

# **Arbitrage Conditions, CAP Rates, and Segmentation in the Housing Market: A Micro Study**

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## **Abstract**

Students of real estate and urban economics are taught to use models of rental markets that assume a linear relation between net rental income and asset value. The slope of this relation is the capitalization rate of landlords who are assumed to capitalize net operating income using a CAP rate that does not vary with the size of the unit to get an estimate of house value. This approach is based on an assumed arbitrage equilibrium in which landlords are free to divide a given amount of interior space into a small number of large units or large number of small units. Such an arbitrage equilibrium requires uniformity in the CAP rate. To the extent that real estate practitioners remember the economics that they were taught, this should be a case of “nature following art” and the empirical literature should support the standard classroom model of a constant CAP rate. Recently Glaeser and Gyourko (2007) have argued that rents are too low for such no arbitrage condition to hold in residential real estate market.

This paper formulates a model of a segmented rental housing market in which CAP rates are concave in unit value. The model is calibrated to user cost computations for Washington, D.C. and predicts that CAP rates should fall as value exceeds \$600,000. Based on the theory a specially matched micro data set on rents and values of equivalent units is collected and the relation between CAP rates and value is tested. The theory is confirmed as CAP rates decline significantly as value exceeds \$600,000. This result has important implications for research on real estate pricing in general and for research on bubbles and inefficiency in residential real estate markets in particular.

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<sup>1</sup> I would like to thank Professor Anthony Yezer for his unfailing inspiration and guidance with this paper. I also thank the helpful comments from participants of George Washington University Department of Economics Micro Seminar. None are responsible for the errors.

## **I. Introduction and motivation**

Rent to asset price ratios of any good in a competitive market should be governed by a no arbitrage condition based on user cost of the supplier. Producers should be indifferent between renting and selling consumer durables and the arbitrage condition expressing that indifference is the capitalization rate (CAP rate) of net rental income. While computation of the capitalization rate can be complex, particularly given that tax treatment of residential real estate, the general position in the literature is that the CAP rate should not vary with the value of the unit.

While the constant CAP rate assumption has many proponents, some recent literature has come to question its empirical validity. Glaeser and Gyourko (2007) conclude that observed rents appear to be too low to justify the asset prices observed in housing markets if the CAP rate is based on the user cost of landlords. Verbrugge (2008) finds that rents and asset prices fail a cointegration test in many markets. Garner and Verbrugge (2008) observe that, based on surveys of owner occupants in the Consumer Expenditure Survey, estimated rental equivalence is concave in estimated house value.

This paper provides evidence, based on a data collected for carefully matched housing units, that the CAP rate is not constant and that rent increases a decreasing rate with value. It then proposes a model to explain the failure of the CAP rate to be constant. This explanation reconciles the standard textbook model of residential real estate and the observed data. The key to this model is to recognize that arbitrage equilibrium of landlords determines the measured CAP rate for lower value units and arbitrage equilibrium for occupants determines the measured CAP rate for higher valued units.

## *II. The case for constant CAP rates and landlord arbitrage*

It is not difficult to find empirical support for the position that CAP rates for consumer durable purchases are constant. Taking automobiles as an example of a major consumer durable, it is possible to observe the ratio of lease to purchase prices as a function of vehicle price. The observed CAP rate is virtually constant reflecting arbitrage equilibrium for manufacturers whose decision to lease is based on the opportunity cost of selling capitalized at a corporate rate that does not vary across models with different selling prices. The empirical validity of this proposition was verified by going to individual manufacturers' websites. Major producers, including Buick, Chrysler, Acura, and Porsche, provide estimated leasing prices which indicate a virtually constant capitalization rates for their own vehicles. A sample of ten automobile manufactures was selected based on data availability. The estimated relation between lease and purchase prices produced a function that was slightly convex at the upper end of the market.<sup>2</sup> The departure from linearity, while statistically significant, was not economically significant, representing less than one-percent difference (for vehicles worth 30 thousand dollars) in monthly leasing cost and demonstrating the proposition that the relation between lease (i.e. rental) and purchase (i.e. asset) prices is approximately linear for automobiles.

The same seller arbitrage condition found in automobile and other durable goods markets motivates the assumption that CAP rates are constant within a local housing market. Indeed, models of real estate pricing where CAP rate does not vary with value are standard, see, for example, DiPasquale and Wheaton (1996) as shown in Figure 1. Further the expression for user cost, on which the CAP rate is based, assumes that the

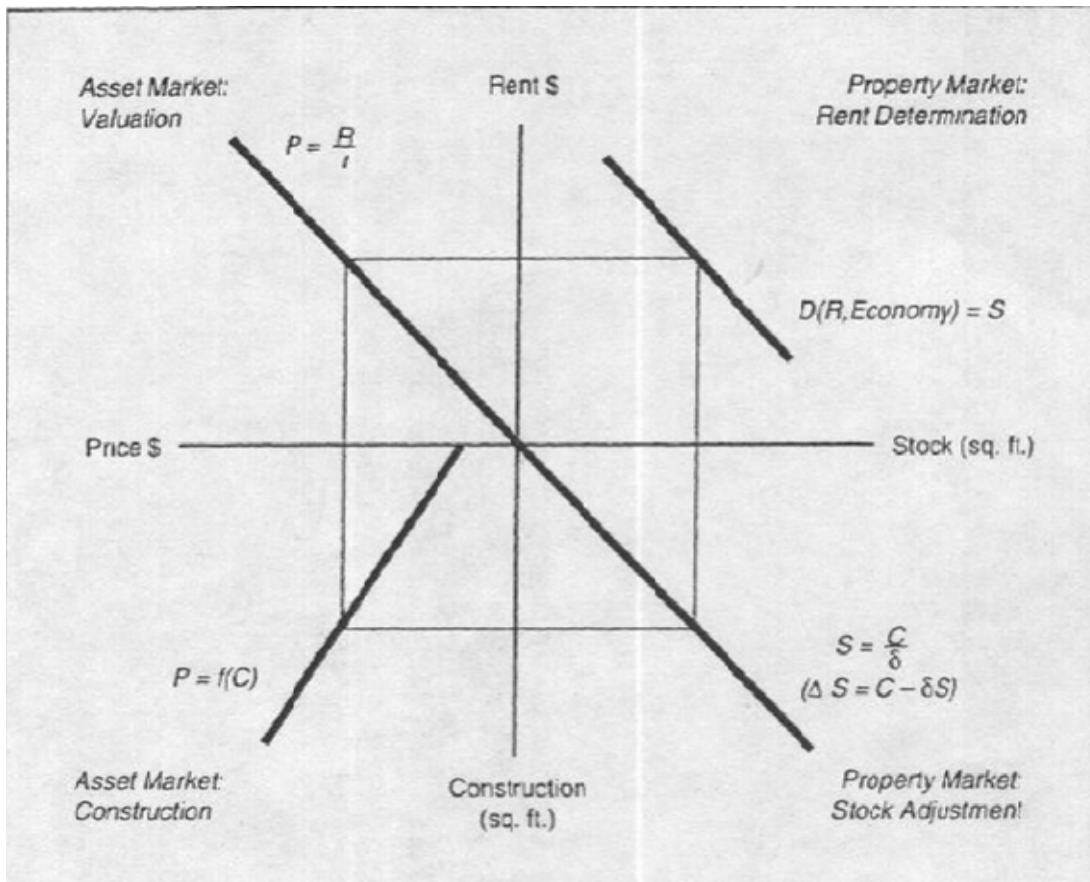
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<sup>2</sup> The possible pricing strategy of over quoting leasing cost to push sales of expensive vehicles is ignored due to the assumptions that the strategy should only play a role in market capitalization rates in the short run.

user cost per unit of housing capital is constant, provided that the expected rate of appreciation does not vary with value itself.

