Title: The Functional Calculus for Operators; Operators Having a Square Root

Abstract: The idea is to look at an operator T on a Hilbert space $\mathcal{H}$ and to define a map $f \mapsto f(T)$ from an algebra of functions $\mathcal{A}$ into $\mathcal{B}(\mathcal{H})$, the algebra of all bounded operators from $\mathcal{H}$ into itself, such that the algebraic operations are preserved and some type of continuity condition holds. For certain operators the algebra can be very big, but there is one algebra that works for any operator. If you will, this is the minimal algebra. If T is any operator and $\mathcal{A}$ is the algebra of all functions that are analytic in some open set containing the spectrum of T, there is such a functional calculus. [The spectrum of T is the set of complex numbers $\{\lambda \in \mathbb{C} : T - \lambda$ does NOT have a continuous inverse$\}$. When T is a $d \times d$ matrix, this is the set of eigenvalues.] This functional calculus can be used to give a sufficient condition that T has a square root; that is, there exists an operator A such that $A^2 = T$. This opens the door to seeing fruitful interactions between two areas of analysis, operator theory and analytic function theory. Can we characterize the set of all operators having a square root?