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Measuring the natural interest rate in Brazil

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1. Introduction

Over the last years, Brazil has experienced a pronounced decline in real interest rate[1]. The reduction in the sovereign risk premium and the falling inflationary surprises (gap between expected and actual inflation) have contributed significantly to drive down natural rates in Brazil.

Recently, in face at the global financial crisis, Brazil's Central Bank has cut the Selic rate aggressively, achieving a single-digit for the first time in memory. Since then, there has been an intense debate concerning the “appropriate” level of the real interest rate in Brazil, essentially if this current rate represents a non-accelerating inflationary level.

One hand, many experts argues that the global crisis has created a window of opportunity for Brazil to bring real rates down – and to keep them there, at a permanently lower level. In another hand, several researchers argues that the recent aggressive monetary easing is an emergency cyclical response, bringing rates below the current neutral level, as a means of avoiding a sharper fall in economic activity. As conditions normalize in the economy, interest rates will eventually need to rise again, as the central bank then focuses on the 2011 inflation outlook.

The aim of the study is to contribute to the debate by estimating the natural rate of

[1] *Ex antes* real interest rate (benchmark interest rate deflating by expected CPI inflation next to 12 months), peaked at levels close to 13.5% in July, 2001, then narrowing all the way to current 4.1% in late 2009.

interest in Brazil defined as the nominal interest rate that keeps inflation steady, using a dynamic model for the Taylor rule intercept. Despite the difficulties involved in the precise determination of equilibrium real interest rates, the real interest rate gap — the difference between the real equilibrium rate and the rate set by the central bank — can thus serve as a leading indicator of future inflationary or deflationary pressures in the economy. Results obtained show the natural rate of interest in Brazil is now probably lower than before – but probably higher than current real rates.

In addition to this introduction, this paper has three other sections. The first section provides a theoretical overview of the definition of the natural rate of interest on which the estimates to be made should be based on. The second section shows the econometric model used and its results for the natural interest rate. The last chapter contains conclusions.

2. The Natural Interest Rate

The definition of the natural rate of interest dates back to Wicksell (1936), who described the natural rate of interest in at least three dimensions:

(1) the rate of interest that equates savings with investment;

(2) the marginal productivity of capital;

(3) the rate of interest that is consistent with aggregate price stability.

These assumptions influenced many studies in the 20th century. Friedman (1968) and Phelps (1968), for instance, used such properties to define the natural rate of unemployment.

The influence of Wicksell's work can also be seen in the current New Keynesian equilibrium models. In this theory, the natural rate is obtained from the equilibrium of markets with flexible prices and rational expectations.

According to the New Keynesian framework, the natural rate of interest has three properties:

(i) it is a rate that is defined on a period-by-period basis;

(ii) it is obtained in real terms; and,

(iii) it is subject to fluctuations in the short and in the long run.

Therefore, even in case of a long-run equilibrium, the natural rate can shift over time

due to structural changes in the economy, i.e., the natural rate cannot be defined as a constant long-term interest rate.

A remarkable difference between the natural rate in the New Keynesian approach and the previous models is the equilibrium of the former, which is consistent with a short-term price stability system. Wicksell and other authors who preceded such approach regarded the natural rate as the interest rate towards which the economy would tend in the long run. Thus, the New Keynesian natural rate is a more complete description of the economy in a monetary policy arrangement that has price stability at shorter horizons as one of its major goals, such as the inflation targeting regime. In other words, it's not a level of interests towards which the economy will unfalteringly steer in the long run, but rather a time-varying interest rate that relies on dynamic economic factors and is compatible with equilibrium.

In this regard, one of the contributions of the New Keynesian paradigm is the possibility to obtain an explicit expression for the natural rate by associating the natural rate with usual (but sometimes unobservable) economic variables. According to Woodford (2003) framework, the natural rate of interest derives from equilibrium between the aggregate demand and a supply curve (Phillips curve), considering a Taylor rule.

As the natural rate of interest is a variable that cannot be directly observed, it can be estimated by different methods. A direct and simple way to calculate equilibrium rates is to filter *ex-post* real interest rates of high frequency movements to avoid transitory

shocks to the economy. Borio et al (2000) suggested the use of a Hodrick-Prescott filter with a very high parameter λ to smooth the trend series. Another measure of natural real interest rates is potential output growth. Many central banks use this measure as a rule of thumb. This measure of the potential output can be understood as a linear trend on the GDP series.

Another possibility to obtain the equilibrium real interest rate measures is to estimate reaction function for Central Banks. Based on the principle that Central Banks seek to achieve their inflation targets by keeping a balanced economic growth, the intercept of the reaction function could be understood as the equilibrium interest rate.

3. ECONOMETRIC ESTIMATIONS

The subject of the present study is to estimate the level of the natural rate of interest in Brazil between 2000 and 2009. As the natural rate of interest is a variable that cannot be directly observed, it can be estimated by different methods. In this paper, statistical filters are used for the *ex ante* and *ex post* real interest series. Then, an estimation of a dynamic Taylor rule is performed.