**Figure 1**

**CAP Rate and User Cost in Housing Market from DiPasquale and Wheaton (1996)**



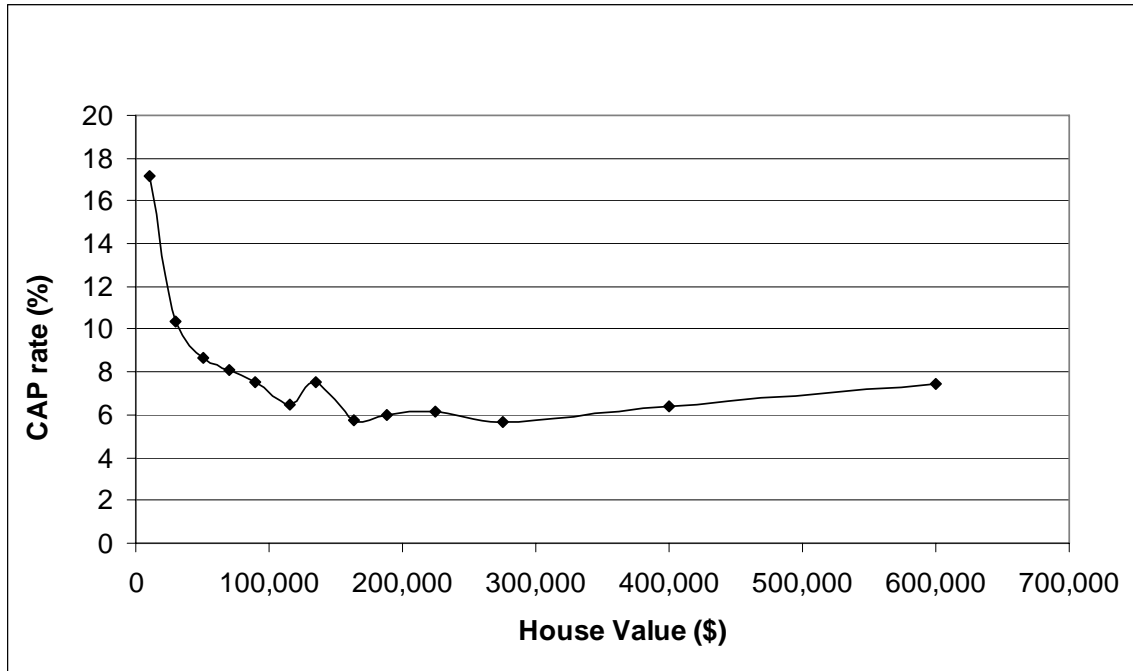
A commonly accepted method for estimating CAP rates, developed by Robin Phillips (1988), implicitly assumes that the CAP rate is not a function of value. Tests for efficiency in asset pricing following the tradition of Case and Shiller (1989) also share this assumption. Finally there is a substantial literature on valuation of rental property that capitalizes net rental income by a constant user cost to get value based on the assumption that the landlords are in a no arbitrage equilibrium. For a recent example, see Benjamin, Chinloy, Hardin, and Wu (2008).

Smith and Smith (2006) used a very micro level data set similar to that collected for this study to estimate housing value using rents and other economic fundamentals. They concluded that actual house prices deviated from predicted value based on observed rents by more than forty percent in five of the cities studied but maintain the assumption that equilibrium requires that rent be linear in value.

The Bureau of Economic Analysis (BEA) estimated CAP rates for households as a function of housing value using the 1991 Residential Finance Survey. The resulting CAP rates, shown in Figure 2, are currently used to transform owner's value into implicit rental income for incorporation in the National Income and Product Accounts (NIPA). Overall the CAP rate appears convex in value but that is largely the product of very low value units where depreciation and operating cost may be unusually large as a proportion of gross rent. The BEA CAP rates have been included in this section on arguments for a linear relation between rent and value because they are virtually constant once unit value exceeds \$75,000 and are certainly not concave as those who argue for a variable CAP rate contend in the next section. The debate over the relation between value and CAP rate has important implications for the measurement of national income.

**Figure 2**

**BEA CAP Rate Measure Based on 1991 Residential Finance Survey**



A final point made by those who believe that rents are linear in value is that the proportion of renters falls with value. It seems counterintuitive that the probability of renting would fall with value if rent was actually concave in value so that the relative price of rent was falling at the same time that the proportion of renters was falling. Failure to observe a constant rent/value relation is then attributed to hidden heterogeneity in the housing units, particularly differences in expected appreciation between rental and owner units that are associated with differences in value. Alternatively differences in the ratio of operating costs to value are sometimes cited as a possible source of variation in CAP rates measured using the gross rent-to-income ratio.<sup>3</sup>

<sup>3</sup> The most common argument here is that, for very low value rentals, credit risk and depreciation are both higher due to the characteristics of the occupant. However these effects disappear well before the value segments of the market studied here (the lowest house value in the micro data this analysis is based on is over 100 thousand dollars).

### *III. The case for variable CAP rates and/or failure to arbitrage by landlords*

A recent paper by Glaeser and Gyourko (2007) concludes that observed rental and asset prices of housing are inconsistent with a no arbitrage equilibrium for landlords. They suggest that part of the apparent problem may be that some determinants of the CAP rate, including maintenance cost, risk, risk preference, and expected appreciation, are difficult to measure. However, they argue that current rents are too low to support asset prices for rental units that equal the asset prices of comparable owner units, thus violating the no arbitrage condition.

Curiously, the literature which assumes CAP rates are constant contains results which appear to violate the assumption. Although Phillips (1988) developed a much-used technique for estimating CAP rates based on pooled regressions of both rental and owner occupied units under the assumption that the rent/value ratio was constant within a given housing market, she found that the expected real rate of appreciation in housing asset prices did not have the expected negative effect on CAP rates. Specifically, she reported that the estimated coefficient of various measures of expected appreciation in a CAP rate equation was significantly greater than zero but also well below unity. Indeed the estimated value was -0.01. This value implies that total return to housing investment across markets is directly related to the CAP rate which is a violation of the inter-area no-arbitrage condition of equality in total return that should characterize the equilibrium of housing markets. Similar results were obtained by Hamilton and Schwab (1985) using a different empirical technique for measuring CAP rates.

Blackley and Follain (1996) estimated a four-equation structural equation of the U.S. housing market and simulated the relation between rents and user cost over time. They found that

only half of a change in user cost is reflected in rents and the lag on the adjustment process is very long, with only one-third of the adjustment completed in the first ten years. These and subsequent research have established the failure of rent-to-asset price ratios to fall with estimates of expected appreciation has been well established as a stylized fact in the literature.

