmonization of fiscal and monetary policies of the analyzed countries with those of the Euro Zone economies. The strong asymmetry between lower and upper tail of the four exchange rates affects the principles of Optimal Monetary Zone and also could lead to spread-off asymmetric shocks within European Union. This is a more important issue from macro-prudential point of view especially in the actual context of economical and financial crisis when we are assisting to an increase of the divergence within the Union that could cause moral hazard and adverse selection.

AKGNOWLEDGEMENT

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Annexes

Basic Stats	EUR/CZK	EUR/HUF	EUR/PLN	EUR/RON
Mean	-0.000121	0.000033	-0.000008	0.000385
Median	-0.000133	-0.000080	-0.000254	0.000000
Maximum	0.031908	0.065272	0.052507	0.123554
Minimum	-0.032471	-0.029232	-0.038318	-0.072379
Std. Dev.	0.004370	0.005827	0.006967	0.007282
Skewness	0.068426	1.236.087	0.400931	1.902.772
Kurtosis	8.944.810	16.109.895	7.309.036	42.754.610
Jarque-Bera	4228.41	21283.56	2297.29	190724.80
Probability	0.00000	0.00000	0.00000	0.00000
Observations	2870	2870	2870	2870

Table 1: Descriptive Statistics

Augmented Dickey Fuller Unit Root Test				
Null Hypothesis : Variable has a unit root				
Variable	EUR/CZK	EUR/PLN	EURRON	EUR/HUF
Test statistic	-55.9964	-54.2840	-63.7256	-53.9378
Test critical values				
1%	-3.9612	-3.9612	-3.9612	-3.9612
5%	-3.4114	-3.4114	-3.4114	-3.4114
10%	-3.1275	-3.1275	-3.1275	-3.1275
*MacKinnon(1996) one-sided p-values				

KPSS Unit Root Test				
Null Hypothesis : Variable is stationary				
Bandwidth :24 (Newey -West automatic) using Bartlett kernel				
Variable	EUR/CZK	EUR/PLN	EURRON	EUR/HUF
Test statistic	0.0382	0.0625	0.1982	0.0247
Test critical values				
1%	0.2160	0.2160	0.2160	0.2160
5%	0.1460	0.1460	0.1460	0.1460
10%	0.1190	0.1190	0.1190	0.1190
*Kwiatkowski-Phillips-Schmidt-Shin (1992,Table 1)				

Table 2: Unit Root Tests

FIGARCH-Estimated Parameters		EUR/CZK	EUR/HUF	EUR/PLN	EUR/RON
Constant term	c	-0.00016	-0.00011	-0.00033	0.00000
		0.00220	0.03660	0.00030	0.97330
AR	φ	-0.07059	-0.07955	-0.06796	-0.02887
		0.00000	0.00010	0.00050	0.10600
Constant term	ω	0.01702	0.02306	0.01178	0.00731
		0.00000	0.00000	0.00140	0.00000
ARCH	α	0.17749	0.16704	0.25687	0.11893
		0.00000	0.00000	0.00000	0.00000
GARCH	β	0.47379	0.59040	0.59362	0.86377
		0.00000	0.00000	0.00000	0.00000
Fractionally term	d	0.40330	0.58514	0.41957	0.95059
		0.00000	0.00000	0.00000	0.00000
Student distribution of the errors	DoF	3.96390	3.37011	8.27102	3.89760
		0.00000	0.00000	0.00000	0.00000

Table 3: Estimated Parameters using FIGARCH

Ljung-Box Test for serial correlation									
Standardized Residuals					Squared Standardized Residuals				
	EUR/CZK	EUR/HUF	EUR/PLN	EUR/RON		EUR/CZK	EUR/HUF	EUR/PLN	EUR/RON
H	0	0	0	0	H	0	0	0	0
P-value	0.6577	0.8627	0.5580	0.7703	P-value	0.9844	1.0000	0.9867	0.6712
Q-stat	21.6182	17.5042	23.3359	19.5440	Q-stat	12.2488	0.6964	11.9862	21.3812
Critical Value	37.6525	37.6525	37.6525	37.6525	Critical Value	37.6525	37.6525	37.6525	37.6525
Confidence level: 5%									
Lag:25									
Null Hypothesis : No serial correlation									

