CENTER FOR ECONOMIC RESEARCH
DISCUSSION PAPER NO. 00-02.

Are Business Cycle Dynamics the Same Across Countries? Testing Linearity Around the Globe

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March 2000

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* A preliminary version of this paper was presented at the Western Economic Association International meetings in June 1999 and at George Washington University. We thank Sei-Wan Kim and Inchul Kim for research assistance, and Simon Potter for comments and suggestions that improved the paper. Jansen thanks the Private Enterprise Research Center for research support.

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1. Introduction

The study of macroeconomic fluctuations has been enhanced recently by the exploration of the nonlinear dynamics that generate those fluctuations. Although there has long been a recognition that business cycles are asymmetric (e.g., Neftci (1984) and others), with state dependent dynamics, relatively little work was done characterizing the nature of those dynamics. Recent advances in nonlinear econometric techniques, however, have permitted a broader exploration of business cycle characteristics.

In this paper we contribute to the literature on nonlinear dynamics by investigating real GDP dynamics for a wide range of countries. We first test for the existence of nonlinearities and, where appropriate, estimate one or more types of nonlinear models. By examining GDP dynamics for both high and middle income countries, we can investigate the degree of commonality in business cycle movements across economies with different structures. In this way, we can help determine the degree to which it is appropriate to use common characterizations when describing business cycle behavior across countries.

Our results suggest that there is a great deal of heterogeneity in the dynamics of real output growth. About one fourth of the countries we tested appear to have linear business cycle dynamics, setting them apart from the rest of the sample. Among the three quarters of countries that produce evidence indicating nonlinear dynamics, there remains a substantial amount of heterogeneity.

We attempted to fit two different classes of nonlinear models, and each was found applicable to a number of countries. Moreover, even within one of these specific types of nonlinear model, we found significant differences in the dynamics described by the country-specific models.

To preview our results, we found substantial evidence in favor of rejecting linearity and employing nonlinear dynamics for aggregate economic performance as measured by real GDP. However, we found cases were linearity could not be rejected. We also found a substantial amount of heterogeneity in the characterization of that nonlinearity, with some countries characterized by one or another of the smooth transition autoregressive (STAR) models, either the exponential STAR or the logistic STAR models, and still others by Beaudry and Koop's CDR model. Moreover, within each class of models there were material differences in the estimated parameters, indicating that the are substantially different dynamics generated within each class of models for each country. While our result is a broad survey of countries and does not dwell long on any one, it seems clear to us that cross-country heterogeneity of the nonlinearities is a key feature of the data.

2. Testing for Linearity

Because we are investigating data on a wide range of countries, some with potential data collection and construction problems, we initially plotted the data in both level and logs. We then examined the plots to identify potentially problematic data sets. This process revealed a number of data problems with the IFS CD-ROM data. Some were easy to correct, including unexplained spikes in the data for Portugal in 1992.4 (which was corrected by looking at a later issue of the IFS CD), a large spike in the first period for the Philippines (which was corrected by deleting the first observation, 1980.1), an unexplained spike in the Netherlands data in 1997.2 (again corrected by looking at a subsequent issue of the IFS CD). Other problems were corrected by getting alternative data directly from central government or central bank sources as we did for Germany, Japan, and Taiwan. In a few cases the data problems led us to drop countries for which real GDP data was available. These included Argentina, for which the data series covered 1968 - 1997 but had missing values for 1991 - 1993, and Denmark, which only had 40 observations. Our rough rule of thumb was to require at least 50 observations, already a small number for a study that estimates nonlinear time series models. Hong Kong, with only 48 observations, was dropped, as was Malaysia (32 observations) and Turkey (48 observations). Israel was dropped because the data showed a large and unexplained change in the behavior of the series between the pre-1981 period and post 1981, although in retrospect we could have fit the series from 1981 - 1998. Korea also exhibited a troubling pattern of continually declining variance beginning early in the sample and declining almost linearly with time. We report some tests on Korean data and eventually managed to estimate a model that seems sensible. In the end, however, we do not have much confidence in our estimated nonlinear model for Korea.

Our preliminary data analysis, especially our minimal sample size criterion and our "no holes in the data" criterion, cut our sample of countries from 32 to 26 (including Korea but excluding Israel). The countries and the time periods for which data are available are listed in Table 1.

The issue of using seasonally unadjusted or seasonally adjusted data is one that is seemingly ever-present, and was especially problematic for us because we could not always get both types of data. We handled this choice as follows. When available on the IFS-CD, we used seasonally adjusted data. For those countries that did not have seasonally adjusted data available, we used seasonally unadjusted data.

Given that our focus is testing for the presence of nonlinearity for a large number of countries, we take a straightforward approach to dealing with seasonality. While we preferred using seasonally adjusted data to sidestep these important issues, when these were not available we used seasonal dummies to control for seasonality. This is admittedly just one of a number of approaches for dealing with seasonality, and it implicitly assumes that seasonality is deterministic. An alternative is to use one of the approaches that assume the seasonal mechanism is stochastic. In that case one might test for seasonal unit roots and employ seasonal differencing (Hylleberg, Engle, Granger and Yoo(1990), Frances(1994) or Andrade, Clark, O'Brien and Thomas (1999)).

There are strengths and weaknesses to both approaches. For example, if seasonality is deterministic and seasonal differencing is applied, over-differencing will result. Alternatively, employing deterministic seasonal adjustments in the face of stochastic seasonality will cause erroneous descriptions of the distributions of regression statistics like the coefficient of determination. Analysis of the consequences of mis-specifying the seasonal structure can be found in da Silva Lopes, (1999) and Franses, Hylleberg, and Lee(1995). Moreover, Franses and Vogelsang (1998) show that the finding of seasonal unit roots may be due to mean shifts in the seasonal series. Finally, Franses (1996) describes an approach that allows for stochastic trends to occur at both the seasonal and nonseasonal frequencies. But all of these approaches add additional layers of complications to a study of nonlinearity in real output across many countries.

Following determination of the list of countries with seemingly reliable data, we examined the basic properties of the individual time series by testing each for the existence of a unit root both in log-levels and in log first differences. We used the augmented Dickey-Fuller test. When the data were in levels we used a model with four lags, a time trend and, where appropriate, seasonal dummies. The model in first difference did not include a time trend. In all cases we were able reject the hypothesis of no unit root in the log levels of real output but in no case were we able to reject that hypothesis for the log first differences.

The next step was to test for the presence of nonlinearities. To do so, we first estimated a linear AR model for each of the countries. To specify lag lengths we used the SIC. We imposed a maximum lag length of eight quarters. This value allows for fairly long dynamics while mitigating the amount of data lost for initialization. Suppose that the data series for a particular country had T observations. After losing an observation for differencing, we then lose eight more for the initialization process. This means that the series used for estimating the linear model actually contained observations running from observation 10 through observation T, and this can be a relatively small number when T can be as small as 50.

Using data for each country, we estimated a simple linear autoregressive model with up to eight lags, and seasonal dummies where needed. Summary statistics for the autoregressive models, along with summary statistics for the quarterly growth rate data, are provided in Table 2, while the AR lag lengths are indicated in Table 3. For a few countries, the SIC criterion suggested lag lengths that left statistically significant serial correlation in the residuals. In this case we would attempt to find longer lag lengths sufficient to eliminate the serial correlation. This occurred for Chile (SIC picked 5 lags but 6 were needed to eliminate serial correlation), Norway (SIC picked 3 lags but 5 were needed to eliminate the serial correlation), Spain (SIC picked 1 lag but 5 were needed to eliminate the serial correlation), and the U.K. (SIC picked 1 lag but 3 were needed to eliminate the serial correlation). For Korea, serial correlation is indicated, but the larger problem is a tremendous heterogeneity in the variance that was difficult to model.