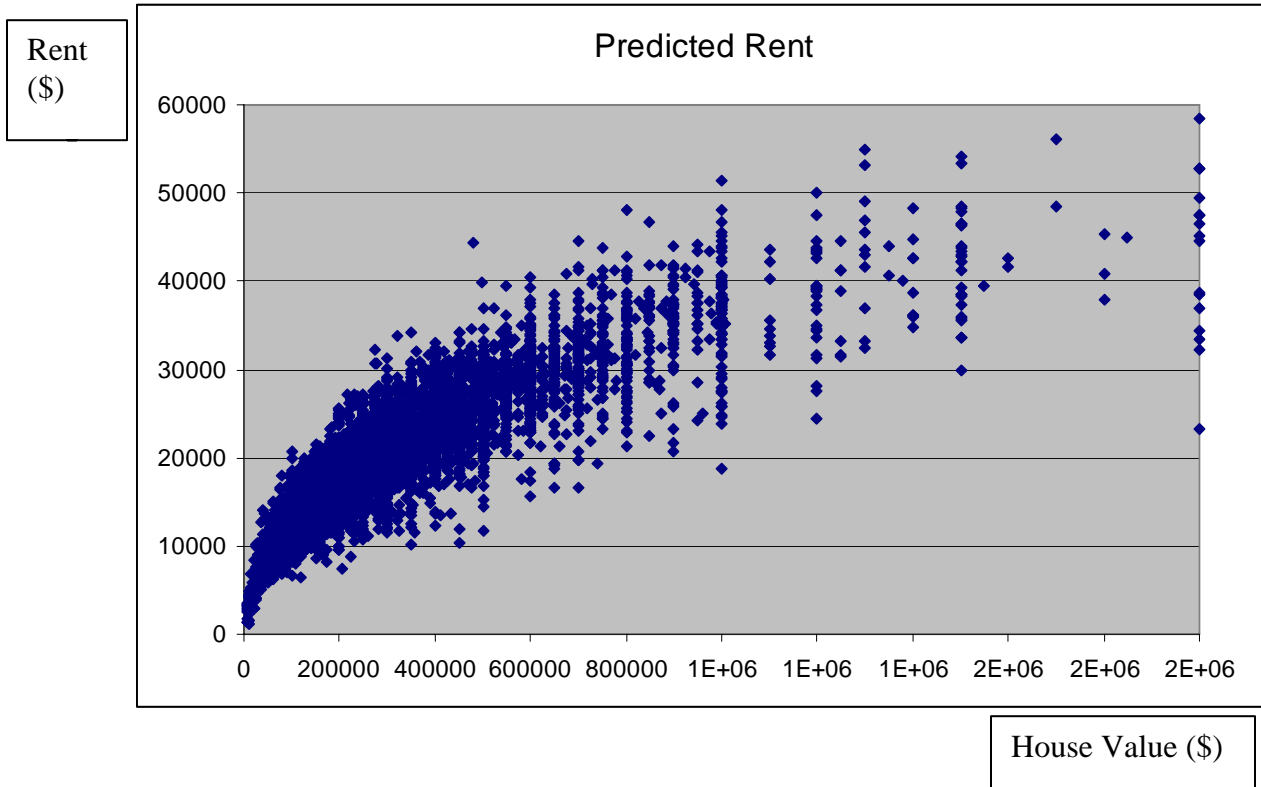
Recently Verbrugge (2008) constructed user cost and rent series for single family units by MSA, region, and nationally for the U.S. over time. In contrast to expectations, he found in all tests that compared to various rental price series, asset prices are characterized by excess volatility, differing trend, and even failure to be cointegrated with rents. These differences do not appear to be explained by volatility in his measure of user cost. Furthermore, Garner and Verbrugge (2008) have used owner's estimates of rental equivalence and value reported in the Consumer Expenditure survey to study the determinants of reported rental equivalence. As is evident from the scatter diagram shown as Figure 3, they find that rental equivalence is concave in value.<sup>4</sup> While owner estimates of rental equivalence and value are not transactions prices, it is difficult to see why owners of higher value units should report significantly lower rent-to-value ratios, particularly given that the question is asked in four successive interviews as part of successive waves of the Consumer Expenditure Survey.

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<sup>4</sup> It is tempting to dismiss owner's estimates of what their unit would rent for as uninformed speculation. Certainly the relation between rental equivalence could arise due to estimation error by the owner-respondents. But the survey collects data for 4 successive quarters and the argument is that, by the fourth quarter, owners have attempted to determine the rental price of their units.

Figure 3

Annual Rental Equivalence versus House Value by Garner and Verbrugge (2008)



Overall, it appears that the case against the constant CAP rate assumption in which rent/asset price ratios reflect a no arbitrage equilibrium of landlords is quite strong and deserves serious testing and explanation.

**IV. A Model With Variable CAP Rates and No Arbitrage Equilibrium**

Titman (1982) developed a model of market equilibrium in which the homeownership rate fluctuates with the distribution of user costs within the population based primarily on differences in marginal tax rates compared to the relatively constant user cost of landlords. Households with user cost below that of landlords choose to own and those with higher user costs rent. Given the positive association between income and marginal tax rate, income and user cost should vary inversely and hence homeownership rates should rise with income.

The model proposed here is based on the fundamental insight in Titman except that it has a segmented housing market. It relies on the fact that user cost of households varies inversely with income due to the tax treatment of owner-occupied housing and the relatively lower borrowing cost facing households with higher income. Furthermore, because housing consumption rises with income (even for rental housing) and also varies inversely with user cost, housing value should increase with income.<sup>5</sup> Palmon and Smith (1998) found a negative correlation between user cost and physical size of housing units for single family housing units in Houston.

Rental housing is supplied by normal landlords (hereafter “landlords”) whose decisions are based on a conventional CAP rate computation and by “inframarginal landlords”, whose supply of rental housing is perfectly inelastic.<sup>6</sup> The number of inframarginal landlords is small and they are assumed, for analytical convenience, to be distributed uniformly across the housing stock. In one segment of the market, where housing value is lower, most households have user cost higher than that of landlords and the demand for rental housing is far greater than the supply from inframarginal landlords. Therefore, the CAP rate does not vary with value in this segment of the housing stock and it is based entirely on the user cost of landlords. Households choose tenure comparing their personal user cost and CAP rate determined by landlords.

In the other segment of the market, house value is sufficiently large so that virtually all households have user cost lower than that of landlords. Accordingly, normal landlords are not active in supplying rental housing in this segment. However, inframarginal landlords are active. The observed CAP rate is lower than that for landlords and is based on the CAP rate of the marginal household at which the supply of housing from inframarginal landlords clears the

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<sup>5</sup> These points are related because quantity of housing consumed by owners is based on user cost.

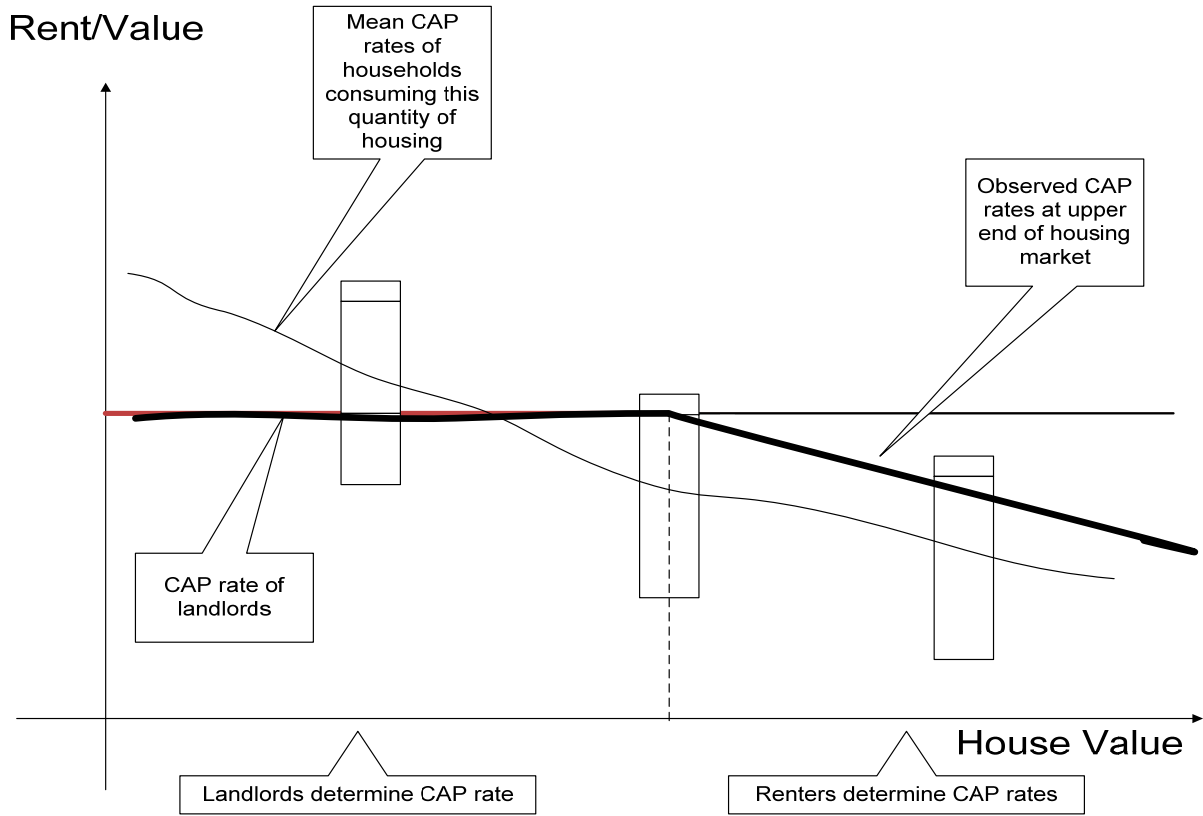
<sup>6</sup> Inframarginal landlords may be owner-occupants who wish to retain ownership of a housing unit for any number of idiosyncratic reasons. There may be a personal attachment to a particular unit, a desire to avoid a taxable event associated with sale of the unit, etc.

