LINEAR ALGEBRA/ADVANCED CALCULUS QUALIFYING EXAM SYLLABUS

This is one exam with two parts: linear algebra and advanced calculus.

Linear Algebra

- Vector spaces: dependent and independent sets of vectors, bases, dimension, subspaces, sums and intersections of subspaces.
- Matrices: row and column spaces, null space, elementary transformations, elementary matrices, row echelon matrices, systems of linear equations, invertible matrices, inverses, LDU factorization.
- Determinants: definition and properties, Cramer's rule.
- Linear transformations: the matrix of a linear transformation, change of basis, linear functionals, dual basis and dual space.
- Linear operators: similarity of matrices, characteristic polynomial, eigenvalues, eigenvectors, minimum polynomial, the Cayley-Hamilton theorem, diagonalization, invariant subspaces, Jordan and rational canonical forms.
- Inner products: orthogonality, the Gram-Schmidt orthonormalization process, projections, orthogonal matrices, the spectral theorem (for symmetric matrices).

Advanced Calculus

- Topology of metric spaces: connectedness, compactness, completeness, the Bolzano-Weierstrass theorem, the Heine-Borel theorem.
- Continuity: continuous functions and compactness, continuous functions and connectedness, the intermediate value theorem.
- Differentiation: the mean value theorem, Taylor's theorem.
- The Riemann-Stieltjes integral: the fundamental theorem of calculus, the mean value theorem for integrals, necessary/sufficient conditions for Riemann-Stieltjes integrability.
- Uniform convergence: tests for convergence of numerical series, uniform convergence and continuity, uniform convergence and integration, uniform convergence and differentiation, uniform convergence on compact sets, Weierstrass' M-test, power series, Abel's theorem.
- Functions of several variables and vector analysis: continuity and differentiability of functions $f : \mathbb{R}^n \to \mathbb{R}^m$, the inverse function theorem, the implicit function theorem, the contraction mapping theorem, differential forms (wedge product, exterior differentiation), closed and exact forms, Poincaré's lemma, Stokes's theorem in $\mathbb{R}^n$.

Linear Algebra References:

- F. R. Gantmacher, *The Theory of Matrices*, 2 volumes, Chelsea, 1959.
- W. H. Greub, *Linear Algebra*, fourth edition, Springer-Verlag, 2004.
- I. N. Herstein and David J. Winter, *Matrix Theory and Linear Algebra*, Macmillan, 1988.
- K. Hoffman and R. Kunze, *Linear Algebra*, second edition, Prentice Hall, 1971.
- Gilbert Strang, *Linear Algebra and its Applications*, third edition, Brooks Cole, 1988.

Advanced Calculus References:

- R. C. Buck, *Advanced Calculus*, third edition, McGraw-Hill, 1978.
- W. Rudin, *Principles of Mathematical Analysis*, third edition, McGraw-Hill, 1964.
- M. Spivak, *Calculus on Manifolds*, Benjamin/Cummings, 1965.
- A good source for differential forms is Chapter 7 of V. I. Arnold, *Mathematical Methods of Classical Mechanics*, Springer-Verlag, 1978.

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