For Japan, we also had a severe problem. Plots of Japanese data and results from prior studies strongly suggests a trend break in the Japanese log-real GDP data in the middle of the 1970s. Since we worked with differenced data, this translated into a linear AR model for growth rates with a level shift specified as occurring in 1974.1. While we were unable to find any lag length to eliminate the serial correlation in Japanese data without the trend break, we found a simple AR(1) model with a level shift, and no significant serial correlation.

Again, we emphasize that specifying and estimating the nonlinear models is merely a starting place, and failure of a linear model to pass routine residual diagnostic tests does not necessarily invalidate nonlinear models estimated from a starting point of a linear model with bad residual diagnostic test results. At the same time, given the immense number of possible nonlinear models, we feel it is important to start with a well-specified linear model before moving on to linearity testing and estimation of nonlinear models. This is especially important because linearity tests of the type used here should begin with well-specified linear models. Poorly specified linear models may lead to apparent rejections of linearity that are in fact rejections of the poorly specified linear null. At the same time, one of the reasons residual diagnostics might indicate problems with the residuals from a linear model is that a nonlinear is a better representation of the data. Attempting to fit a linear model to data generated from a nonlinear model is one of the classic reasons for finding problems with residuals of the linear model.[1]

We employed four tests of linearity. The first three make use of the residuals from the linear models but do so in somewhat different ways. The fourth test follows a completely different approach.

The first test is a Lagrange multiplier type test that has been proposed in the smooth transition model literature (e.g. Terasvirta (1994), Terasvirta and Anderson (1992)). In this test, which we call the LM-STR test, the residuals from the autoregressive model are regressed on lag values of the growth rate and interactive terms with the "delay variable." The delay variable implies a specific nonlinear model (described later) and LM-STR test thus has an explicit alternative hypothesis. The delay variable here will be lagged values of the quarterly growth rate and we test for delay variables up to one year (four quarters) in the past. The residual model to be estimated is thus:

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[1]For example, see J. Johnston, Econometric Methods, McGraw Hill, 3rd ed, 1984, p. 309.

$$\hat{e}_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \sum_{i=1}^{p} \lambda_{1i} y_{t-i}\hat{y}_{t-d} + \sum_{i=1}^{p} \lambda_{2i} y_{t-i}\hat{y}_{t-d}^{2} + \sum_{i=1}^{p} \lambda_{3i} y_{t-i}\hat{y}_{t-d}^{3} + \varepsilon_t \qquad \textbf{(1)}$$

The test for linearity is the test of the null hypothesis that:

$\lambda_{1j} = \lambda_{2j} = \lambda_{3j} = 0$, for all j.

Rejection of the null indicates the presence of nonlinearities.

The second test for non-linearity is Ramsey's RESET test. In this test of linearity, we regress the residuals from the linear models on lagged quarterly growth rates and powers of those growth rates:

$$\hat{e}_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i g_{t-i} + \sum_{i=1}^{p} \sum_{j=2}^{3} \nu_{ij} g^{j}_{t-i} \qquad \textbf{(2)}$$

The test for linearity is a test of the null hypothesis that the $\nu_{ij} = 0$ or all I, j. This test has a general alternative hypothesis.

The third and last test using residuals that we performed is the LST test, based upon a polynomial expansion. As with the other two, this test uses the residuals from the linear models and regresses them on lagged values for the growth rate. In this test, however, the residuals are regressed upon interactions terms of lags of varying lengths:

$$\hat{e}_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i g_{t-i} + \sum_{i=1}^{p} \sum_{j=1}^{3} \theta_{ij} g_{t-i} g_{t-j} + \sum_{i=1}^{p} \sum_{j=1}^{3} \sum_{k=1}^{3} \delta_{ijk} g_{t-i} g_{t-j} g_{t-k} \qquad \textbf{(3)}$$

The test for linearity is: $\theta_{ij} = \delta_{ijk} = 0$, for all I, j, and k.

A fourth last test of linearity follows a different path. This test is based upon the "current depth of recession" (CDR) model first developed by Beaudry and Koop (1993). The CDR model is based upon idea that business cycles are asymmetric, with economic dynamics in recessions differing from those in expansions and the estimated model thus includes an asymmetric response term. The nonlinearity is generated by a variable that measures the strength or depth of the current recession. The variable is calculated as the distance between current real GDP and the previous peak level of real GDP. To measure this distance one defines a measure called Current Depth of the Recession (CDR), where:

$$CDR_t = \max\{\ln Y_{t-j}\}_{j \geq 0} - \ln Y_t . \qquad \textbf{(4)}$$

With this definition in place, one estimates an augmented autoregressive model of real GDP growth as:

$$g_t = \alpha + \sum_{i=1}^{p} \phi_i g_{t-i} + \sum_{j=1}^{q} \theta_i CDR_{t-j} + \varepsilon_t \qquad \textbf{(5)}$$

where α is a drift parameter. To interpret this equation, consider the case in which the lag on the CDR term is one. If the coefficient on the CDR variable is positive ($\theta_1 > 0$), economic growth is greater when CDR is positive than when it is zero. Intuitively, this means that growth is faster when the economy is in the recessionary and recovery periods of the business cycle, until it recovers to its previous peak, than when it is growing above its previous peak (the expansionary phase). In this case, economic growth responds more strongly to negative shocks than to positive ones. In addition, a positive CDR coefficient implies that positive shocks have more persistent effects on output than negative shocks. Clearly, if the coefficient on CDR is negative, just the opposite is true. In such a case, when the economy enters a recession it tends to be mired there. To test for linearity one simply tests whether the T coefficients are jointly significantly different from zero.

The CDR model is somewhat misnamed in that the CDR term is non-zero for both the recessionary phase of the business cycle, when output is below its previous peak and falling, and for the recovery phase, when output is below its previous peak but rising. As an alternative, Bradley and Jansen (1997) estimated a model that splits the business cycle into three regions, the recession, recovery, and expansion. Thus Beaudry and Koop's CDR term is effectively split into a term that measures the depth of recession during the time GDP is declining, and a term that measures the distance from the previous peak during the time GDP is growing but still below the previous peak. Bradley and Jansen report that their modified model - which encompasses the Beaudry and Koop specification - receives some support in data for a few of the countries they examine, but for others the restrictions implied by the original Beaudry and Koop specification cannot be rejected.

Another modification of Beaudry and Koop's model that has appeared in the literature is Pesaran and Potter's (1997) version that includes of an "overheating" term to match, on the growth side, the effect of CDR on the recession side.

For seasonally unadjusted data, the CDR model was modified to account for the regular seasonal behavior of real GDP growth. In the modified version, the nonlinear response to output below the previous peak is calculated on a quarter-by-quarter value.

Thus, a recession occurs when the first quarter is below last year's first quarter, when the second quarter this year is below last year's second quarter, and so on. In this case the SCDR term (for seasonal CDR) was defined as:

$$SCDR_t = \max\{\ln Y_{t-4j}\}_{j \geq 0}^{[t/4]} - \ln Y_t . \quad \textbf{(6)}$$

Then the SCDR model is:

$$g_t = \alpha + seasonal\ dummies + \sum_{i=1}^{p} \phi_i g_{t-i} + \sum_{j=1}^{q} \omega_i SCDR_{t-i} + \varepsilon_t. \qquad \textbf{(7)}$$

The results of the four linearity tests are presented in Table 3. The lag lengths for each of the autoregressive models are presented there as well as the probability values for each of the four tests. We also present the delay value that for the LM-STR test that gives the smallest p-value for the test of linearity, as well as the order of the autoregressive model used for the CDR test, where p is length of the AR terms and q is the length of the CDR terms.