housing market. In this segment of the market the CAP rate is determined by the marginal households who are in arbitrage equilibrium between renting and owning. Because of the positive association between income and housing consumption, the observed rent-to-asset price ratio falls throughout this segment of the housing market. The net result is the apparently counterintuitive finding that both the fraction of renters and the rent-to-value ratio tend to fall as the value of the housing unit rises.

Housing market equilibrium produced by this model is illustrated in Figure 4. The CAP rate set by landlords is constant and average user cost of households is decreasing with value of housing consumed by households. At each housing value, user cost of households consuming that amount of housing services has a distribution around the mean which is drawn as uniform for purposes of illustration. Households make their tenure choices by comparing their user cost and market CAP rate. The share of housing supplied by inframarginal landlords comprises the small area at the top of each bar.

Figure 4

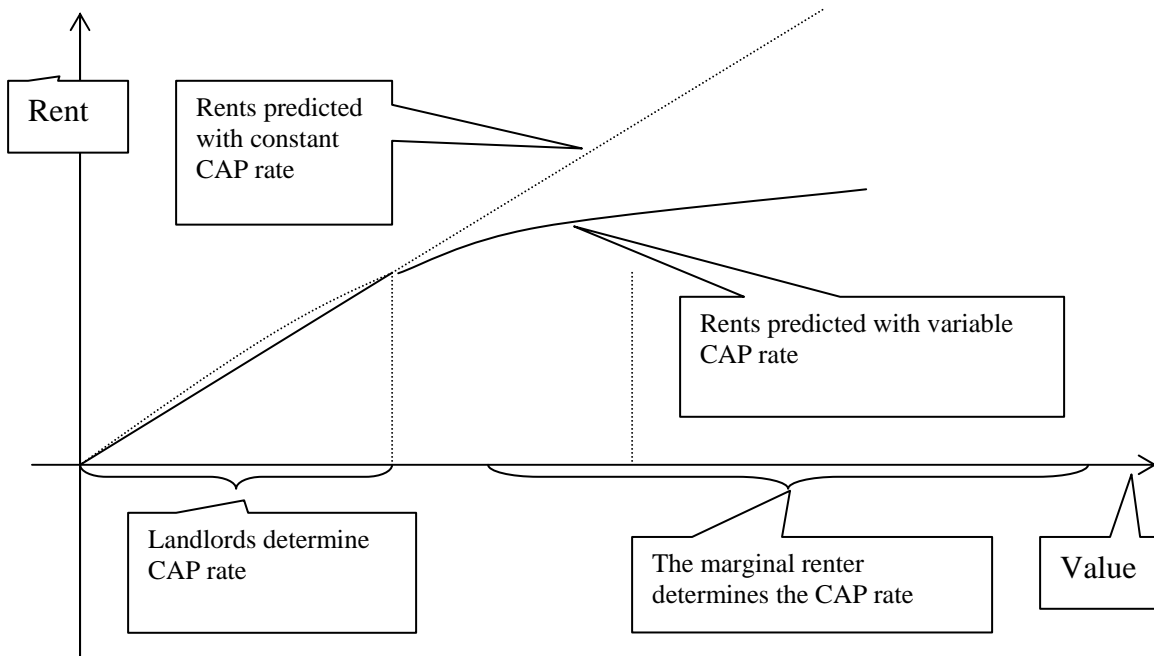
CAP Rate, User Cost, and Inframarginal Rent



The market capitalization rate is still set by landlords until the CAP rate of households falls sufficiently low so that inframarginal landlords can meet the entire demand for rental housing. This occurs at house values sufficiently high that the mean household CAP rate is significantly below that of landlords. As the quantity of housing, and hence its value, increases beyond this point, rental housing is supplied by inframarginal landlords and it is priced based on the user cost of the marginal household renting in that quantity segment of the market. Thus there are two segments of the housing market and a no arbitrage equilibrium holds in each. In the lower value segment, there is a no arbitrage equilibrium based on CAP rates of landlords. In the high value segment of the housing market the no arbitrage equilibrium is based on the CAP rates of the

marginal households who rent. If this prediction of the model holds in the data, there should be a rent curve that is concave in house value which is show in figure 5.

**Figure 5**  
**Expected Relation between Rent and House Value**



It is possible calculate the CAP rates of landlords and consumers that are shown in Figure 4. Because the subsequent section will test the theory using a dataset collected specially in 2008 from Washington, DC, user cost and CAP rate are calculated for the Washington, DC metropolitan area with most recent data in 2007.

Landlords are assumed to set the price of housing correctly following the valuation model proposed by Ling (1992). If, for each unit of housing service  $h$ , landlords can charge  $r$ , then the sales price of a property producing  $h$  units of housing services should be set at:

$$\begin{aligned}
p_H * h = & \sum_{t=1}^N \frac{r * h * (1 + \rho)^t}{(1 + k)^t} + \sum_{t=1}^N \frac{\tau_i * \delta_t}{(1 + k)^t} + [(1 - \tau_i) * c] \sum_{t=1}^N \frac{(1 + \pi_0)^t}{(1 + k)^t} \\
+ & \frac{p_H * h * [(1 - B) * (1 - \pi)^N] * (1 - \tau_c) + \tau_c * AB_N}{(1 + k)^N}
\end{aligned} \tag{1}$$

where  $p_H$  is price level for each unit of housing service,  $\rho$  is general inflation,  $\tau_i$  is income tax rate,  $\delta$  is annual tax depreciation deduction,  $c$  is tax deductible operating cost, which increase at the rate of  $\pi_0$ . Property owners incur a cost  $B$  at time of sale and properties appreciate at a rate of  $\pi$ . The capital tax rate is  $\tau_c$  percent and  $AB$  is the adjusted tax basis of the property at time of sale and  $k$  is the weighted average cost of capital:  $AB_N = r * h - \sum_{t=1}^N \delta_t$  and  $k = (1 - \tau_i)i + (1 - \tau_{RP})RP$ , where  $RP$  is the risk premium and  $\tau_{RP}$  is the tax rate associated with the risk premium. Equation (1) is the basis for the landlord CAP rates shown in Figure 7. Values from Washington, DC appropriate for a 1 year holding period were used to compute the landlords' CAP rate.<sup>7</sup> The CAP rate of landlords based on equation (1), consistent with the textbook example in Figure 1, does not vary with the amount of housing services in the unit (i.e. the CAP rate is not a function of  $h$ ). Figure 7 shows CAP rate for landlords computed using the Ling (1992) formula shown above as equation 1 and applying parameters appropriate for Washington, D.C. in 2007 as shown in Appendix 1.

Next, it is necessary to determine the user cost of households as a function of house value in Washington, D.C. for 2007. User cost is calculated using average household characteristics of households in each house value interval based on the American Housing Survey (AHS) 2007 national survey for the Washington, D.C.