Table 4: Ljung Box Tests for serial correlation

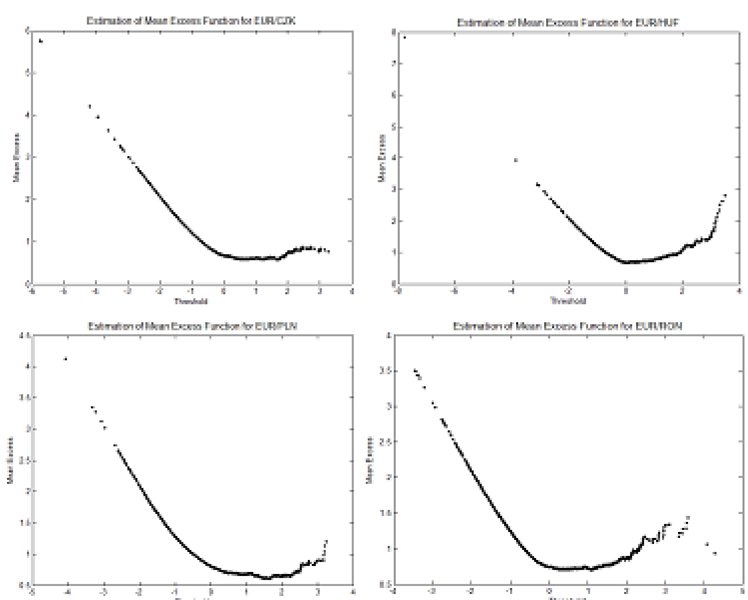

Figure 1: Mean Excess Function for the four currencies

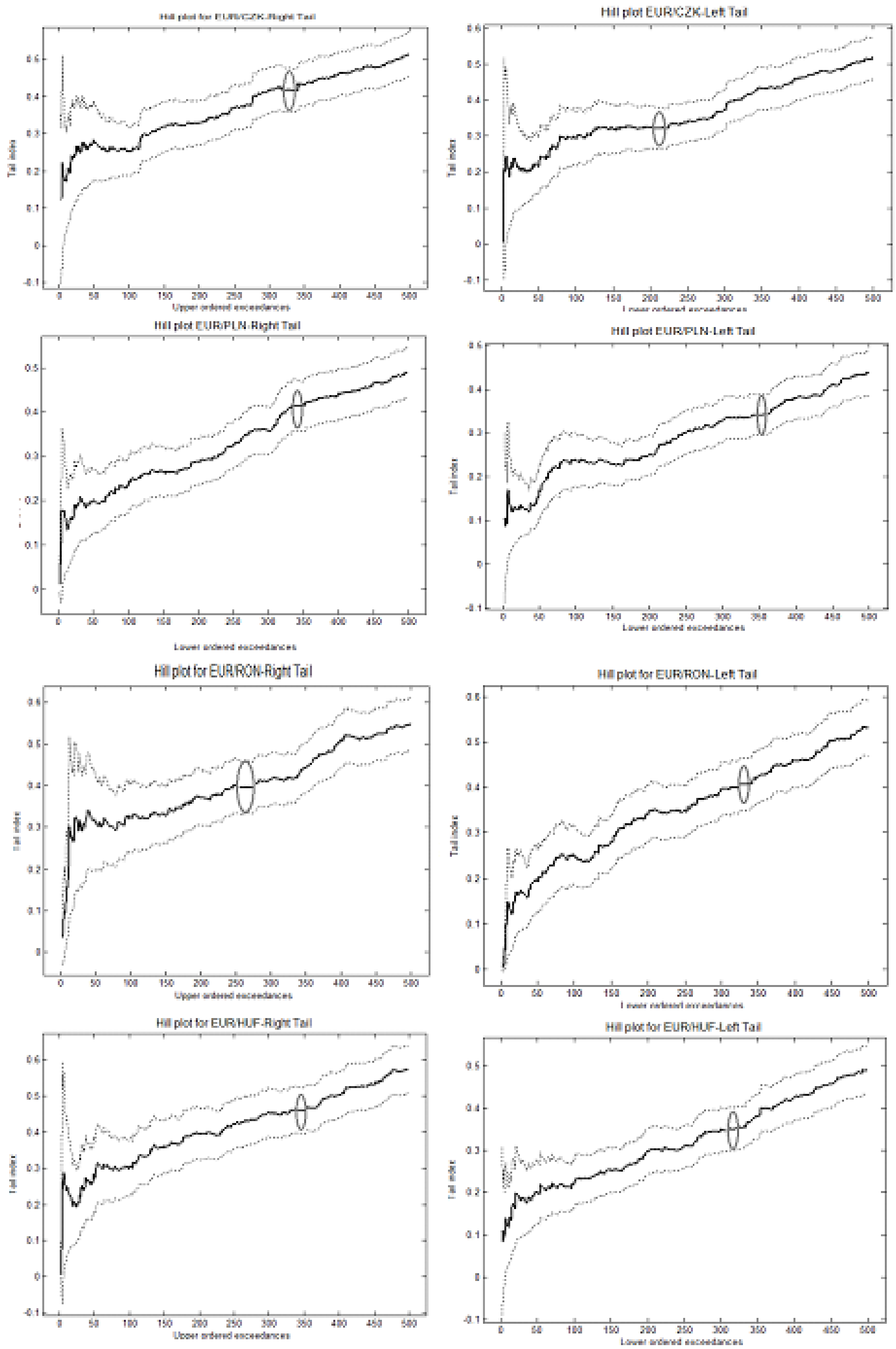

Figure 2: Hill Plots for the four currencies.

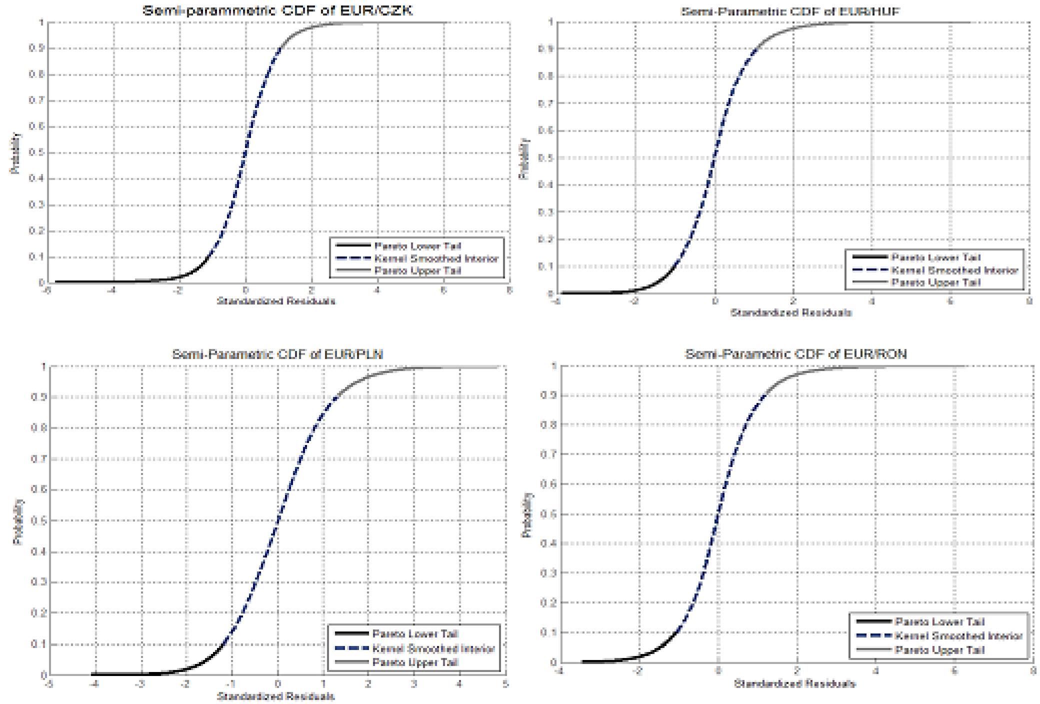

Figure 3: Semi Parametric CFD for filtered innovations

DoF	**DoF CI**	
16.8876	11.9560	21.8191

Correlation Matrix using T-copula

	EUR/CZK	EUR/HUF	EUR/PLN	EUR/RON
EUR/CZK	1.0000	0.3052	0.3451	0.1467
EUR/HUF	0.3052	1.0000	0.4877	0.2356
EUR/PLN	0.3451	0.4877	1.0000	0.3395
EUR/RON	0.1467	0.2356	0.3395	1.0000