The results of testing are summarized in Table 4. Of the 26 countries tested only 6 fail to produce any evidence rejecting linearity. Thus, this group of countries (Finland, Portugal, Peru, Spain, Singapore, and New Zealand) needs no further investigation, they share a linear dynamic process for real output growth. The remaining 20 countries produced some evidence suggesting nonlinear dynamics.

None of the countries showed rejection of the null hypothesis in all four tests. For five countries we found rejection of linearity in three of the four tests. Four of these (Korea, Austria, Japan, and the U.K.) rejected linearity for the three tests based upon the residuals from the AR model, but did not have a statistical significant CDR term. This can be interpreted to mean that nonlinearity is present but not in a form that is consistent with the specific type of asymmetry required by the CDR model. The fourth country, Germany, rejected linearity for LM-STR and LST tests as well as for the CDR model.

Eight countries produced two rejections of linearity. In all cases, one of the rejections was in the LM-STR test, with most of the second rejections occurring in the LST test. Only once was the “confirming” result provided by the RESET test. Finally, seven countries produced only one rejection of linearity. Most of the countries rejected linearity in the LM-STR test but two countries (U.S. and France) found no rejection with any of the residual based tests but did produce a statistically significant CDR term. This result seems puzzling, since for most countries the LST or LM-STR test gives the rejection of linearity rather than the CDR test. One answer is provided by Jansen and Oh (1999), who simulate a data generating process equivalent to the estimated CDR model for the U.S. and report that, for this DGP, the three residual-based tests have all have low power for sample sizes like that we have for the U.S., around 160 observations.

In terms of the tests, the LM-STR test produced, by far, the most evidence against linearity. Of the twenty-six countries tested, eighteen rejected the null of linearity with the

LM-STR test. This can be contrasted with the RESET test for which only five rejections were found. In addition, only five sets of significant CDR terms were found.

3. Fitting the Nonlinear Models

The results presented in the previous section indicated that there is material evidence of nonlinearity in economic growth rates around the world. In this section we attempt to model that nonlinearity. We do so by estimating both smooth transitions autoregressive (STAR) models and current depth of the recession (CDR) models. We estimate STAR models only for those countries that rejected the null hypothesis of linearity in the LM-STR test, but we estimated a CDR-type model for all of the data series. This difference arises because the estimation of the CDR model is joint with testing for nonlinearity. As discussed above, if the CDR coefficient is significantly different from zero, then one has tested for nonlinearity. At the same time, one has estimated the nonlinear model.

A. STAR Models

For our purposes, STAR models come in two types, logistic smooth transition autoregressive models (LSTAR) and exponential smooth transition autoregressive models (ESTAR). Both types of models are based upon traditional threshold autoregressive (TAR) models in which the autoregressive process changes as the values of the switching variable change. For example a threshold model is given by:

$$y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \left(\sum_{i=1}^{p} \beta_i y_{t-i} \right) I_t + \varepsilon_t \qquad \textbf{(8)}$$

where:

$$I_t = \begin{cases} 1 & \text{if } y_t < c \\ 0 & \text{if } y_t \geq c \end{cases} \qquad \textbf{(9)}$$

One drawback of the threshold models is that they are discrete, in the sense that the process being described switches instantly from one regime - one set of dynamics - to another as a threshold is crossed. In the neighborhood of the threshold, the model dynamic changes discontinuously as the threshold is crossed. Because of the possibility that the transition across regimes occurs more smoothly than specified by the threshold model, the STAR models specify a more general transition function:

$$y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \left(\sum_{i=1}^{p} \beta_i y_{t-i} \right) F(\hat{y}_{t-d}) + \varepsilon_t \qquad \textbf{(10)}$$

where a lagged value of real GDP growth is the transition variable and d is the length of the delay for the switching variable. In the LSTAR model the transition function is a logistic transition function in which:

$$F(\hat{y}_{t-d}) = \left[1 + e^{-\gamma(\hat{y}_{t-d} - c)}\right]^{-1}, \quad \gamma > 0 \qquad \textbf{(11)}$$

The value of the transition function will depend upon the degree and direction by which y_{t-d}

deviates from c, the switching value of the transition variable. It is the estimated value for c that defines the rough line of transition between two regimes. If the value for c splits the observed data into a relatively infrequently observed period and a relatively frequently observed period, then we can talk about the “normal” regime and a regime embodying the extraordinary dynamics. For example, as y_{t-d} - c get large, the value of $e^{-((y_{t-d} - c)}$ approaches zero. As a result, the value of the transition function approaches one and the dynamics of y_t are generated by both the $"_i$ and $\$_i$ in equation (3). In addition, as the value of y_{t-d} - c get very small (large negative values for y_{t-d}), then the value of $e^{-((y_{t-d} - c)}$ approaches infinity. The value of the transition function in this case approaches zero and the dynamics are described by the $"_i$ only. Note that estimation of the value for c defines which regime occurs with relatively lower frequency. If the estimate for c is negative and large in absolute value, relative to the distribution of y_{t-d}, then the atypical period will occur when the transition function has a value at or close to zero. This means that the atypical dynamics will be described by the $"_i$ alone. In sum, the logistic specification or LSTAR model describes a stochastic process which is characterize by an alternative set of dynamics for *either* large or small values of the transition function.

It is also possible that the estimated transition function F(y) will be so shallow that it never achieves either extreme value, zero or one, over the range of data in a sample. In that case there is little gained by talking about regimes, since the estimated transition function never places the model in one regime or the other over the relevant sample period.

In contrast to the logistic specification, the exponential specification generates alternative dynamics for *both* large and small values for the transition variable. In the exponential or ESTAR model, the transition function is given by:

$$F(\hat{y}_{t\ d}) \qquad \left[1 \quad e^{\ ((y_{t\ d}\ \ c)^2}\right], \quad (>0. \qquad \textbf{(12)}$$

Consider extreme values for the transition variable in this specification. If the value of y_{t-d} - c is very large or very small, then the value of $e^{\ ((y_{t\ d}\ c)^2}$ approaches zero and the value of the transition function approaches 1. In these instances the dynamics will be described by both the "$_i$ and the $$_i$. When the value of y_{t-d} - c is zero, the value for $e^{\ ((y_{t\ d}\ c)^2}$ is one and the value for the transition function is zero. In these cases, the dynamics will be described by only the "$_{i.}$.

For both the LSTAR and the ESTAR models, the value of the transition function will take on values between zero and one as the transition variable takes on intermediate values, and the range of this period of intermediate values depends on estimated parameters of the transition function. The rate of transition between the two regimes is governed by the value of (, the adjustment parameter. As (increases in value, the rate of transition between the two regimes increases. In the limit, as (approaches infinity, the models degenerate into a traditional TAR models. Alternatively, if (were zero then the models degenerate to linear AR models (in which the "$_i$ and the $$_i$ parameters cannot be separately estimated).

To identify the type of model to be estimated, we return to the linearity test which also provides a specification tests for the choice between LSTAR and ESTAR. For this purpose, Terasvirta and Anderson present a set of hypothesis tests for the coefficients in equation (1) above:

$$H_{o1}\text{:}\quad 8_{3i} \quad 0\,,\ i \quad 1, . . . \, p$$

$$H_{o2}\text{:}\quad 8_{2i} \quad 0 \quad \text{given} \quad 8_{3i} \quad 0\,,\ i \quad 1, . . . \, p \qquad \textbf{(13)}$$

$$H_{o3}\text{:}\quad 8_{1i} \quad 0 \quad \text{given} \quad 8_{2i} \quad 8_{3i} \quad 0\,,\ i \quad 1, . . . \, p$$

Terasvirta and Anderson then propose a set of decision rules to make use of these tests:

1. If H_{o1} is rejected, select an LSTAR model.
2. If H_{o1} is not rejected and H_{o2} is rejected, then select and ESTAR model.
3 H_{o1} is not rejected and H_{o2} is not rejected but H_{o3} is rejected then select an LSTAR model.[2]

In addition, Terasvirta (1994) recommends checking the hypothesis test by examining the p-values associated with testing each of the null hypotheses. If the p-value from testing H_{o2} is the smallest, select the ESTAR model, otherwise select the LSTAR. Specifically, he states that (Terasvirta (1994; 210)) “It is again better to compare the relative strengths of the rejections. If the model is a LSTAR model, then typically H_{o1} and H_{o3} are rejected more strongly than H_{o2}. For an ESTAR model, the situation may be expected to be the opposite.”

