Nuclear Physics with lattice QCD

Paulo Bedaque & NPLQCD collaboration

(Beane, Orginos, Savage, Luu, Pallante, Parreno …, Sato, Walker-Loud)

- Goals of the program
- How it works and basic problems
- Current program and some results
- Future directions

The theory of strong nuclear forces does not describe (yet) nuclear forces …

Matter of principle and matter of practice

0, 1 baryon

- hadron masses
- decay constants
- weak matrix elements
- pion scattering
- "exotic" particles
- …

2 or more baryons

- NN phase shifts
- hyperon interactions
- pion-nucleon couplings
- electroweak "exchange currents"
- three-body forces
- …

QCD reduced to quadratures

sea quarks

$$<|\,\bar{q}...q(x)\ \ ...\ \ \bar{q}...q(y)|> = \int DA\, e^{-S[A]} \det(\gamma.(\partial + A) + m)\ \ (\gamma.(\partial + A) + m)^{-1} ... (\gamma.(\partial + A) + m)^{-1}$$

always the same

depends on the operator considered

valence quarks

$$\simeq \sum_{\{A\}} (\gamma.(\partial + A) + m)^{-1} ...\ (\gamma.(\partial + A) + m)^{-1}$$

generate configurations >> quark propagators >> observables

Extracting physics from euclidean space : masses are "easy"

$$< 0 | \pi(t, \vec{k} = 0)\, \pi^{\dagger}(0, \vec{k} = 0) | 0 > = \sum_n e^{-Ht} < 0 | \pi(0, \vec{0}) | n >< n | \pi^{\dagger}(0, \vec{0}) | 0 >$$

$$\underset{t \to \infty}{\longrightarrow} \quad e^{-m_{\pi} t} < 0 | \pi(0, \vec{0}) | \pi >< \pi | \pi^{\dagger}(0, \vec{0}) | 0 >$$

some operator with quantum numbers of the pion, made of quarks and gluons, for instance:

$$\overline{q}(0, -\vec{p})\gamma_5 \tau^a q(0, \vec{p})$$

lowest energy state with the quantum numbers of the pion

Extracting physics from euclidean space : scattering is "impossible"

$$<0|\pi(t,-\vec{k})\,\pi(t,\vec{k})\pi^\dagger(0,-\vec{k})\,\pi^\dagger(0,\vec{k})|0>=\sum_n e^{-Ht}<0|\pi(0,-\vec{k})\,\pi(0,\vec{k})|n><n|\pi^\dagger(0,-\vec{k})\,\pi^\dagger(0,\vec{k})|0>$$

$$\underset{t\to\infty}{\longrightarrow}\; e^{-2m_\pi t}<0|\pi(0,-\vec{k})\,\pi(0,\vec{k})|\pi\pi \text{ at rest}><\pi\pi \text{ at rest}|\pi^\dagger(0,-\vec{k})\,\pi^\dagger(0,\vec{k})|0>$$

uninteresting off-shell amplitude

No scattering from infinite volume euclidean amplitudes

(Maiani-Testa “theorem”)

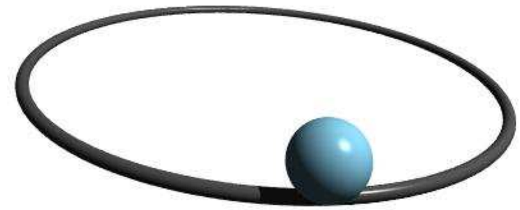
Scattering through finite volumes: the Luscher method

(Marinari, Hamber, Parisi, Rebbi)

one particle

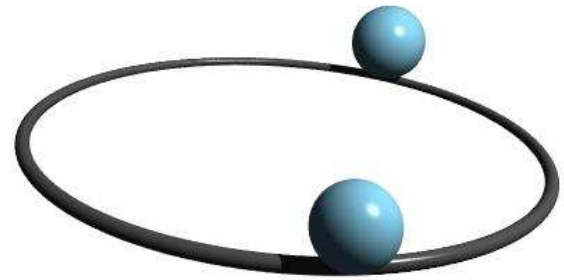
Periodic boundary conditions: box is a torus