Correlation Matrix using Gaussian-copula

	EUR/CZK	EUR/HUF	EUR/PLN	EUR/RON
EUR/CZK	1.0000	0.2907	0.3318	0.1355
EUR/HUF	0.2907	1.0000	0.4700	0.2246
EUR/PLN	0.3318	0.4700	1.0000	0.3309
EUR/RON	0.1355	0.2246	0.3309	1.0000

Empirical Kendall's *tau*

	EUR/CZK	EUR/HUF	EUR/PLN	EUR/RON
EUR/CZK	1.0000	0.1929	0.2176	0.0902
EUR/HUF	0.1929	1.0000	0.3222	0.1435
EUR/PLN	0.2176	0.3222	1.0000	0.2242
EUR/RON	0.0902	0.1435	0.2242	1.0000

Theoretical R using Kendall's tau

	EUR/CZK	EUR/HUF	EUR/PLN	EUR/RON
EUR/CZK	1.0000	0.2984	0.3351	0.1412
EUR/HUF	0.2984	1.0000	0.4848	0.2235
EUR/PLN	0.3351	0.4848	1.0000	0.3449
EUR/RON	0.1412	0.2235	0.3449	1.0000

Empirical Spearman's *rho*

	EUR/CZK	EUR/HUF	EUR/PLN	EUR/RON
EUR/CZK	1.0000	0.2837	0.3168	0.1343
EUR/HUF	0.2837	1.0000	0.4629	0.2122
EUR/PLN	0.3168	0.4629	1.0000	0.3273
EUR/RON	0.1343	0.2122	0.3273	1.0000

Theoretical R using Spearman'*rho*

	EUR/CZK	EUR/HUF	EUR/PLN	EUR/RON
EUR/CZK	1.0000	0.2960	0.3303	0.1405
EUR/HUF	0.2960	1.0000	0.4800	0.2217
EUR/PLN	0.3303	0.4800	1.0000	0.3410
EUR/RON	0.1405	0.2217	0.3410	1.0000

Table 5: Correlation Matrices

Conditioned Dependence with Canonical Vine Copula		
Pair	**SJC**	
	Upper tail	**Lower tail**
EUR/PLN-EUR/CZK	0.1504	0.0815
EUR/PLN-EUR/HUF	0.1710	0.1389
EUR/PLN-EUR/RON	0.0265	0.0088
EUR/CZK-EUR/HUF|EUR/PLN	0.2714	0.1619
EUR/CZK-EUR/RON|EUR/PLN	0.0739	0.0278
EUR/HUF-EUR/RON|EURPLN,EUR/CZK	0.1031	0.0569
Log Likelihood	742.526	

Table 6: Canonical Vine-Copula

Pair	**Parameters of Constant Correlation with SJC Copula**		
	EUR/PLN-EUR/CZK	**EUR/PLN-EUR/HUF**	**EUR/PLN-EUR/RON**
τ^{Lower}	0.1396 **(0.0000)**	0.2001 **(0.0000)**	0.1249 **(0.0000)**
τ^{Upper}	0.1880 **(0.0000)**	0.3490 **(0.0000)**	0.1867 **(0.0000)**
AIC	-400.0751	-836.2473	-344.2178
BIC	-400.0751	-836.2411	-344.2116
NLL	-178.7288	-361.7337	-165.6595