The results of model specification tests are given in Table 5. Of the eighteen countries tested, eleven were determined to be LSTAR models and seven were found to

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[2] Recall that if none of the null hypothesis can be rejected, then linearity cannot be rejected and we would use the linear AR model.

be ESTAR models.[3] In interpreting the STAR models there are two coefficients that determine the nature of the process switching output growth between regimes. The first coefficient is "(" which determines the speed of adjustment between the two regimes and "c" which determines the centering of the area over which the transition takes place.

Consider first the LSTAR models.[4] For purposes of comparison, we can divide the countries into two groups, those with a "high" value for c and those with a "low" value for c. The first set of countries have a value for c between 2.7 percent and 4.6 percent and their transition functions are pictured in Figure 1. The second set of countries have a value for c between -1.5 and 2.2 percent and their transition functions are pictured in Figure 2.

Recall that c is the value at which the transition function is equal to one half, the economy is halfway between the two regimes and this value forms a rough line of demarcation. For South Africa, Switzerland, and Japan this means that the output growth rate switches from one dynamic model (F=0) to another dynamic model (F=1) when the output growth rate is about 4.0 percent. For the UK, the direction of switch is the opposite. Finally, the values for (determine how quickly the switch is made. Compare South Africa and Switzerland. The value for c for South Africa is 4.6 percent and for Switzerland it is 4.0 percent. But the speed of transition is very different with Switzerland having a much higher

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[3] The data for two of the countries that specify an ESTAR model, Korea and Greece contain huge, and potentially suspicious seasonal patterns. For example, in Korea GDP in one quarter is routinely 50 percent below the levels for other quarters. In these two cases, the ESTAR model is simply trying to fit these extreme seasonal dynamics and tell us nothing about business cycle dynamics. For this reason, we chose not to include these two model in our interpretations of the results.

[4] The estimated coefficients for the LSTAR models are given in Appendix A.

value for (. This means that Switzerland will switch between regimes quickly and will have relatively few values in the transition region. For Switzerland, the L-STAR model is basically a threshold or TAR model, with very little smoothing of the transition. South Africa, in contrast, has a very slow transition function and in almost all periods F is between zero and one. As the range of South Africa growth rates is -2.7% to 6.0%, so the transition function varies from about .05 to .65. It approaches zero for the two or three observations that near -2.7%, but it never nears one within our sample of South African data.

Switzerland, in contrast, has data that ranges from -5.1% to 6.9%. For data points below 3%, the transition function for Switzerland is zero, and this includes the majority of the Swiss data. For data points above about 4.25% the transition function for Switzerland is one, and this occurs for a very small part of the sample. Data points in the transition range, with 0<F<1, are also few in number. Observations above 3% make up only about 5% of the Swiss data.

The change in dynamics will be governed by the estimated parameters. In the Swiss case, negative and low positive value of growth will set the transition function equal to zero and dynamics will be governed by an AR 1 coefficient equal to .1550. Large positive growth rates will cause the transition function to approach one and the dynamics will be governed by both the original AR coefficient and the AR coefficient multiplied by the transition function. In these relatively rare periods, the dynamics are generated by an AR coefficient equal to -2.737. This regime is obviously unstable, but the overall model is stable, because of sign of the coefficient. A large positive shock will be quickly offset (due to the negative coefficient) in the atypical regime, driving output growth back to is stable regime.

Another country with a gradual transition function is Germany. The range of German data is -4.7% to 3.9%. Only when output growth is below -2.5% does the German transition function approach zero, and the transition function is about .85 at the upper end of the range of German data. Further, while Germany has one observation at -4.7%, the second smallest value is above -3%. Thus, for most of the observations the transition function is above zero, and it is never at unity. This demonstrates an advantage of the LSTAR model over the simple threshold models. The German LSTAR model allows quite a richer set of dynamics than a TAR model without requiring completely separate regimes. Indeed, the interpretation of separate regimes must be made carefully, since there is no observed data in the range where F equals one. Although conceptually there are two regimes defined by F=0 and F=1, within sample the F=0 regime almost never occurs, and the F=1 regime never occurs. This is important to keep in mind when interpreting the model.

Now let's consider the ESTAR models. ESTAR models are a bit different from LSTAR models in that they don't define two separate regimes divided in to high and low growth regions. Rather, the ESTAR model describes a region of growth rates in which the dynamics are generated by the linear AR process. For values of growth outside this region, either positive or negative, alternative dynamics kick in until output growth returns to is original region.[5]

[5] The estimated coefficients for the ESTAR models are given in Appendix B.

The center point of the simple dynamic region is given by the c parameter and that is point at which the transition function is equal to zero. When the economy's growth rates are very close to c, then the linear model essentially describes the dynamics. As the value of the growth rate deviates from c, the transitional dynamics become more important until eventually, the growth rate gets so large or small that the transition function equals one and both sets of coefficients set the dynamic path.

The width of the region of "normal" dynamics is set by the gamma parameter. The smaller the parameter the wider the region that is covered by the linear dynamics or a relatively small contribution from the nonlinear dynamics. As Figure 3 shows ESTAR models can show a wide spectrum of characteristics. At one extreme is Canada which has a relatively narrow region for which the transition function is less than one. For Canada, the dynamics are primarily determined by both sets of coefficients. The Canadian data ranges from a low of -1.9% growth to a high of 3.5%. For values above 1% - and these make up almost half of the Canadian data - the transition function is one. For values below 1% but above -2%, a bit more than half of the Canadian data, the transition function varies from slightly below one to zero and back. At the lower endpoint of the data range, -1.9%, the transition function has not returned completely back to one.

At the other extreme is Mexico in which the transition function is never equal to one. In fact, the range of Mexican growth rates is -8.1% to 9.7%, and in this range the transition function is near zero. In Mexico's case, the value for gamma is very small, so Mexico is quite often in the transition regime as the value for transition function is close to zero for a wide range of GDP growth rates. However, the coefficients in the transition regime are very large compared to the "normal" regime. The constant in the transition regime is -165.14 as compared with 0.65 and the first order coefficient is 24.5 as compared with .5102. Thus, although the transition function is small, it is being multiplied by some large coefficients. What is the net outcome?

Investigating the deterministic dynamics shows the following. When there is a 1,2 or 3, standard deviation positive shock, the model quickly (after 2 or 3 periods usually) returns to the simple dynamics state. Although the negative response in the first period after the shock is enhanced by the transitional dynamics, the opposite occurs in the subsequent period. This leads to a smaller reaction in the following period and so on. In no small part, the large negative constant causes the product of transition function and the coefficients to be negative, offsetting the positive shock at even numbered lags. With large enough positive shocks, the lagged term takes over and the large coefficient overwhelms the original AR coefficient. With a positive shock equal to 3 standard deviations - shocks of a size that never occurred in our sample - the period after the shock is still positive instead of negative due to the transitional dynamics. Thus, the ESTAR terms damp the oscillation created by the negative AR term.