Energy levels at $E_n = \sqrt{\left(\frac{2\pi n}{L}\right)^2 + m^2}$



two particles

for L >> a : $\Delta E = \frac{4\pi a}{ML^3}\left(1 + c_1 \frac{a}{L} + \cdots\right)$



In the a<<L limit we can work in perturbation theory

$$\Delta E \simeq \int dr_1 dr_2 |\psi(r_1, r_2)|^2 V(r_1 - r_2) = \frac{1}{L^3}\int dr\, V(r) = \frac{4\pi a}{ML^3}$$

In general (R<<L)

$$\sqrt{M\Delta E}\cot\delta(\Delta E) = \frac{1}{\pi L}\mathcal{S}\left(\frac{M\Delta E L^2}{4\pi^2}\right)$$

known function

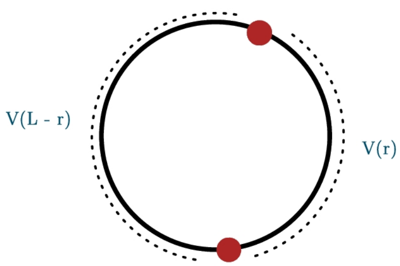
Can learn about the deuteron in boxes smaller than the deuteron

Problems

- Large volumes ?
- Small pion masses ?
- Unquenched ?
- Large statistics ?

Large volumes: fitting two nucleons inside a box

pions "around the world" are not a problem



$$\Delta E = \frac{1}{L^3}\int_{L^3} dr V_L(r) = \frac{1}{L^3}\int dr\, V(r) = \frac{4\pi a}{ML^3}$$

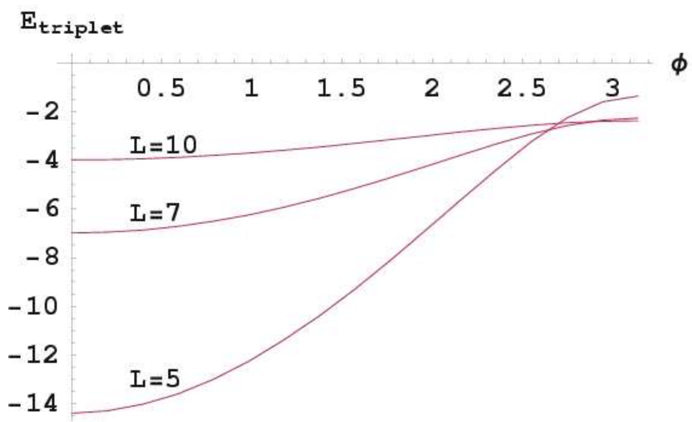
(P.B & I. Sato)

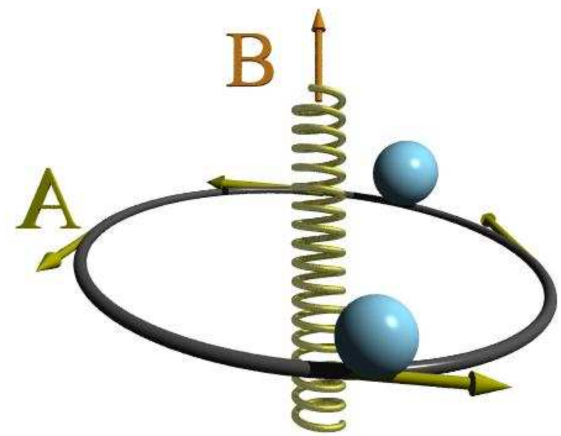
much more complicated analysis:

realistic m_π, L=5 fm ~ 1% error

$\delta(\Delta E<0)$ or $\delta(\Delta E > 350$ MeV) : unphysical region

Twisted boundary conditions = Aharonov-Bohm (P.B., 04)





