Stationary Disk Assemblies on Inhibitory Vesicles

Thesis Abstract

We assume that a vesicle is modeled by a two-dimensional smooth manifold M which is compact without boundary.

In our model, minimization of the free energy reduces to a problem finding the minimizers of a functional $\mathcal{J}$ defined on subsets of M whose area is m and whose characteristic functions have bounded variation. Let $|D\chi_E|$ be the total variation measure of χ_E and $|D\chi_E|(M)$ be the size of M under this measure. Then

$$\mathcal{J}(E) = |D\chi_E|(M) + \frac{\gamma}{2}\int_M \left|(-\Delta)^{-\frac{1}{2}}(\chi_E - m)\right|^2 dx. \tag{1}$$

Our goal is to identify minimizers (local or global) of $\mathcal{J}$, given the area constraint m. If a critical point E has a smooth boundary, then the Euler-Lagrange equation is

$$\mathcal{H}(\partial E) + \gamma(-\Delta)^{-1}(\chi_E - m) = \lambda. \tag{2}$$

On the left side of above equation, $\mathcal{H}(\partial E)$ stands for the geodesic curvature of the boundary of E.

Our main result is showing the following:

- The existence of many geodesic disc pattern under a certain parameter range.
- The centers of E are determined by Green's function.

We will mathematically construct a disk assemblies using a fixed point argument and then applying Lyapunov-Schmidt reduction by the following steps:

- Start with an approximate solution and estimate the energy of system.
- Calculate the first and second variation (linearized operator) of the energy functional at the approximate solution.
- Control the deviation from exact geodesic disk.
- Determine the radii and locations of droplets.
- Investigate the role played by the Gauss curvature of M.