The same is true for small negative shocks. Despite the large negative constant, there is enough positive response from the AR term for the two be offsetting. However, If the negative shock gets large enough -- about 2.75 times the standard deviation, then the negative shock and the negative constant are reinforcing, the value of transition function goes to one and GDP growth goes off to negative infinity. Again, shocks of this size were never observed in our sample, so the model's instability for shocks outside the range of values observed within sample is somewhat less troubling than it would otherwise be.[6]

B. CDR Model

Although we estimated CDR models for all twenty-six countries, only five (Germany, Taiwan, Sweden, France and the US) had statistically significant CDR terms.[7] The nature of the nonlinearity is governed by the CDR coefficients. For most countries, including France, Germany, and the U.S., the CDR coefficients are positive. This means that, other things equal, the economy's growth rate is faster after a recession. Another way to interpret this is that it suggests recessions would be briefer than expansions, since recessions engender strong positive growth in subsequent quarters.

For Japan, the CDR term is positive in the period prior to 1974.1 and zero thereafter. That is, Japan exhibited this faster growth after a period of recession up till 1973.4, but after that its economy shows little tendency to grow more quickly - or less quickly for that matter - after the occurrence of a recession. Interestingly, Japan had few recessions prior to 1974, so this term basically captures the response to only a couple of very minor 1-period negative growth quarters. Finally, Sweden shows more complicated dynamics, with a strong positive response to the first lag of SCDR and a nearly-equal negative response to the second lag of SCDR.

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[6] This does not mean the model is predicting such an outcome. The above description is just heuristic. The correct way to calculate the impulse response is stochastically as suggested by Potter (2000).

[7] The coefficients for those models with significant CDR terms is given in Appendix C.

The results for Germany also look a bit problematic, especially at first glance, since the constant term is negative. However, it is a very small magnitude (-0.04%) and is not statistically significant. Restricting it to zero has no important effect on the other coefficients estimates, and changes the log likelihood value by .014. Even a zero constant may be a concern, but the CDR model assigns much of the impetus to growth to the role of growing out of a recession, and this effect occurs strongly in the German data. Given the distribution of stochastic shocks that the CDR model estimates have impacted Germany over the sample period, there are enough periods of negative real GDP growth, and a strong enough estimated response to these recessions in the CDR model, to impart a positive growth rate to German real output.

4. Impulse Response Functions

We report a small number of impulse response functions, in order to illustrate some of the dynamics from these models. Calculating impulse response functions for nonlinear models is more involved than calculating impulse response functions in linear AR models. The most important difference is that the impulse response functions are state-dependent, and hence vary with the initial conditions at the time of the shock. For example, in an L-STAR model the impulse response function will depend on the value of the transition function F, as well as the values of the lagged AR terms entering the model directly or via multiplication by F. For the CDR model, the impulse response function will depend on the value of the CDR term.

To calculate nonlinear impulse response functions we use Potter's (2000) definition:

$$NIRF_n(v,y_t,y_{t-1},...) = E[Y_{t+n}|Y_t = y_t+v, Y_{t-1} = y_{t-1}, Y_{t-2} = y_{t-2},...] - E[Y_{t+n}|Y_t = y_t, Y_{t-1} = y_{t-1}, Y_{t-2} = y_{t-2},...],$$

where v is the impulse and lower-case letters represent realized values of the random variable.

This works as follows. For a shock at time t, we calculate the baseline forecast by drawing randomly from the history of residuals, using these to calculate the forecast of output from time t forward to t+36. We averaged the realizations over 1,000 iterations to get the baseline path. Then we repeated the exercise imposing a specific shock v at time t, usually of one or two standard error magnitudes. We again drew from the history of residuals to calculate the forecast for period t+1 through t+36, again iterating 1,000 times and averaging. The difference between these paths is the impulse response function.

Figure 4 presents impulse response functions for Germany for both the L-STAR and CDR models, and for two different dates, 1976.2 and 1984.1. Clearly all four graphs show the asymmetry of responses that one expects and typically gets from nonlinear models. In all four cases, positive shocks generate more persistence in above-baseline output levels into the future, while negative shocks exhibit less persistence and sometimes even lead to eventual above-baseline output in the future. For example, in the CDR models negative shocks of one, two, or two and one-half standard errors all lead to long-run output levels that are about 2% less than the baseline path, while positive shocks tend, especially for larger shocks, generate output levels above the baseline path by more than the value of the shock. Thus a positive 2% shock leads to an increase in the level of output over baseline that is nearly 4%. These results are similar for the initial values corresponding to both 1976.2 and 1984.2.

The L-STAR models generate even more interesting patterns. There is an even greater tendency for negative shocks to not lead to long-run declines in the level of output. In 1976.2 none of the negative shocks cause output to fall in the long run below the baseline path, and the largest negative shock, about a -2.5 standard error shock, led output to eventually surpass the baseline output level after only a few quarters. For shocks in 1984.1 this effect is still there but is attenuated, so that only the largest negative shock leads to a long-run increase in output above the baseline path. The smaller one and two standard error shocks cause output to fall in the long run below the baseline path. Note that even for these, it is the larger two standard error shock (-2.2%) that causes long run output to be closer to the baseline path than the one standard error shock.

For both 1976.2 and 1984.1, positive shocks are permanent and largely equal in size to the shock itself. That is, a 2.2% shock tends to have output in the long run that is about 2% higher than the baseline path.

These impulse responses indicate somewhat different dynamic responses to a shock depending on whether we model German output growth as following a CDR or L-STAR model. Certain broad features - the greater persistence of positive shocks - are common to both models, but the CDR model shows less of a tendency for negative shocks to lead to long - run output levels at or above the baseline path. We emphasize, however, that this result is not a general feature of the CDR model but instead a result that holds for Germany and for the specific states of the world that existed at the dates we investigated. Other CDR models for other countries can and do show negative shocks resulting in eventual output levels in excess of the baseline path.

In Table 5 we illustrate the impulse response functions for Sweden, a country for which we estimated an E-Star model as well as a seasonal CDR model. The dates of the shocks were chosen to be 1985.3 and 1994.1.

For the E-Star model, we again see evidence of asymmetry, especially in the impulse response functions for 1985.3. Negative shocks of all size result in long-run output levels that are between 1% and 2% below the baseline path, and typically the shortfall in output is less than the size of the original shock. Positive shocks also show a diminution of the gap between the output levels following the shock and the baseline path as the horizon lengthens, but this effect is more gradual, so that the largest shock - almost 4.5% - results in output that is about 3% above the baseline path after 36 quarters.

For 1994.1 the impulse response functions for the E-Star model appear much more symmetric, and the effect of both positive and negative shocks is somewhat reduced over time, especially for the larger magnitude shocks. Thus the largest shock, about 4.5%, causes output to be just under 3% above baseline after 36 quarters, while the smallest shock, about 2%, causes output to be about 1% above baseline after 36 quarters.

For the seasonal CDR model, the shocks in 1985.3 and in 1994.1 exhibit a fair amount of symmetry. The point estimates are that there is a slight tendency, almost undetectable visually, for positive shocks to be more persistent.

Thus for Sweden the shocks in 1985.3 are predicted to have asymmetric effects, and with greater persistence of positive shocks, for the E-Star model, while the seasonal CDR model shows near-symmetry. For 1994.1 both models indicate a very large degree of symmetry.

5. Conclusion

In this paper we have examined the dynamic process for real GDP for a set of countries that spans the spectrum of economic development. We undertook a series of linearity test to determine if a common characterization of GDP dynamics could be applied to all or most of the countries. Our results provided substantial evidence in favor of rejecting linearity and employing nonlinear dynamics for aggregate economic performance. Even here however, we found a substantial amount of heterogeneity with some countries characterized by ESTAR models, some countries characterized by LSTAR models and some countries characterized by CDR models. Within each class of models there were material differences in the estimated parameters, indicating that the are substantially different dynamics generated within each class of models.