<sup>7</sup> See the table of parameter values used in Appendix A1.

metropolitan area. Marginal tax rates for 2007 are taken from the appropriate federal and state income tax schedules. Households' user cost follows a standard formula following Green and Malpezzi (2003) with the assumption that households sell properties after one year to match the same assumption for landlords.

$$(i + \rho + \tau_p) * (1 - \tau_i) + c + B - \pi \quad (2),$$

where  $\tau_p$  is the property tax rate.

In order to measure the variation in user cost by house value it is necessary to discover the way income, mortgage interest rates and marginal tax rates vary with house value in the Washington, DC metropolitan area. Table 1 shows that mortgage interest rates from the American Housing Survey tend to fall slightly with income.

Table 1

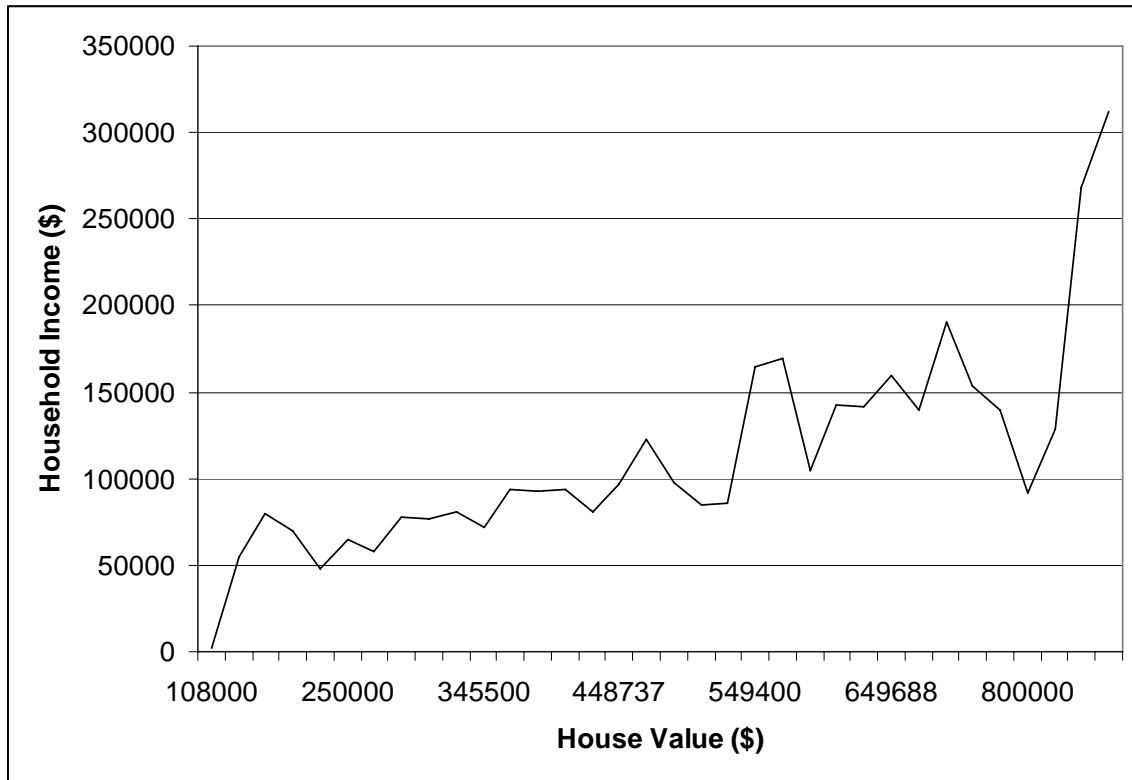
Washington DC Income And Mortgage Rate Distribution By Income Level  
(Calculations based on the 2007 AHS)

<b>Variable</b>	<b>Income Group</b>	<b>Obs</b>	<b>Mean</b>	<b>Std. Dev.</b>
Household Income	Full sample	406	88127	87187
First mortgage rate	<10000	4	5.94	0.79
First mortgage rate	>=10000 & <16050	0		
First mortgage rate	>=16050 & <40000	15	6.61	1.51
First mortgage rate	>=40000 & <65000	34	6.53	1.15
First mortgage rate	>=65000 & <131450	71	6.41	0.98
First mortgage rate	>=131450 & <200300	37	5.79	0.80
First mortgage rate	>=200300 & <357700	9	5.68	0.94
First mortgage rate	>=357700	8	5.13	0.55

Figure 6 demonstrates the empirical relation between housing consumption in Washington and household income.

**Figure 6**

**Household Income and Housing Consumption for Washington DC Metropolitan Area in 2007**



Average user cost of households is computed as a function of house values using equation 2 above and applying the mortgage interest rates in Table 1 along with the appropriate marginal tax rates of both federal and state income tax for Washington, DC based on the relation between income and house value in Figure 6. The result is shown in Figure 7 where user cost for households is a decreasing function of house value. The main driving force in this relation is the variation in marginal tax rates, although differences in mortgage interest rates reinforce the negative relation.

Figure 7

CAP Rate and Mean User Cost for Washington DC Metropolitan Area in 2007

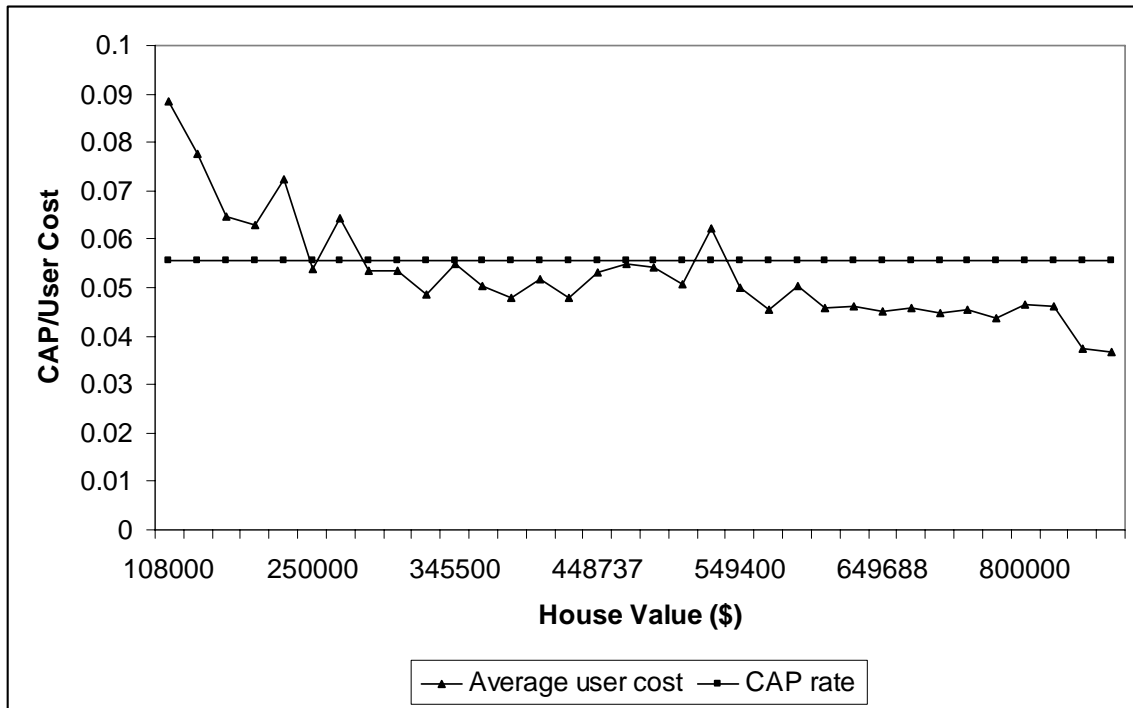


Figure 7 confirms the relation between CAP rates of landlords and households that are the basis for the theoretical model presented in Figure 4.