Nuclear Physics with Lattice QCD

S. Beane, T. Luu, K. Orginos, E. Pallante, A. Parreno,
M. Savage, …

Initial strategy:

- Use MILC lattices (dynamical, improved staggered)
- domains wall fermions (good chiral symmetry) on the valence sector
- software (QDP++, Chroma) written under SciDac
- initially borrow propagators from LHPC, currently generate our own

Ingredients for our calculation:

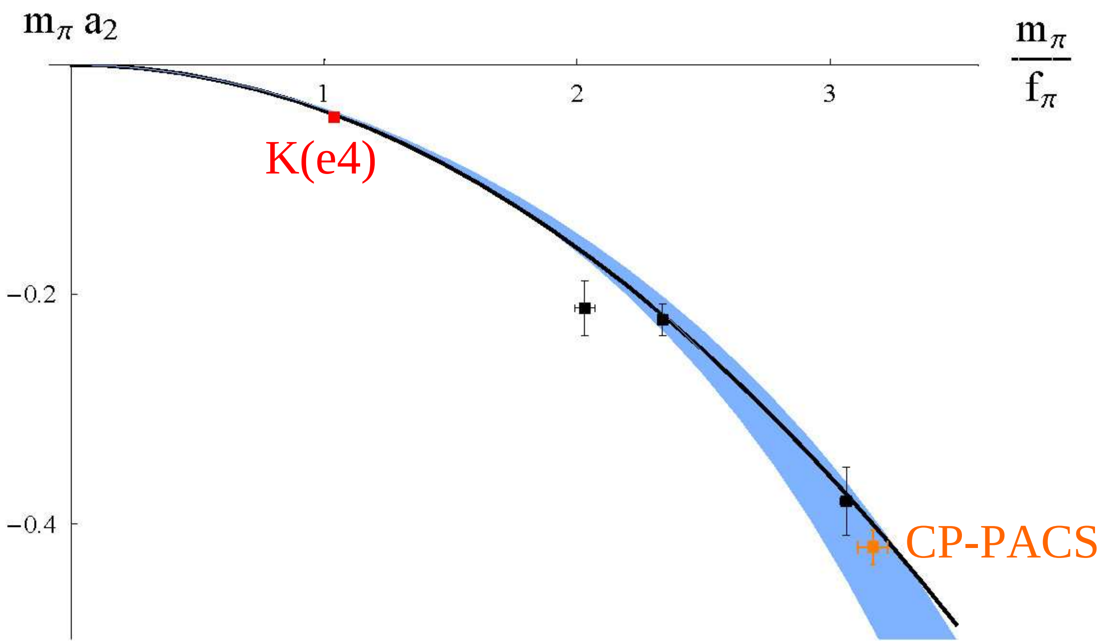
Improved (asqtad) staggered lattices from MILC

- a = 0.125 fm, m_u=m_d=0.007, 0.010, 0.020, m_s = 0.050 (physical): m_π = 294, 348, 484 MeV
- HYP-smeared

Domain-wall fermions with masses tuned to the (lightest) sea pion generated by LHPC (and currently by us)

- domain-wall height = 1.7
- L_5 =16
- quark masses adjusted (to few %) to match lightest sea pion (m_u=m_d=0.0081, 0.0131, 0.0313)
- Dirichlet boundary condition at $t-t_0$=22