We also found that the LSTAR and ESTAR models can be used to fit fairly complex dynamics even if the economy does not follow a strict regime switching path. For some countries, the “switch” was more a matter of degree than of discrete change, yet the economy’s dynamics were still state-dependent.

In general, our results emphasize that cross-country business cycle behavior is complex and cannot be model by simple univariate models. Care must be taken to investigate the possibility that those dynamics are non-linear and one should be very cautious in generalizing results based upon U.S. data.

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Table 1. Countries Included in this Study

Country	Period	Number of Observations
countries with only seasonally unadjusted data		
Austria	1964.1 - 1995.4	128
Chile	1980.1 - 1997.4	72
Finland	1970.1 - 1996.2	106
Greece	1975.1 - 1991.1	65
Korea	1960.1 - 1998.1	153
Mexico	1980.1 - 1997.3	71
New Zealand	1982.2 - 1997.4	63
Norway	1966.1 - 1997.4	128
Peru	1979.1 - 1998.1	77
Philippines	1980.4 - 1998.1	70
Portugal	1977.1 - 1996.4	80
Singapore	1984.3 - 1997.1	51
Sweden	1969.1 - 1998.1	117
Switzerland	1965.1 - 1998.1	133
Taiwan	1977.1 - 1995.4	76
countries with seasonally adjusted data		
Australia	1959.3 - 1998.1	155
Canada	1957.1 - 1998.1	165
France	1970.1 - 1998.1	113
Germany	1960.1 - 1998.4	156
Italy	1960.1 - 1997.4	152
Japan	1955.2 - 1988.4	175
Netherlands	1977.1 - 1998.3	87
South Africa	1960.1 - 1998.1	153

Spain	1970.1 - 1998.1	113
United Kingdom	1957.1 - 1998.1	165
United States	1957.1 - 1998.1	165

Data from IMF's International Financial Statistics CD-ROM, April 1999, except German data is from the Bundesbank, Japanese data is from Bank of Japan, and data for Taiwan is from the Central Bank of Taiwan.

Table 3 AR Specification and Linearity Test Results

Country	AR Lag	LM-STR Test Results		Reset p-value	LST p-value	CDR	
		p-value	Delay			p, q	p-value
countries with only seasonally unadjusted data							
Austria	7	.00	4	.00	.05	4,1	.72
Chile	5	.02	1	.40	.54	4,1	.17
Finland	4	.08	--	.22	.11	4,1	.77
Greece	3	.04	3	.45	.02	3,1	.33
Korea	4	.00	3	.01	.00	4,1	.48
Mexico	1	.02	1	.21	.01	2,1	.08
New Zealand	2	.10	--	.28	.31	2,2	.16
Norway	5*	.00	4	.07	.00	3,1	.33
Peru	1	.16	--	.86	.36	1,1	.11
Philippines	4	.01	3	.09	.35	4,1	.54
Portugal	1	.24	--	.33	.19	1,1	.54
Singapore	1	.28	--	.43	.89	1,1	.30
Sweden	4	.02	4	.32	.11	4,2	.01
Switzerland	1	.00	4	.00	.18	4,2	.07
Taiwan	5	.00	3	.51	.24	4,2	.01
countries with seasonally adjusted data							
Australia	1	.03	3	.67	.62	1,1	.09
Canada	1	.04	3	.58	.73	1,1	.30
France	2	.18	--	.06	.50	2,1	.02
Germany	4	.00	1	.07	.00	4,1	.00
Italy	1	.02	1	.10	.03	2,1	.14

Japan	3	.00	1	.03	.02	3,1	.85
Netherlands	1	.01	2	.73	.12	1,1	.39
South Africa	2	.02	1	.96	.05	2,1	.66
Spain	5*	.24	--	.56	.35	2,1	.16
United Kingdom	3*	.00	3	.04	.03	1,1	.07
United States	1	.09	--	.85	.18	2,1	.01

Table 2 Summary Statistics

Country	Quarterly Growth Rate		AR Model				
	Mean	Standard Error	Residual SE	R^2	Log Likelihood	Serial Corr. p-value	ARCH p-value
countries with only seasonally unadjusted data							
Austria	0.7118	9.0783	1.2888	.9815	-194.95	.55	.48
Chile	1.2461	4.5123	2.2509	.7779	-135.06	.27	.60
Finland	0.6372	6.0006	2.0690	.8894	-212.58	.44	.72
Greece	0.2723	14.2281	2.0109	.9820	-125.45	.12	.80
Korea	2.1752	41.6489	4.4502	.9891	-426.87	.02	.10
Mexico	0.5262	4.7546	2.2959	.7805	-152.66	.17	.95
New Zealand	0.5606	2.9261	2.5909	.2825	-139.09	.56	.97
Norway	0.9095	4.1910	2.3257	.7124	-271.41	.16	.89
Peru	0.2921	8.5470	4.4013	.7492	-214.98	.47	.83
Philippines	0.4578	10.3293	2.3446	.9542	-141.07	.07	.92
Portugal	0.8053	2.2951	1.9606	.3081	-160.61	.93	1.00
Singapore	1.7253	3.3242	1.7274	.7525	-93.67	.95	.86
Sweden	0.4697	11.5602	1.8595	.9758	-224.24	.11	.10
Switzerland	0.4634	1.6568	1.6523	.0360	-249.12	.11	.22
Taiwan	2.0371	3.7189	2.1813	.6958	-149.10	.14	.43
countries with seasonally adjusted data							
Australia	0.9287	1.3510	1.3521	.0049	-262.25	.38	.00
Canada	0.9030	0.9919	0.9635	.0622	-224.22	.45	.01
France	0.6048	0.7085	0.6487	.1771	-106.96	.36	.88
Germany	0.6775	1.3512	1.2579	.1565	-246.37	.34	.39
Italy	0.8559	1.1664	1.1049	.1087	-226.80	.57	1.00
Japan (no break)	1.3139	1.3258	1.1244	.2934	-260.67	.00	.04
Japan (w break)	1.3139	1.3258	1.0480	.3935	-247.61	.13	.42
Netherlands	0.5560	0.9128	0.9164	.0041	-112.18	.71	.01
South Africa	0.7487	1.2471	1.2124	.0675	-240.22	.19	.02
Spain	0.7027	0.6047	0.2373	.8532	5.15	.50	.97

U. K.	0.5961	1.0744	1.0646	.0406	-236.51	.11	.01
United States	0.7124	0.9136	0.8732	.0921	-208.18	.48	.12

APENDIX A

ESTIMATED LSTAR MODELS

1. Seasonally Unadjusted Data

Austria: LSTAR

g_t = -9.4908 + 814.0508*D2 + 812.3616*D3 + 813.9720*D4 - 0.1619*g_{t-1} - 0.3923*g_{t-2} + 0.2234*g_{t-3}

-0.0374*g_{t-4} + 0.4265*g_{t-5} - 0.8917*g_{t-6} - 0.0105*g_{t-7} + (-803.5194 - 0.1467*g_{t-1} + 0.2707*g_{t-2} - 0.1928*g_{t-3}

+0.7698*g_{t-4} - 0.3496*g_{t-5} + 0.7025*g_{t-6} -0.2247*g_{t-7})/{1+exp[-(14.8145/9.0783)*(g_{t-4} + 1.5875)]} + e_t

residual s.e. = 1.0728, R2 = .9884, Log Likelihood = -167.16 (120 obs.), Q(8)=6.64 (p=.58)

Chile: LSTAR

g_t = -1.3970 - 1.5935*D2 - 2.8731*D3 + 1.4632*D4 + 0.0246*g_{t-1} - 0.1664*g_{t-2} + 0.4143*g_{t-3} - 0.0010*g_{t-4}

+ 0.5909*g_{t-5} +(5.4232 - 0.4299*g_{t-1} -0.2788*g_{t-2} - 0.7028*g_{t-3} + 0.3393*g_{t-4} + 0.6913*g_{t-5})/