Table 7: Static SJC estimations

Pair	**Parameters of Tyme-Varying Correlation with SJC Copula**						
	ω^{Upper}	α^{Upper}	β^{Upper}	ω^{Lower}	α^{Lower}	β^{Lower}	NLL
EUR/PLN-EUR/CZK	-0.4794 **(0.4145)**	-5.6886 **(0.0030)**	2.0210 **(0.0361)**	0.3325 **(0.7718)**	-8.0219 **(0.0165)**	-0.4893 **(0.8386)**	-200.0396
EUR/PLN-EUR/HUF	-1.9735 **(0.0000)**	-0.4101 **(0.0000)**	4.0994 **(0.0000)**	0.5589 **(0.5917)**	-11.5395 **(0.0096)**	1.1421 **(0.3904)**	-424.9870
EUR/PLN-EUR/RON	0.9933 **(0.2552)**	-7.3540 **(0.0087)**	-3.4692 **(0.1862)**	-0.3253 **(0.8518)**	-7.6146 **(0.1462)**	1.8227 **(0.6118)**	-177.8138

Table 8: Time varying SJC -Copula

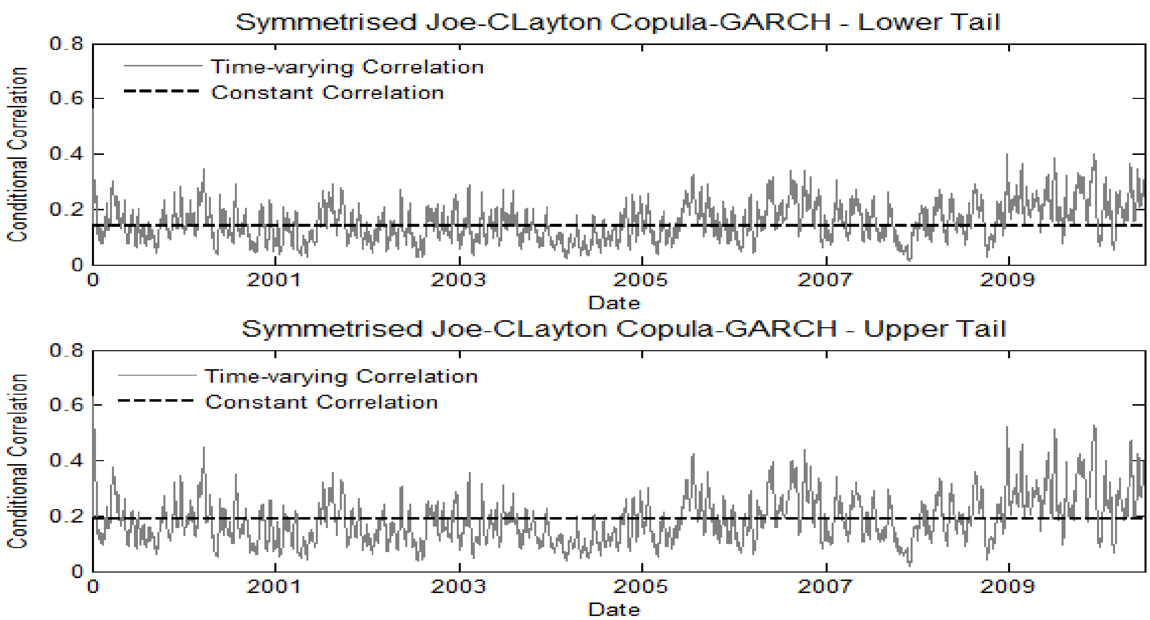

Figure 4: EUR/CZK-EUR/PLN SJC Copula GARCH

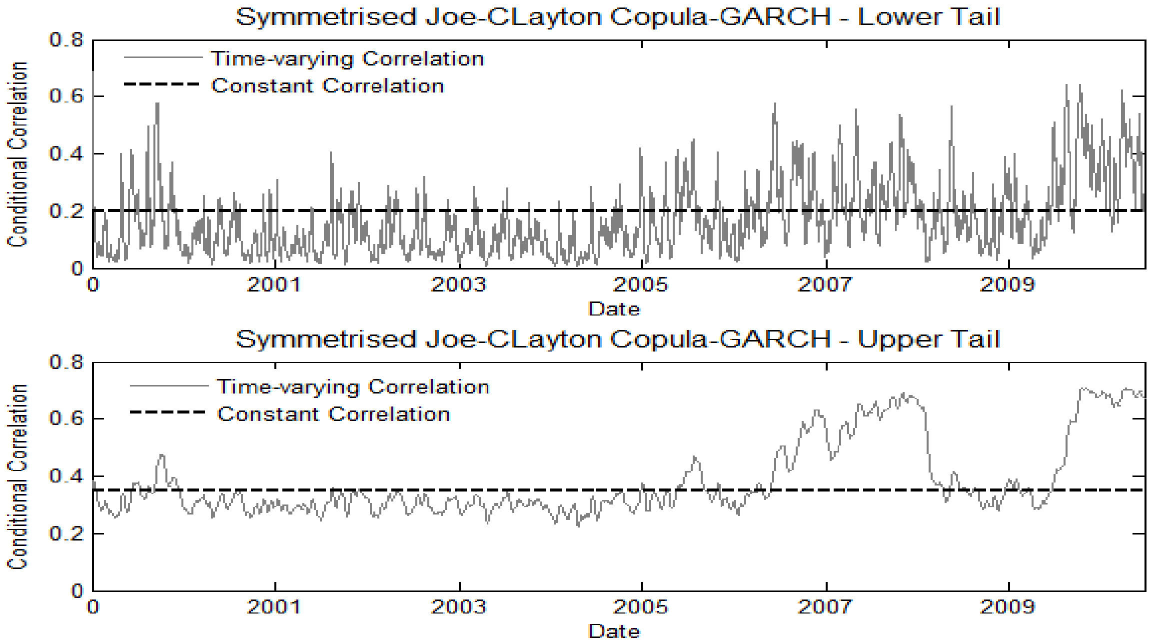

Figure 5: EUR/HUF-EUR/PLN SJC Copula GARCH

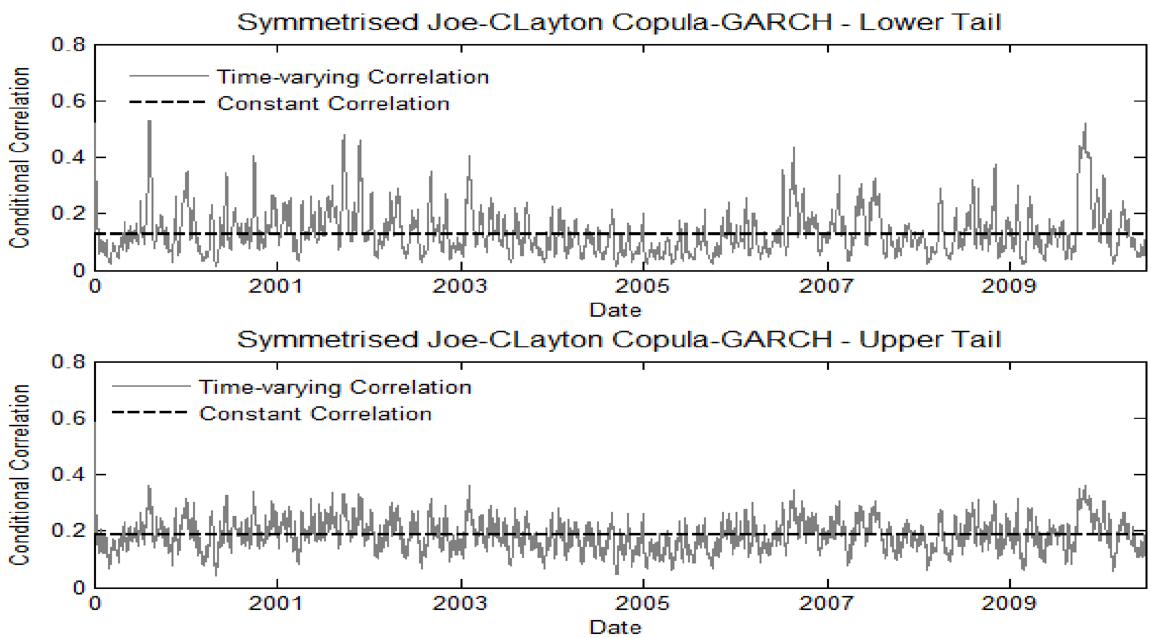

Figure 6:EUR/RON-EUR/PLN SJC Copula GARCH