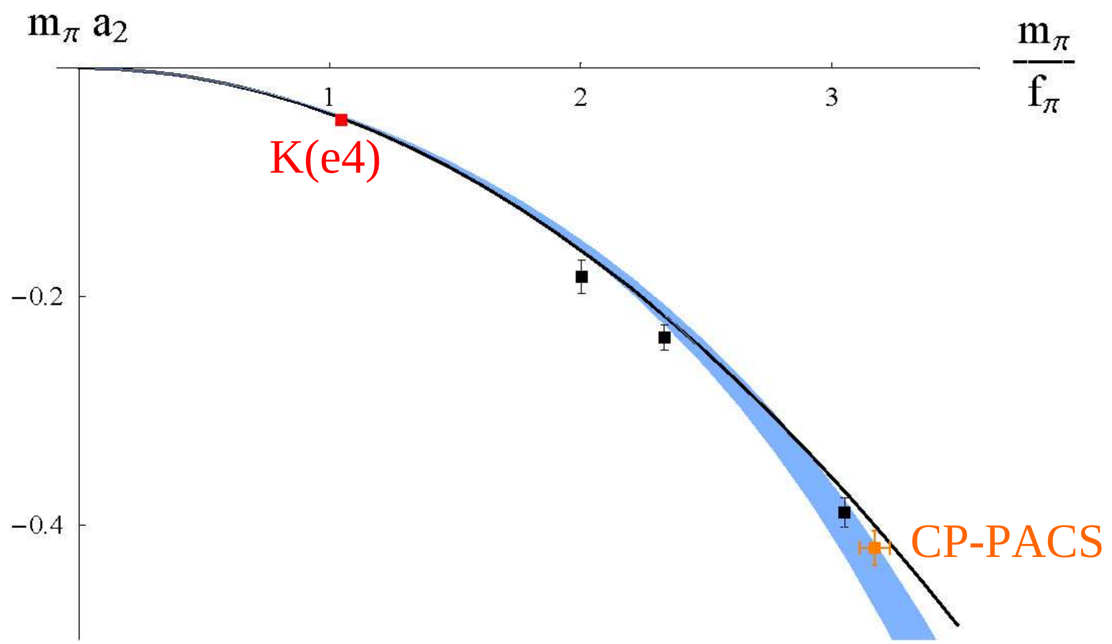
I=2 $\pi\pi$ scattering



$$m_\pi a_2 = -\frac{m_\pi^2}{8\pi f_\pi^2}\left[1 + \frac{3m_\pi^2}{16\pi^2 f_\pi^2}\left(\log\frac{m_\pi^2}{\mu^2} + l_{\pi\pi}(\mu)\right)\right]$$