**V. Empirical Test of the relation between CAP Rates and Asset Value**

**Micro Data**

Data were collected consistent with a natural experiment performed by the housing market in which the effects of heterogeneity in structures were removed by careful matching of physical and geographic characteristics of housing units. Rental records and sales records were matched at the zip code level for the District of Columbia and Maryland parts of metropolitan area around Washington, DC. Advertised rents were

collected from Craig’s List (www.craigslist.com) and Condo.com from August 2008 through the beginning of September 2008. Sales price data and property information came from county tax records for properties that are sold from Jan 1<sup>st</sup> 2008 through June 30<sup>th</sup> 2008.

Each pair of properties is the same type, either single family or multifamily, in the same zip code, within close proximity, usually on the same block by physical address, and finally similar in size and other physical characteristics.

For single family housing units, more than 46 pairs in 40 zip codes were matched, and for multifamily housing units, 102 pairs in 50 zip codes were matched. Most of these matches were located in Montgomery County, Prince George’s County and the District of Columbia with a small number in Fredrick County. Single family housing unit pairs are hard to find in the heart of the city and multifamily housing unit pairs generally could not be found in extremely wealthy neighborhoods. Appendix 2 further discusses the data.

Summary statistics for the micro data is show in Table 2. A scatter plot of the matched rent vs. house value is shown in Figure 8. This matched-pair data on posted prices may be compared to the graph of owner estimates of rental equivalence and value in Figure 3. It seems apparent that the survey data indicate greater concavity than the market prices.<sup>8</sup>

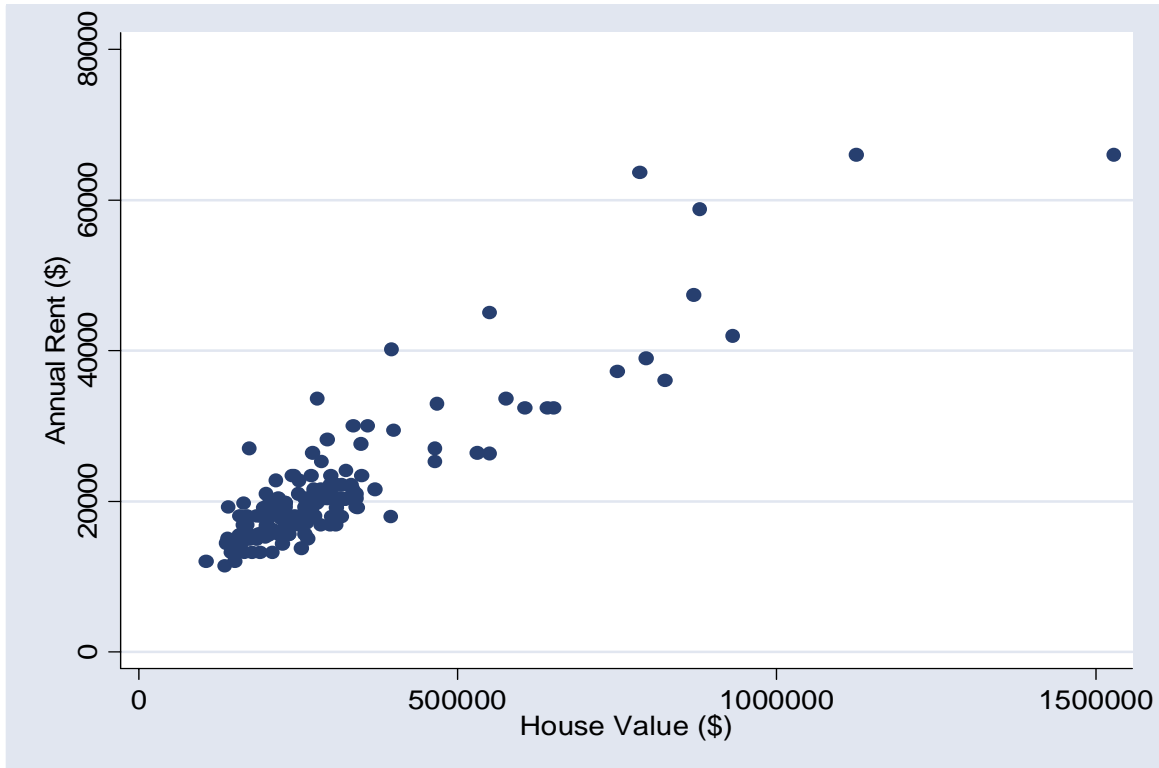
**Table 2**  
**Descriptive Statistics from the Matched D.C. Housing Data**

<b>Single Family</b>					
<b>Variable</b>	Obs	Mean	Std. Dev.	Min	Max
<b>House value</b>	46	415993	225312	165000	1125000
<b>Annual rent</b>	46	27949	11739	17100	66000
<b>Individual unit CAP rate</b>	46	0.0732	0.0216	0.0436	0.1553
<b>Multifamily</b>					
<b>Variable</b>	Obs	Mean	Std. Dev.	Min	Max
<b>House value</b>	102	255737	164652	105000	1528569
<b>Annual rent</b>	102	18840	6866	11400	66000
<b>Individual unit CAP rate</b>	102	0.0795	0.0166	0.0432	0.1371

<sup>8</sup> Because I do not have access to the BLS rental equivalence data, I cannot test for differences in the degree of concavity of the rent-value relation between the two data sources.

Figure 8

Scatter Plot of Matched House Value and Rent



### CAP Rates and Asset Value

The relation between asset and rental prices for these matched pairs of units can now be estimated with and without fixed effects at the Zip code level. It is assumed that the appreciation rates are the same for these carefully matched housing units within the same zip code, because the effects of physical and geographic heterogeneity have been removed. The test for concavity in the rent-value relation can proceed by direct estimation of equation:

$$R_i = \beta_1 A_i + \beta_2 A_i^2 + \varepsilon \quad (3)$$

where  $R_i$  is the rent for a housing unit  $i$ ,  $A_i$  is the asset price of the matched housing unit and  $\varepsilon$  is an identically and independently distributed disturbance. The segmented market model developed above predicts a concave rent curve with respect to value in

which  $\beta_1$  is positive and  $\beta_2$  is negative. The conventional view of a constant CAP rate predicts  $\beta_2 = 0$ .

Table 3 shows ordinary least squares estimates of the parameters of equation 3:

**Table 3**  
**OLS Estimation of Equation 3**

<b>Value</b>	<b>Multifamily</b>	<b>Single Family</b>	<b>Combined</b>
<b>Value</b>	0.0809*** <sup>9</sup>	0.0806***	0.0806***
<b>Value2</b>	-2.64E-08***	-2.75E-08***	-2.75E-08***

Estimation results fit the prediction of the model with segmentation. The second order effect of asset values on rents is negative and significant in all the equations whether estimated separately using single family or multifamily housing units or combined. Interestingly, estimation results show that housing type is not significant in determining the rent/value ratio at different levels of housing value.<sup>10</sup> This is consistent with the theory and indicates that effects of unobserved heterogeneity were removed in the matching process as effectively for single as for multifamily housing.<sup>11</sup>

Figure 4 and Figure 5 show the different predictions of the model with market segmentation for the relationship between rents and assets prices for single family and multifamily housing units. Figure 9 and Figure 10 show the actual rent/value relation strongly supports the model with segmentation in which the market CAP rate is decreasing at the upper end of housing market.

The concavity in the value/rent function appears to be less extreme than that found in Consumer Expenditure Survey data shown in figure 3 where owner's estimates of value and rental equivalence are related but the general functional form is similar.