{1+exp[-(60.4280/4.5123)*(g_{t-1} - 0.4061)]} + e_t

residual s.e. = 2.1246, R2 = .8329, Log Likelihood = -133.56 (66 obs.), Q(8)=7.47 (p=.49)

Norway: LSTAR

g_t = 1.8499 - 2.7794*D1 - 1.5069*D2 - 0.0517*D3 + 0.3204*g_{t-1} + 0.1789*g_{t-2} + 0.1998*g_{t-3} + 0.4385*g_{t-4}

- 0.5269*g_{t-5} +(0.6830 - 0.9903*g_{t-1} -0.7058*g_{t-2} - 0.5636*g_{t-3} - 0.1890*g_{t-4} + 0.7674*g_{t-5})

/{1+exp[-(27.9886/4.19)*(g_{t-4} + 2.7957)]} + e_t

residual s.e. = 2.1341, R2 = .7750, Log Likelihood = -256.44 (122 obs.), Q(8)=5.98 (p=.65)

Philippines: LSTAR

g_t = 2.5008 - 15.7884*D2 + 11.0046*D3 - 13.8048*D4 - 0.0614*g_{t-1} - 0.1128*g_{t-2} - 0.0980*g_{t-3} + 0.0444*g_{t-4}

+ (5.9395+ 0.1274*g_{t-1} -0.0932*g_{t-2}+ 0.3784*g_{t-3}+ 0.7133*g_{t-4})/{1+exp[-(38.4604/10.3293)*(g_{t-3} -0.7180)]}+e_t

residual s.e. = 2.1283, R2 = .9670, Log Likelihood = -130.61 (64 obs.), Q(8)=12.23 (p=.14)

Switzerland: LSTAR

g_t = 0..2711 + 0.2782*d2 + 860E-05*D3 - 0.1528*D4 + 0.1550*g_{t-1}

+ (2.8907 - 2.8924*g_{t-1})/{1+exp[-(8.6174/1.6555)*(g_{t-4} - 4.0527)]} + e_t

residual s.e. = 1.5156, R2 = .2147, Log Likelihood = -230.18 (128 obs.), Q(8)=8.43 (p=.39)

Taiwan: LSTAR

g_t = 1.1620 - 0.3904*D2 + 6.3528*D3 + 0.9582*D4 - 0.4814*g_{t-1} + 0.0342*g_{t-2} - 0.3557*g_{t-3} + 0.3700*g_{t-4}

- 0.4044*g_{t-5} +(-3.5750 + 4.9988*D2- 4.5749*D3 + 14.2125*D4 + 0.8727*g_{t-1} + 0.1267*g_{t-2} + 0.4644*g_{t-3}

- 01073*g_{t-4} + 0.1555*g_{t-5})/{1+exp[-(2.6674/3.7189)*(g_{t-3} - 2.7490)]} + e_t

residual s.e. = 2.0291, R2 = .7843, Log Likelihood = -137.08 (70 obs.), Q(8)=1.80 (p=.99)

2. Seasonally Adjusted Data

Germany: LSTAR

g_t = -2.9086 -1.1433*g_{t-1} + 1.7646*g_{t-2} + 2.7423*g_{t-3} - 0.38403*g_{t-4}

+ (3.8374 + 1.1702g_{t-1} - 1.9508*g_{t-2} - 3.1775*g_{t-3} + 1.2079*g_{t-4})/{1+exp[-(1.0131/1.3512)*(g_{t-1} + 2.1831)]} + e_t

residual s.e. = 1.1078, R2 = .3772, Log Likelihood = -223.46 (151 obs.), Q(8) = 4.40 (p=.82)

Japan: LSTAR

Model without trend break:

g_t = 0.1619 - 0.0552*g_{t-1} + 0.3951*g_{t-2} + 0.3490*g_{t-3}

+ (9.3012 - 1.1699*g_{t-1} - 1.1248*g_{t-2} - 0.6447*g_{t-3})/{1+exp[-(2.3101/1.3258)*(g_{t-1} - 3.2656)]} + e_t

residual s.e. = 1.0830, R2 = .3681, Log Likelihood = -251.12 (171 obs.), Q(8)=12.56 (p=.13)

Model with trend break:

g_t = -0.1742 + 0.0354*D -1.5838*g_{t-1} + 1.4129*g_{t-2} + 0.9961*g_{t-3}

+ (0.6860 - 0.1376*D -3.2708*g_{t-1} -3.8728*g_{t-2} - 2.4694*g_{t-3})/{1+exp[-(0.3476/.013258)*(g_{t-1} - 4.0848)]} + e_t

(D_t = 0 for t < 1974.1, else D_t = 1)

residual s.e. = 1.0367, R2 = .4281, Log Likelihood = -242.58 (171 obs.), Q(8)=15.90 (p=.04)

Netherlands: LSTAR

g_t = 0.5396 + 0.0696*g_{t-1} + (0.2242 - 0.9274*g_{t-1})/{1+exp[-(2.7636/0.9164)*(g_{t-2} - 1.7479)]} + e_t

residual s.e. = 0.9120, R2 = .0692, Log Likelihood = -108.34 (84 obs.), Q(8)=6.43 (p=.60)

South Africa: LSTAR

g_t = -2.2814 - 0.4901*g_{t-1} + 0.4336*g_{t-2}

+ (23.9934 - 3.2535*g_{t-1} - 1.3002*g_{t-2})/{1+exp[-(0.5592/1.2471)*(g_{t-1} - 4.5781)]} + e_t

residual s.e. = 1.1778, R2 = .1499, Log Likelihood = -233.28 (150 obs.), Q(8)=8.01 (p=.43)

U.K.: LSTAR

Beginning from linear AR(1) model:

g_t = -1.0696 + 1.5695*g_{t-1} + (1.7438 - 1.6198*g_{t-1})/{1+exp[-(-28.0042/1.0799)*(g_{t-4} - 3.1097)]} + e_t

residual s.e. = 1.0399, R2 = .1019, Log Likelihood = -230.23 (160 obs.), Q(8) = 11.57 (p=.17)

Beginning from linear AR(3) model:

g_t = 0.4702 - 0.0572*g_{t-1} + 0.0542*g_{t-2} + 0.2463*g_{t-3}

+ (16.6654 - 1.0767*g_{t-1} + 0.6004*g_{t-2} - 4.0185*g_{t-3})/{1+exp[-(-33.6883/1.08)*(g_{t-3} - 2.5482)]} + e_t

residual s.e. = 1.0144, R2 = .1623, Log Likelihood = -225.59 (161 obs.), Q(8) = 9.59 (p=.30)

APPENDIX B

ESTIMATED ESTAR MODELS

1. Seasonally Unadjusted Data

Greece: ESTAR

g_t = -3.9098 + 22.3676*D2 + 7.8955*D3 - 4.4308*D4 - 0.2635*g_{t-1} - 0.4505*g_{t-2} - 1.0643*g_{t-3} + (1.3158

+ 0.0321*g_{t-1} - 0.7457*g_{t-2} - 1.2600*g_{t-3})/{1-exp[-(-130.13/14.2281)*(g_{t-3} - 1.4613)^2]} + e_t

residual s.e. = 1.8013, R2 = .9794, Log Likelihood = -115.14 (61 obs.), Q(8) = 3.74 (p=.88)

Korea: ESTAR

NOTE: Data are extremely heteroskedastic, with variability of data apparently decreasing constantly over sample period after first few years. Because of this, data was transformed to correct for heteroskedasticity by multiplying data for observations beyond t=24 (1967.1) by t/24. Because of the need for this transformation, the results are not included in the analysis, but we do report our estimates of the transformed model here.

ng_t = 1.6822 + 1.1375*nD2 + 2.4439*nD3 + 2.9213*nD4 - 0.4216*ng_{t-1} + 4.0666*ng_{t-2} + 0.3542*ng_{t-3}

- 0.7876*ng_{t-4} + (0.4715 - 0.1042*ng_{t-1} - 4.5884*ng_{t-2} - 0.8623*ng_{t-3} + 1.2397*ng_{t-4})

/{1-exp[-(0.6255/81.35)*(g_{t-3} + 48.8958)^2]} + e_t

residual s.e. = 12.07, R2 = .9801, Log Likelihood = -570.74 (148 obs.), Q(8) = 18.77 (p=.02)

For comparison, the linear AR model on transformed data:

ng_t = 1.8963 + 1.1310*nD2 + 1.9678*nD3 + 2.4301*nD4 - 0.4810*ng_{t-1} - .4678*ng_{t-2} - 0.4469*ng_{t-3}

+ 0.4948*ng_{t-4} + e_t

residual s.e. = 13.16, R2 = .9751, Log Likelihood = -587.27 (148 obs.)