$l_{\pi\pi}(\mu)$ ↑ $8(l_1+l_2)+2(l_3-l_4)$

Improved statistics



4 x 0.007, 6 x 0.010, 8 x 0.020

weighted fit: $l_{\pi\pi} = 3.3(6)(3)$

different weigths

$m_\pi a_2 = -0.0426\ (6)(3)(18)$

$l_{\pi\pi}$

1-loop – 2-loop
w/o counterterm

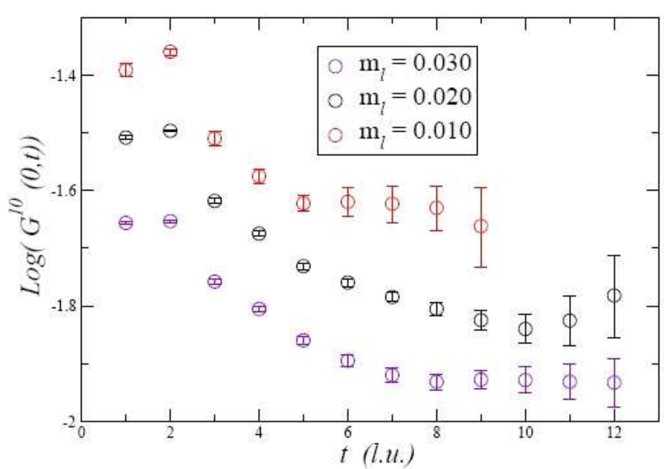
K(e4): $m_\pi a_2 = -0.0454(31)(10)(8)$

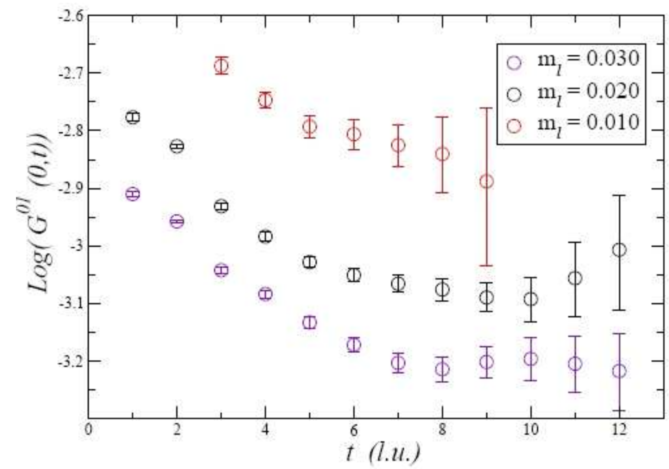
theoretical

χPT predicts discretization errors (a^2) ~ 1% (D. O'Connel, A. Walker-Loud, R. V. Water, J. Chen)

Finite volume ($e^{-m_\pi L}$) ~ 1% (P.B. & I. Sato)

Nucleon-nucleon

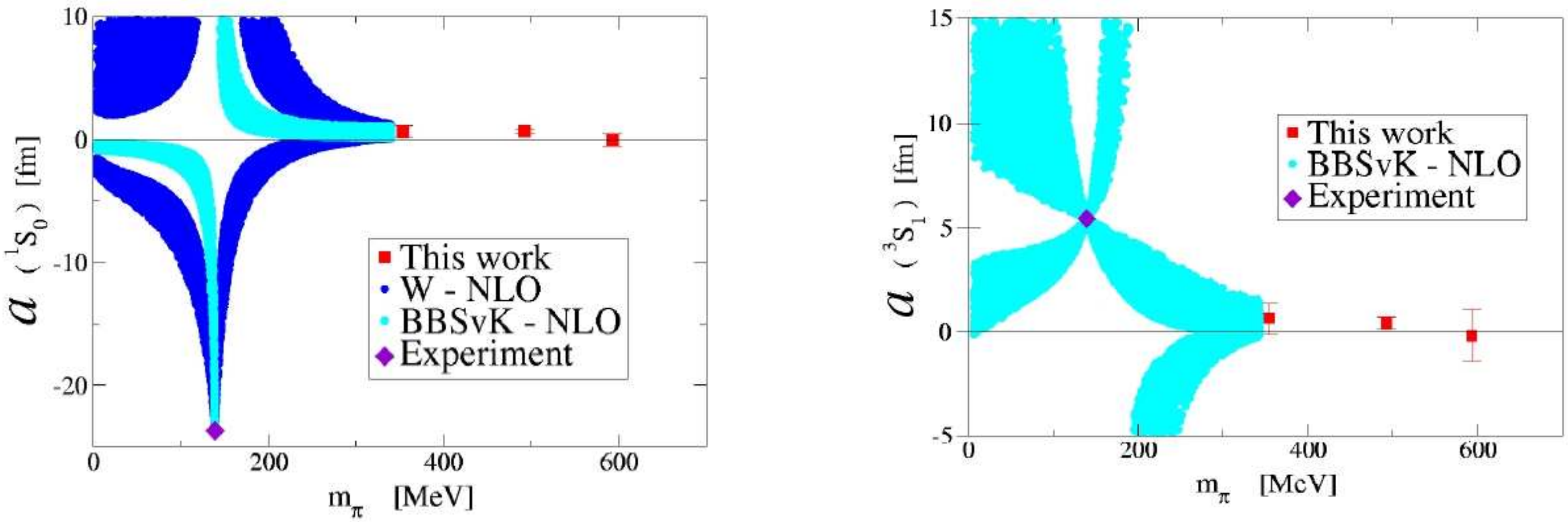




Larger statistical errors: $\frac{\text{signal}}{\text{noise}} \sim e^{-(2M_N - 3m_\pi)t}$