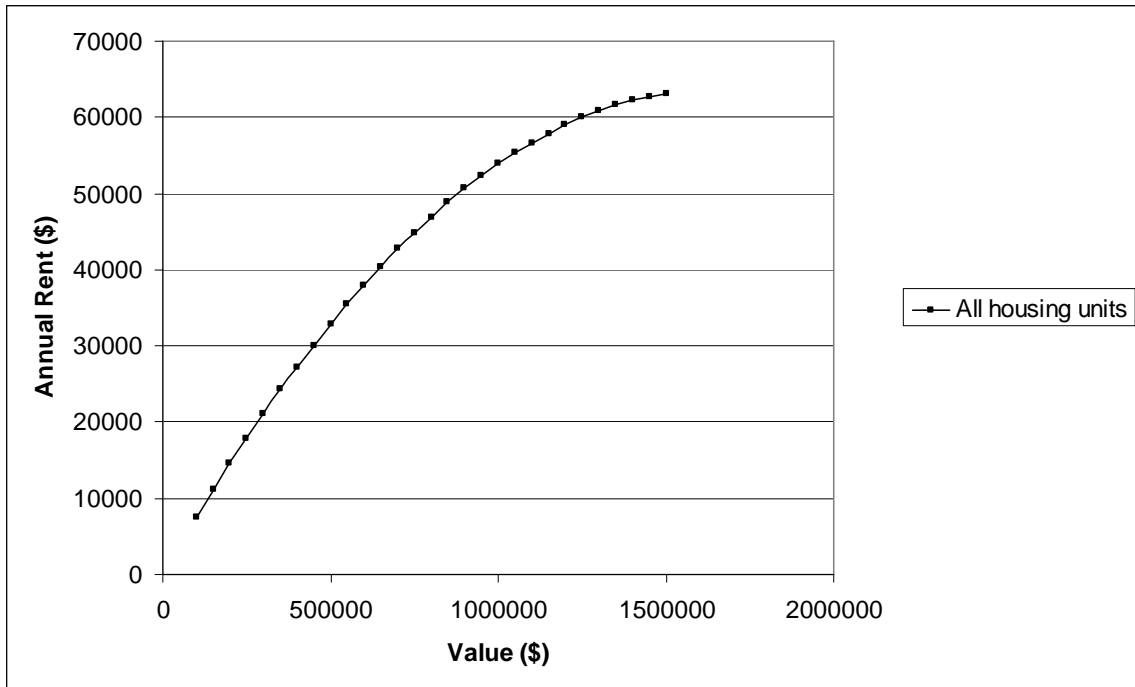
<sup>9</sup> Level of significance for estimates in this paper: \*\*\* 1%, \*\* 5% \* 10%.

<sup>10</sup> Tests of interactive dummy variables indicated that the estimated differences in the parameters for single and multifamily housing are not statistically significant.

<sup>11</sup> Problems of unobserved heterogeneity are potentially greater for single family housing because differences in structure type are larger. In some cases, multifamily unit pairs were actually within the same structure, and in other cases, multifamily units were offered for rent or sale.

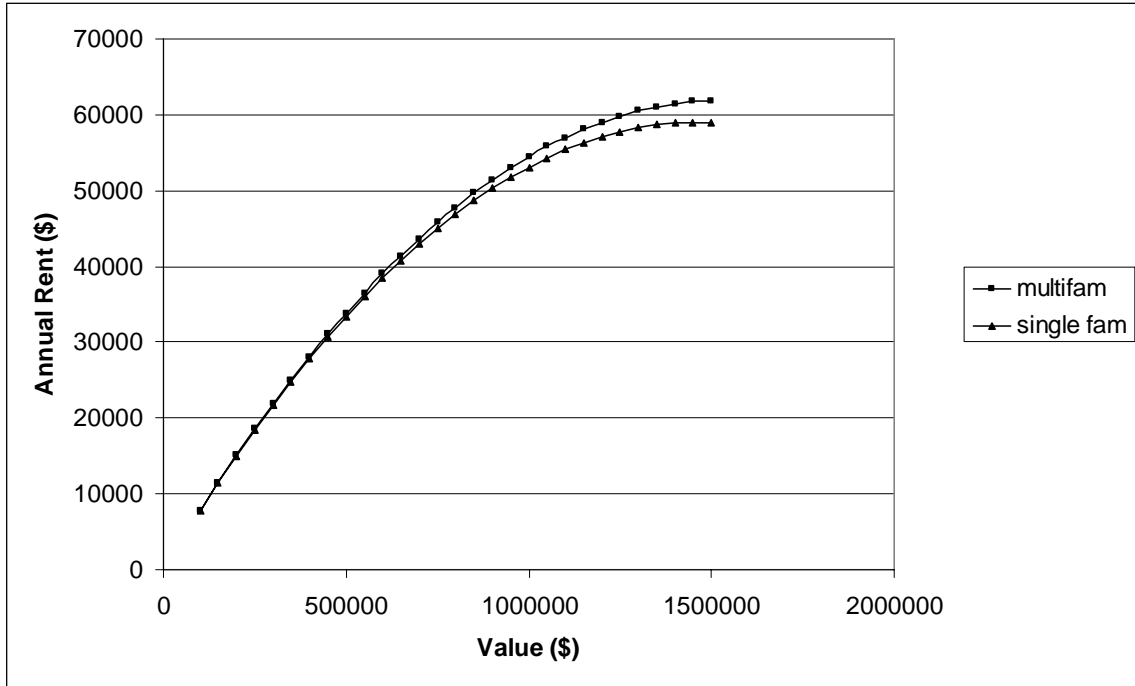
**Figure 9**

**Relation between House Value and Annual Rents Estimated with Full Sample**



**Figure 10**

**Relation between House Value and Annual Rents in Separate Estimations for  
Multifamily and Single Family Housing Units**



A piecewise linear spline function estimation provides an even sharper test of the segmented housing market model. Spline regressions specifically assume that the data are segmented. They are designed to estimate the slopes of the linear relations in different segments of data. The pattern of landlord CAP rates and user cost of owners displayed in Figure 7 suggests that the observed relation between rent and value in a segmented model of the Washington, D.C. housing market should be piecewise linear. To illustrate the estimation process, first assume housing stock is segmented at some housing value  $A^*$  which is called a joint or knot. Housing units that are less expensive are subscripted with 1 and otherwise 2. Rents and house value at two different segments follow different CAP rates but CAP rates within segments are assumed constant:

$$R_{1i} = \beta_1 A_{1i} + \varepsilon \quad (4);$$

$$R_{2i} = \beta_2 A_{2i} + \varepsilon \quad (5).$$

And combined estimation is

$$R_i = [\beta_1 A_i] D_1 + [\beta_2 A_i] D_2 + \varepsilon \quad (6),$$

where  $D_1 / D_2$  is 1/0 when housing units belong to the first segment and 0/1 otherwise.

After some iteration, equation 6 can be expressed as:

$$R_i = \beta_1 X_1 + \beta_2 X_2 + \varepsilon \quad (7),$$

where  $X_1 = A_{1i} D_1 + A^* D_2$  and  $X_2 = A_{2i} D_2 - A^* D_2$ . Equation 7 is used to estimate the slopes of the two segments.

The estimation can be carried out with specific assumptions about the position of joints in the relation or maximization procedures can be designed to search for such joints if they are not known.<sup>12</sup> Two versions of equation 7 are estimated. One imposes the joint based on actual average user cost and market CAP rate calculation shown in Figure 7. Because Figure 7 shows clearly that user cost moves significantly lower than the landlords' CAP rate for housing valued above 600,000 dollars, the model is estimated with a single joint exists at housing consumption of 600,000 dollars. Two alternative Spline models are estimated in which joints are determined endogenously, one with a single knot and the other with two knots. Estimation results are shown in Table 4 are both significant and meet expectations. The CAP rate of higher value housing units is significantly lower than that of lower value units. Figure 11 allows comparison of the estimation results.

