Abstract

A classic result in noncommutative ring theory states that a ring R is an $n \times n$ matrix ring if, and only if, R contains n^2 matrix units $\{e_{ij}\}_{1 \leq i,j \leq n}$, in which case $R \cong M_n(S)$ where S is a subring of R that can be described completely in terms of the matrix units. A lesser known result states that a ring R is an $(m+n) \times (m+n)$ matrix ring,so $R \cong M_{m+n}(S)$ for some ring S, if, and only if, R contains three elements a, b, and f satisfying the two relations $af^m + f^n b = 1$ and $f^{m+n} = 0$. In this talk, we investigate algebras over a commutative ring (or field) with elements c and f satisfying the two relations $c^i f^m + f^n c^j = 1$ and $f^{m+n} = 0$. Surprisingly little is known here about the structure of these algebras and about the underlying ring S for most cases of the integers i, j,m, and n. Questions whether S is non-trivial or not turn out to be surprisingly difficult to answer, let alone describing the structure of these algebras or of S in general.