In my view, the single biggest challenge

Chiral "extrapolation"



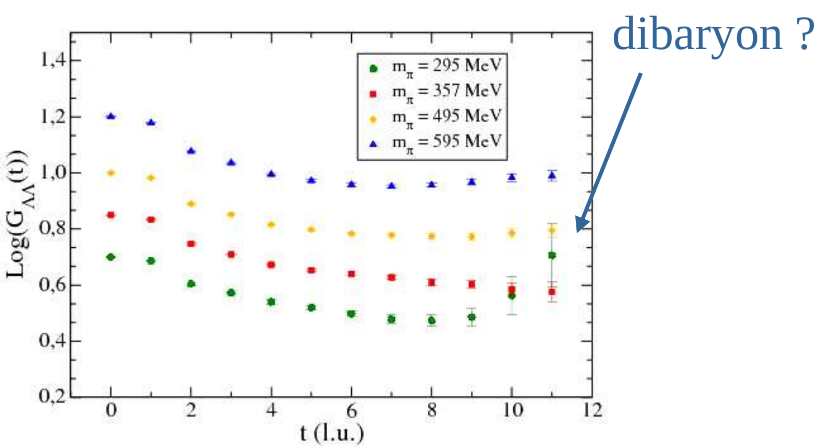
- no anchor at $m_\pi = 0$
- wild behavior of the scattering length with m_q

Other processes

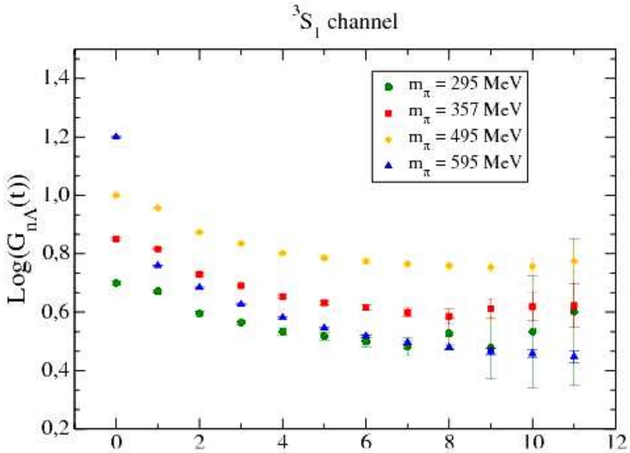
πK:

$$m_\pi a_{3/2} = -0.0574 \pm 0.0026 \pm 0.0058$$
$$m_\pi a_{1/2} = \ \ 0.1725 \pm 0.0017 \pm 0.0156$$

ΛΛ, NΛ, NΣ , ΣΣ, ΞΞ (KK, KN, … to appear soon)