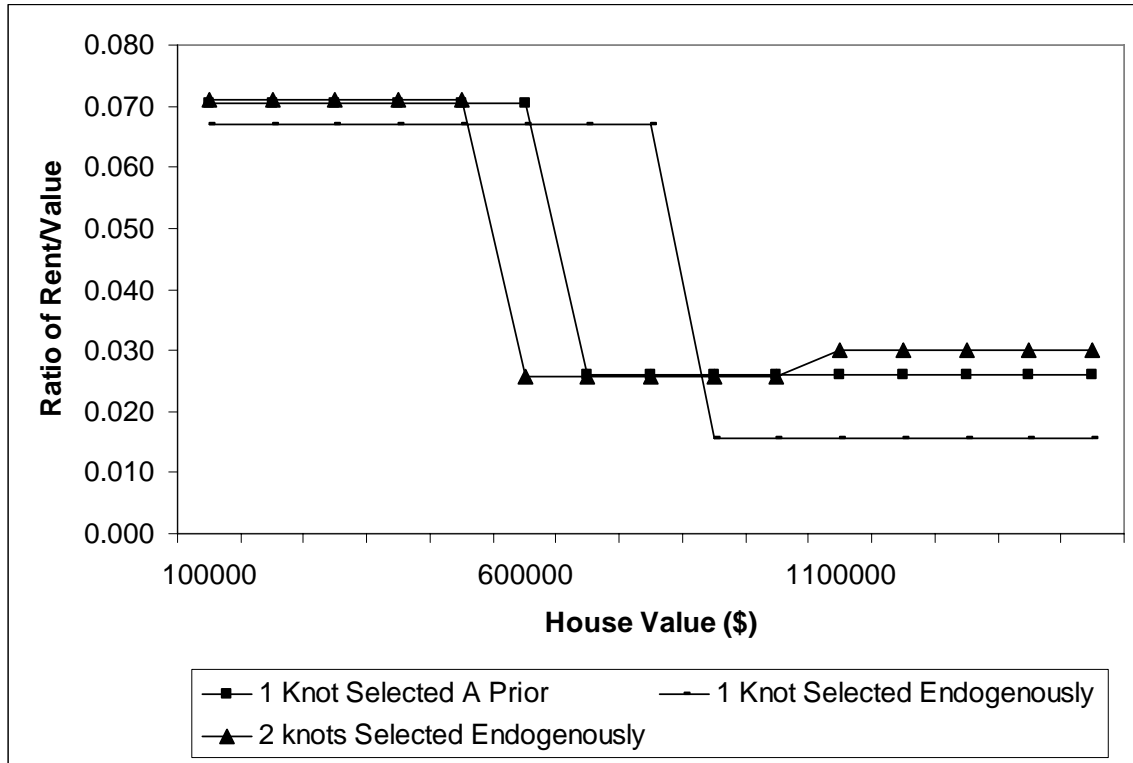
**Table 4**  
**Spline Estimation of Equation 7**

<b>Exogenous Knot</b>		<b>Endogenous Knot(s)</b>	
<b>Single Knot</b>		<b>Single Knot</b>	
<b>Segment (House Value \$)</b>	<b>Estimate</b>	<b>Segment (House Value \$)</b>	<b>Estimate</b>
-600000	0.071***	-816784	0.067***
600000-	0.026***	816784-	0.016**
<b>Double Knots</b>			
<b>Segment (House Value \$)</b>		<b>Estimate</b>	
-579523		0.071***	
579523-1054046		0.026**	
1054046-		0.030**	

<sup>12</sup> See, for example, Marsh, Maudgal, and Raman (1990) for a discussion.

**Figure 11**

**Relation between House Value and Annual Rents in Two Segments of the Same Housing Market**



Comparison of Figures 7 and 11 shows remarkably close agreement between the pattern of CAP rates expected and the observed rent to value relation from the spline estimates. Recall that, for values below 600,000 dollars, CAP rates should be based on user cost of landlords and then fall to rates based on user cost of households. This pattern is found in the rent/value data. Of course the rent/value ratio is based on gross rents received by landlords and the landlords' required CAP rates calculated are lower because they are based on net rental income.

**VI. Conclusion**

The empirical results obtained from the specially collected matched-pair data on rental and owner properties support the predictions of the model of a segmented housing

market in which CAP rates are variable, particularly as house value becomes sufficiently large. The results reject the constant CAP rate assumption across all house values. However, for lower valued units, ranging up to 600,000 dollars in the Washington D.C. dataset, it appears that a no arbitrage equilibrium at a constant rent/value ratio exists. These results have a number of implications. First, there is a substantial segment of the housing market for which the traditional analysis is of asset price based on rents capitalized at the user cost of landlords is appropriate. Second, applying this CAP rate to rents of higher value housing units may give the false impression that these units are overpriced. Third, tests for efficiency in housing market pricing that use rent to value ratios may result in incorrect conclusions unless the rents and values are carefully matched. Fourth, the current CAP rates that are used by the BEA in the determination of national income should be reconsidered.

The segmented market model suggests the shape of the rent to value relation should vary according to the number of inframarginal landlords and the distribution of household user costs for higher income households. As the number of inframarginal landlords increases, the concavity in the rent-value relation should increase. CAP rates for high value units should be highly variable over time across markets because the user cost of renters at high end of the market could vary significantly with factors such as inflation and mortgage rates and the number of inframarginal landlords may also be quite variable across space and time. These implications of the model may be of interest for future research.

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## Appendix

### A1

#### Set of Parameters in Calculating CAP Rate and User Cost for Washington DC

##### Metropolitan Area in 2007

Parameter	Value	Note
$i$	0.06	Landlord interest rate
RP	0.03	Risk premium
$\tau_p$	0.03	Property tax rate
$\rho$	0.015	general inflation
$\tau_p$	0.36	Income tax rate
$\delta$	0.03	Tax depreciation deduction
C	0.02	Operating cost
$\pi_0$	0.045	Annual increase in operating cost
B	0.03	Cost of selling
$\pi$	0.06	House price inflation
$\tau_c$	0.36	Capital gain tax rate
$\tau_{rp}$	0.24	Tax rate associated with risk premium

### A2

#### Data Collection Process and Its Advantage

The special micro data set collected to support the empirical tests reported here builds on an approach taken by Smith and Smith (2006) who attempted to find pairs of single family units matched so that they differed by no more than 100 square feet in interior area, one bedroom, and one half bath. Units were located within one mile. The purpose of this match was to eliminate differences in physical attributes and location that could cause rents and values to differ so that CAP rates could be constructed directly as the ratio of rent to value.

The data used in this study are similar to that of Smith and Smith (2006) but improve on their technique in two ways. First, the matching process does not rely on merging records in commercial database only. Because this study focuses on Washington DC only, neighborhood characteristics and physical characteristics are compared more carefully. The second, and more important improvement, is the inclusion of multifamily housing unit matches. As contrast to single family housing units, multifamily housing units have the advantage of being more homogeneous structures with less margin for heterogeneity in physical characteristics. Given that many pairs are located within one physical building, structural characteristics and proximate surroundings are identical. Even for pairs that are not in the same building, many are located in a single development that was designed and constructed by the same builders so that units in distinct buildings are essentially identical. The reliability of the multifamily data provides not only comparison to but also serves as a double check for the single family housing units analysis

The method of data collection begins with property transfer and tax records from Washington DC and counties in the Maryland suburbs. Real estate transactions occurring between January 1, 2008 and June 30, 2008 were selected.. Home owners can post rental advertisements on internet websites (craigslist.org and condo.com), and these advertised rents were used to determine annual rental income. For each rental record of which address was available, the address was used to find the property in county tax records. In some cases the address on the advertisement was not detailed enough but it can be combined with other information published in the records (for example, owner's name, square footage or other physical characteristics) and used to identify the property in the tax records. County tax records provide the location, and some physical characteristics which allow reasonable comparisons between these rental records and sales records were selected as potential matches in this first step. This is the initial screening for matches. Some records can be identified as good matches during this step, particularly cases where units were in the same structure or same block.

After initial screening, promising pairs that are possibly matches but not in identical locations are subject to a test of distance using Google Map. This is the second

screening. Some properties were found to be located close enough so that they were selected as matches in this second step. In the third step, potential matches, usually not located further than 1 mile or which physical characteristics that were hard to compare, were evaluated using the shoe leather (physical inspection) approach to determine if physical inspection of the exterior indicated that pairs were matches.