Mexico: ESTAR

g_t = 0.6514 + 1.5664*D2 - 3.2710*D3 + 5.2919*D4 - 0.5102*g_{t-1}

+ (-165.1383 + 24.4176*g_{t-1})/{1 - exp[-(0.0013/4.7546)*(g_{t-1} - 0.0505)^2]} + e_t

residual s.e. = 2.2064, R2 = .8100, Log Likelihood = -147.69 (69 obs.), Q(8) = 13.44 (p=.10)

Sweden: ESTAR

g_t = -8.2711 + 15.9587*D2 - ..5985*D3 + 20.0510*D4 - 0.2672*g_{t-1} - 0.5411*g_{t-2} - 0.0977*g_{t-3} - 03.112*g_{t-4}

+ (-1.0457 -0.1095*g_{t-1} +0.8067*g_{t-2} -0.2405*g_{t-3} +0.4922*g_{t-4})/{1-exp[-(0.0416/11.55)*(g_{t-4} + 2.2460)^2]} + e_t

residual s.e. = 1.7493, R2 = .9800, Log Likelihood = -213.50 (112 obs), Q(8) = 5.29 (p=.73)

2. Seasonally Adjusted Data

Australia: ESTAR

g_t =1.1567 - 0.3397**g_{t-1} + (-0.4035 + 0.6298*g_{t-1})*{1- exp[-(0.4849/1.3537)*(g_{t-3} - 1.0281)^2]} + e_t

residual s.e. = 1.3419, R2 = .0501, Log Likelihood = -255.61 (151 obs.), Q(8)=6.65 (p=.58)

Canada: ESTAR

g_t =0.3591 + 0.6834*g_{t-1} + (0.4960 - 0.5902*g_{t-1})*{1- exp[-(1.6070/0.9862)*(g_{t-3} + 0.6956)^2]} + e_t

residual s.e. = 0.9478, R2 = .1053, Log Likelihood = -216.75 (161 obs.), Q(8) = 8.86 (p=.36)

Italy: ESTAR

g_t = 0.2506 + 0.6549*g_{t-1} + (1.3952 - 0.6944*g_{t-1})*{1-exp[-(0.2154/1.1664)*(g_{t-1} - 0.5041)^2]} + e_t

residual s.e. = 1.0800, R2 = .1714, Log Likelihood = -221.33 (150 obs.), Q(8) = 7.65 (p=.47)

APPENDIX C
ESTIMATED CDR MODELS

1. Seasonally Unadjusted Data

Sweden: CDR

g_t = -4.2602 + 7.9709*D2 - 1.4044*D3 + 12.8688*D4 - 0.1828*g_{t-1} - 0.3314$_{t-2}$ - 0.1556*g_{t-3}

+ 0.3902*g_{t-4} + 0.4425*CDR_{t-1} - 0.4572*CDR_{t-2} + e_t

residual s.e. = 1.7877, R2 = .9780, Log Likelihood = -218.75 (112 obs.) Q(8) = 4.33 (p=.83)

Taiwan: CDR

g_t = -0.7024 + 2.1615*D2 + 2.2843*D3 + 2.1926*D4 - 0.1305*g_{t-1} + 0.1090*g_{t-2} - 0.0363*g_{t-3}

+ 0.4701*g_{t-4} + 0.2926*CDR_{t-1} + 0.8345*CDR_{t-2} + e_t

residual s.e. = 2.1522, R2 = .7088, Log Likelihood = -147.59 (70 obs.) Q(8) = 6.85 (p=.55)

2. Seasonally Adjusted Data

France: CDR

g_t = 0.0788 + 0.3334*g_{t-1} + 0.3932*g_{t-2} + 0.3567*CDR_{t-1} + e_t

residual s.e. = .6383, R2 = .2107, Log Likelihood = -104.67 (110 obs.) Q(8) = 5.55 (p=.70)

Germany: CDR

g_t = -0.0398 - 0.1704*g_{t-1} + 0.2659*g_{t-2} + 0.1301*g_{t-3} + 0.4091*g_{t-4} + 0.5165*CDR_{t-1} + e_t

residual s.e. = .0122, R2 = .2078, Log Likelihood = -241.62 (151 obs.) Q(8) = 2.04 (p=.98)

Japan: CDR

Model without trend break: CDR model not indicated.

Model with trend break:

g_t = 2.0903 - 1.7726*D_t + 0.0210*D_t*g_{t-1} + 0.1612*D_t*g_{t-2} + 03428*D_t*g_{t-3} + 4.6690*(CDR_{t-1} - D_t*CDR_{t-1})+ e_t

(D_t = 0 for t < 1974.1, else D_t = 1)

residual s.e. = 1.0480, R2 = .3935, Log Likelihood = -247.60 (171 obs.) Q(8) = 12.41 (p=.13)

U.S.A.: CDR

$g_t = 0.2096 + 0.3856*g_{t-1} + 0.1882*g_{t-2} +0.2852*CDR_{t-1} + e_t$

residual s.e. =. 8620, R2 = .1315, Log Likelihood = -203.79 (162 obs.) Q(8) = 9.03 (p=.34)

Table 4. Summary of Linearity Testing

	Evidence Rejecting Linearity?			
Country	LM-STR	RESET	LST	CDR
Germany	Y	N	Y	Y
Taiwan	Y	N	N	Y
Sweden	Y	N	N	Y
Switzerland	Y	Y	N	N
Japan	Y	Y	Y	N
Philippines	Y	N	N	N
Chile	Y	N	N	N
Austria	Y	Y	Y	N
Netherlands	Y	N	N	N
South Africa	Y	N	Y	N
Australia	Y	N	N	N
UK	Y	Y	Y	N
Italy	Y	N	Y	N
Greece	Y	N	Y	N
Korea	Y	Y	Y	N
Canada	Y	N	N	N
Mexico	Y	N	Y	N
Norway	Y	N	Y	N
US	N	N	N	Y
France	N	N	N	Y
Finland	N	N	N	N
Portugal	N	N	N	N
Peru	N	N	N	N
Spain	N	N	N	N
Singapore	N	N	N	N
New Zealand	N	N	N	N

Table 5. Results of Model Specification Tests

County	Model Indicated
Austria	LSTAR
Chile	LSTAR
Germany	LSTAR (or CDR)
Japan	LSTAR (or CDR)
Netherlands	LSTAR
Norway	LSTAR
Philippines	LSTAR
South Africa	LSTAR
Switzerland	LSTAR
Taiwan	LSTAR (or SCDR)
UK	LSTAR
Australia	ESTAR
Canada	ESTAR
Greece	ESTAR
Italy	ESTAR
Korea	ESTAR
Mexico	ESTAR
Sweden	ESTAR (or SCDR)
France	CDR
U.S.	CDR