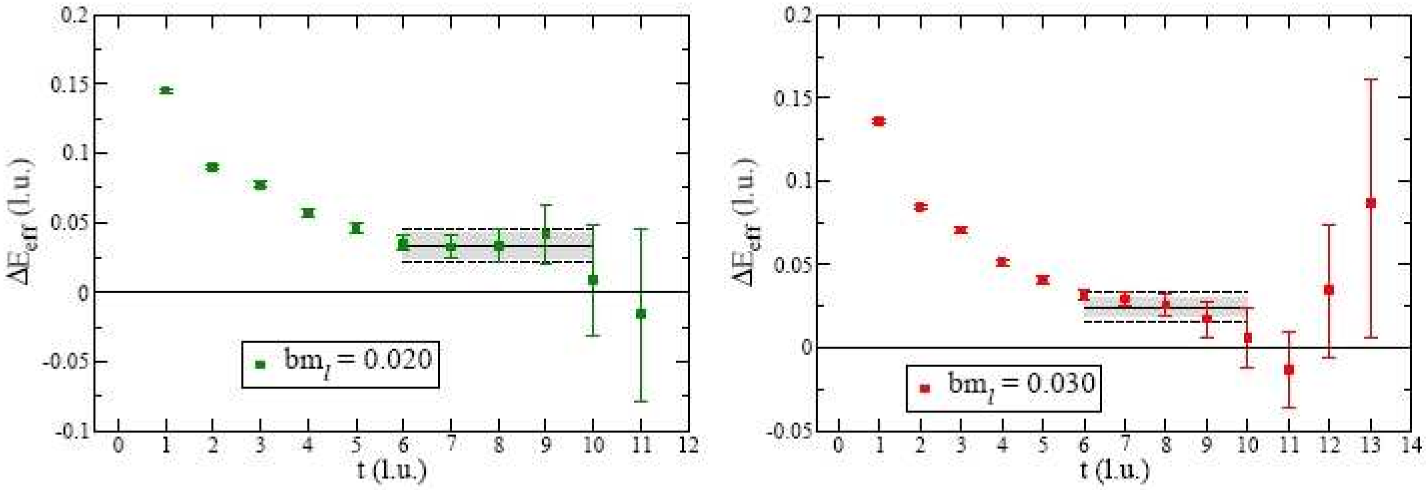
ΛΛ



NΛ triplet

Nucleon-hyperon results

Example: $n\Sigma^-$ triplet



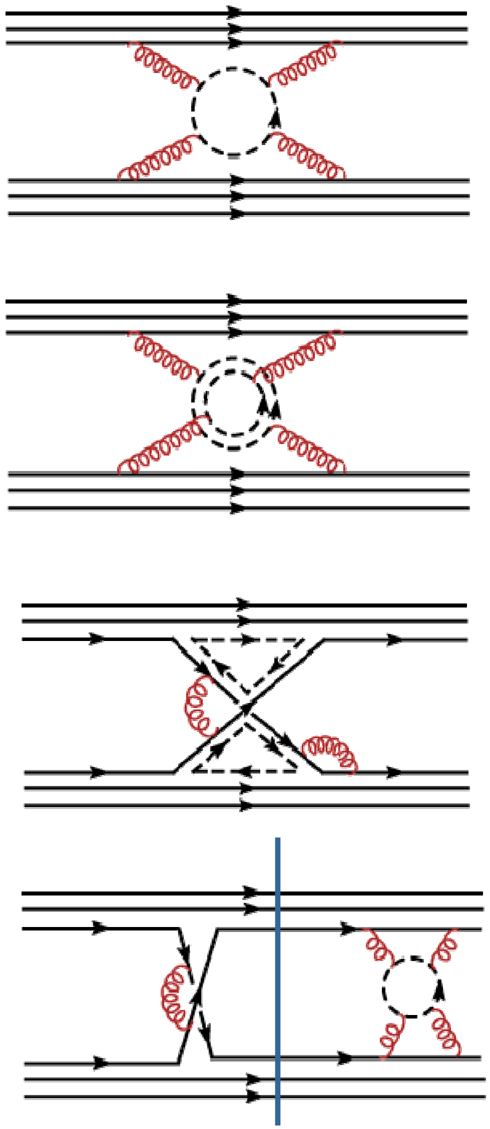
Channel	m_π (MeV)	Range	ΔE (MeV)	$\lvert\mathbf{k}\rvert$ (MeV)	δ (degrees)	$-(k\cot\delta)^{-1}$ (fm)
$n\Lambda$	$592 \pm 1 \pm 10$	8-12	$-9 \pm 8 \pm 20$	–	–	$0.8 \pm 1.4 \pm 0.4$
$^1S_0$	$493 \pm 1 \pm 8$	6-9	$29.8 \pm 5.4 \pm 2.5$	$197 \pm 24 \pm 4$	$-32.3 \pm 8.1 \pm 2.8$	$0.63 \pm 0.12 \pm 0.014$
	$354 \pm 1 \pm 6$	5-9	$56.8 \pm 6.0 \pm 5.5$	$255 \pm 22 \pm 13$	$-53.4 \pm 8.5 \pm 10.1$	$1.04 \pm 0.24 \pm 0.15$
$n\Lambda$	$592 \pm 1 \pm 10$	8-13	$-13 \pm 13 \pm 8$	–	–	$3 \pm 14 \pm 2$
$^3S_1$	$493 \pm 1 \pm 8$	7-11	$-4 \pm 13 \pm 14$	–	–	$(-\infty, \infty)$
	$354 \pm 1 \pm 6$	5-10	$23 \pm 17 \pm 4$	$168 \pm 62 \pm 14$	$-23 \pm 18 \pm 4$	$0.50 \pm 0.26 \pm 0.06$
$n\Sigma^-$	$592 \pm 1 \pm 10$	9-13	$-17 \pm 11 \pm 27$	–	–	$(-\infty, \infty)$
$^1S_0$	$493 \pm 1 \pm 8$	5-9	$24.9 \pm 7.8 \pm 3.0$	$179 \pm 28 \pm 11$	$-27.2 \pm 9.0 \pm 3.8$	$0.57 \pm 0.13 \pm 0.05$
$n\Sigma^-$	$592 \pm 1 \pm 10$	6-10	$38.5 \pm 8.8 \pm 5.0$	$226 \pm 26 \pm 15$	$-44.3 \pm 9.8 \pm 5.4$	$0.85 \pm 0.20 \pm 0.10$
$^3S_1$	$493 \pm 1 \pm 8$	6-10	$53 \pm 14 \pm 5$	$261 \pm 35 \pm 13$	$-58 \pm 15 \pm 5$	$1.19 \pm 0.51 \pm 0.15$

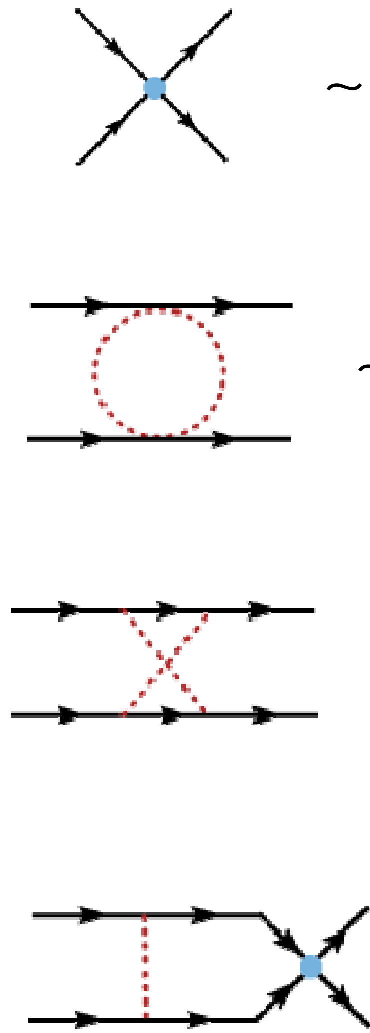
statistical systematic

Conclusion

- Feasibility of interactions in the lattice demonstrated
- Meson-meson is competitive w/ experiments, lattice and χPT hand-in-hand, Gasser-Leutwyler coefficients determined
- Hadron-hadrons much harder (signal/noise, chiral extrapolation)
- Anisotropic Wilson lattices for next year, better signal

Twisting only the valence quarks (P.B. & J. Chen, `04)





$\sim\ e^{-\Lambda_{QCD}L}$

$\sim\ e^{-m_\pi L}$

$\sim\ e^{-m_\pi L}$

dependence on sea b.c. exponentially suppressed