3.1 Hodrick – Prescott Filter

The Hodrick-Prescott filter (HP filter) is a mathematical tool used in macroeconomics especially in real business cycle to separate the cyclical component of a time series from raw data. It is used to obtain a smoothed non-linear representation of a time series, one that is more sensitive to long-term than to short-term fluctuations. The adjustment of the sensitivity of the trend to short-term fluctuations is achieved by modifying a multiplier λ.

The reasoning for the methodology uses ideas related to the decomposition of time series. Let y_t for t = 1,2,…,T denote the logarithms of a time series variable. The series y_t is made up of a trend component, denoted by τ and a cyclical component, denoted by c such that $y_t = \tau_t + c_t$. Given an adequately chosen, positive value of λ, there is a trend component that will minimize:

$$\min \sum_{t=1}^{T} (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2.$$

The first term of the equation is the sum of the squared deviations $d_t = y_t - \tau_t$ which penalizes the cyclical component. The second term is a multiple λ of the sum of the squares of the trend component's second differences. This second term penalizes variations in the growth rate of the trend component. The larger the value of λ, the higher is the penalty.

In the present study, HP Filter is used in *ex ante* and *ex post* real interest rate series. The *ex post* real interest rate series are obtained by deflating the benchmark Selic rate according to the Consumer Price Index (CPI) accumulated throughout twelve months, whereas the *ex ante* real interest rate series are obtained by deflating the benchmark Selic rate according to the expectations CPI for the year T and T +1[2], obtained in the market survey conducted by the Brazilian Central Bank[3].

To evaluate additional instruments of the interest rate used in Brazil, a different measures for *ex ante* real interest rate series are also calculated: i) by deflating the benchmark Selic rate according to the expected CPI for 12 months ahead and ii) by deflating the SWAP 360 rate according to the expected CPI for the year T and T +1.

Figure 1 summarizes central tendency measure results. It's interesting to note that differences between *ex post* real and the *ex ante* real interest rates may be attributed to

[2] This variable was constructed following the suggestion made in Minella et al (2002), where:
$D_{j,t} = \left(\frac{12-j}{12}\right)(E_j\pi_t - \pi_t^*) + \left(\frac{j}{12}\right)(E_j\pi_{t+1} - \pi_{t+1}^*)$, where $E_j\,\pi_t$ = expectation in month j = 1, 2,…, 12 for inflation in year t; π^*_t = midpoint target range for year t; $E_j\,\pi_{t+1}$ =expectation in month j for inflation of year $t+1$; π^*_{t+1} = midpoint target range for year $t+1$.

[3] This survey is fully available at http://www.bcb.gov.br/?FOCUSERIES.

the gap between expected and actual inflation from economic agents.

Figure 1: HP Filter results in the ex ante and ex post real interest rate

	ex post real selic[1]	***ex antes*** real selic[2]	***ex antes*** real selic[3]	***ex antes*** real swap[4]
Mean	8.6	10.1	10.3	10.7
Median	9.2	11.1	11.3	10.5
Maximum	10.4	13.4	13.3	17.1
Minimum	4.3	4.7	5.0	5.3
Standard deviation	1.7	2.6	2.6	3.3
Period:	**nov/01 a Dec/09**	**nov/01 a Dec/09**	**nov/01 a Dec/09**	**nov/01 a Dec/09**

(1) ***ex post*** real selic: deflating the benchmark Selic rate according to the CPI accumulated throughout 12 months
(2) ***ex antes*** real selic: deflating the benchmark Selic rate according to the expectations CPI for the year T and T +1
(3) ***ex antes*** real selic: deflating the benchmark Selic rate according to the expectations CPI for the next 12 months
(4) ***ex antes*** real swap deflating the Swap 360 rate according to the expectations CPI for the year T and T +1

Results suggest that the decisions about the effectively implemented monetary policy were close to the expectations of the economic agents in most time of the analyzed period, since the similarity of results between measures of central tendency (mean and median) of the *ex post* real long-term interest rates and the *ex ante* real interest rates.

Figures 2 to 5 show HP filter results for real interest rates series between November, 2001 and December 2009:

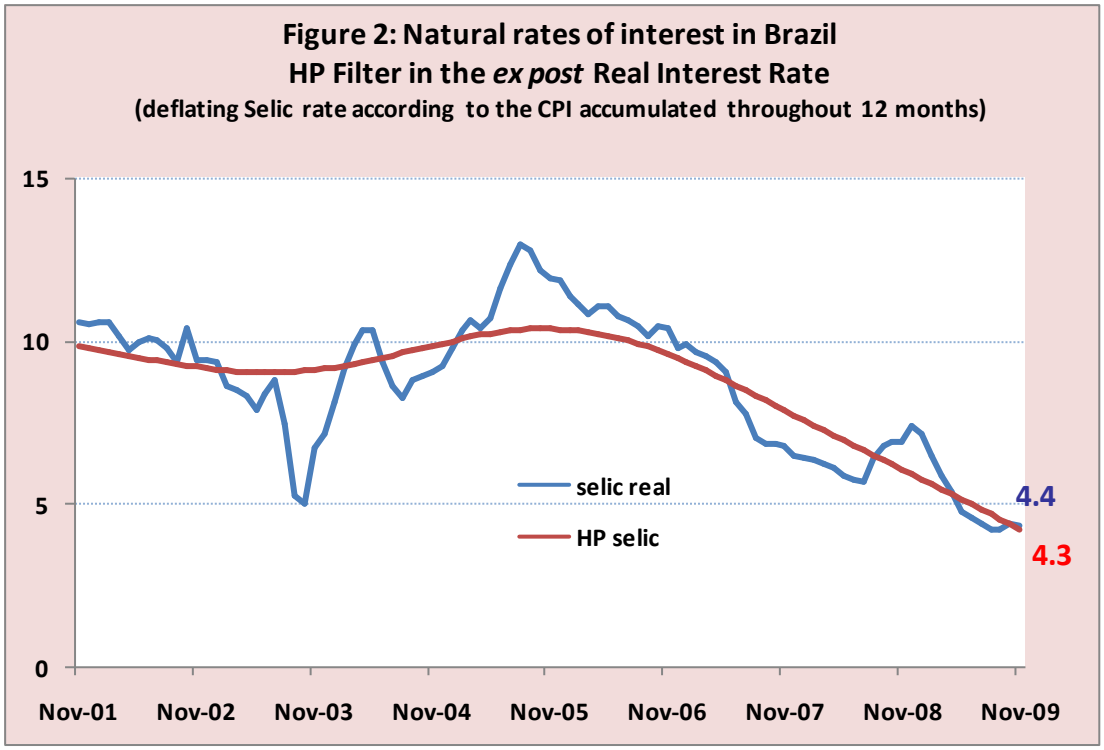


Figure 2: Natural rates of interest in Brazil
HP Filter in the ex post Real Interest Rate
(deflating Selic rate according to the CPI accumulated throughout 12 months)
15
10
5
0
selic real
HP selic
4.4
4.3
Nov-01
Nov-02
Nov-03
Nov-04
Nov-05
Nov-06
Nov-07
Nov-08
Nov-09

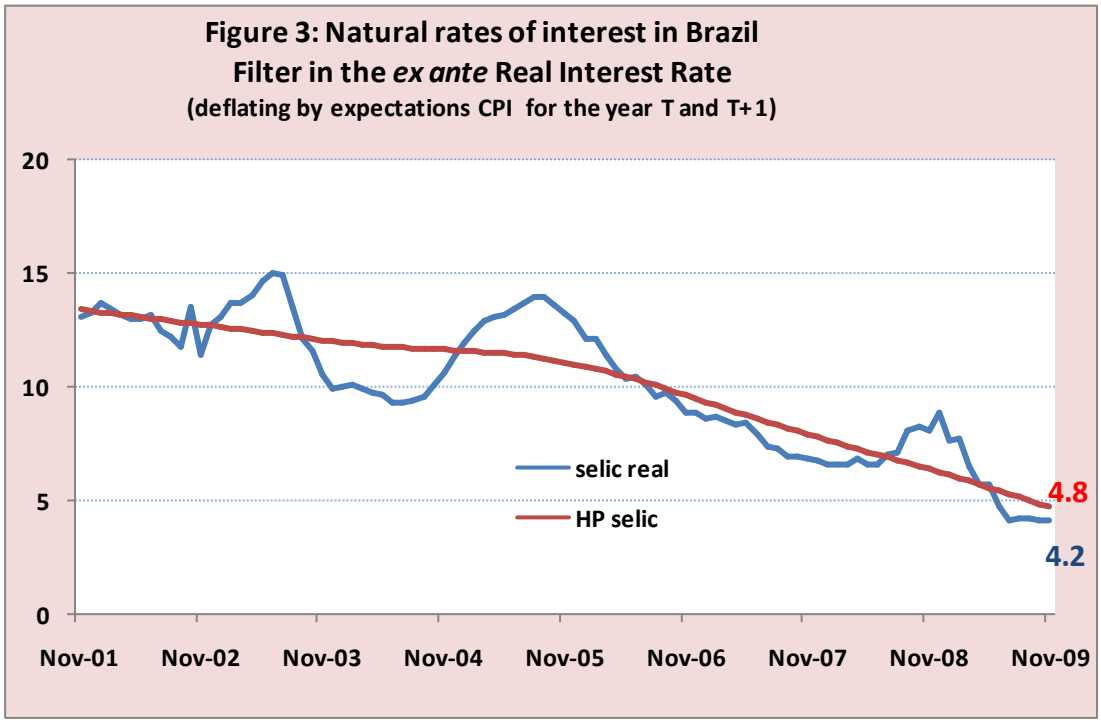


Figure 3: Natural rates of interest in Brazil
Filter in the ex ante Real Interest Rate
(deflating by expectations CPI for the year T and T+1)
20
15
10
5
0
selic real
HP selic
4.8
4.2
Nov-01
Nov-02
Nov-03
Nov-04
Nov-05
Nov-06
Nov-07
Nov-08
Nov-09

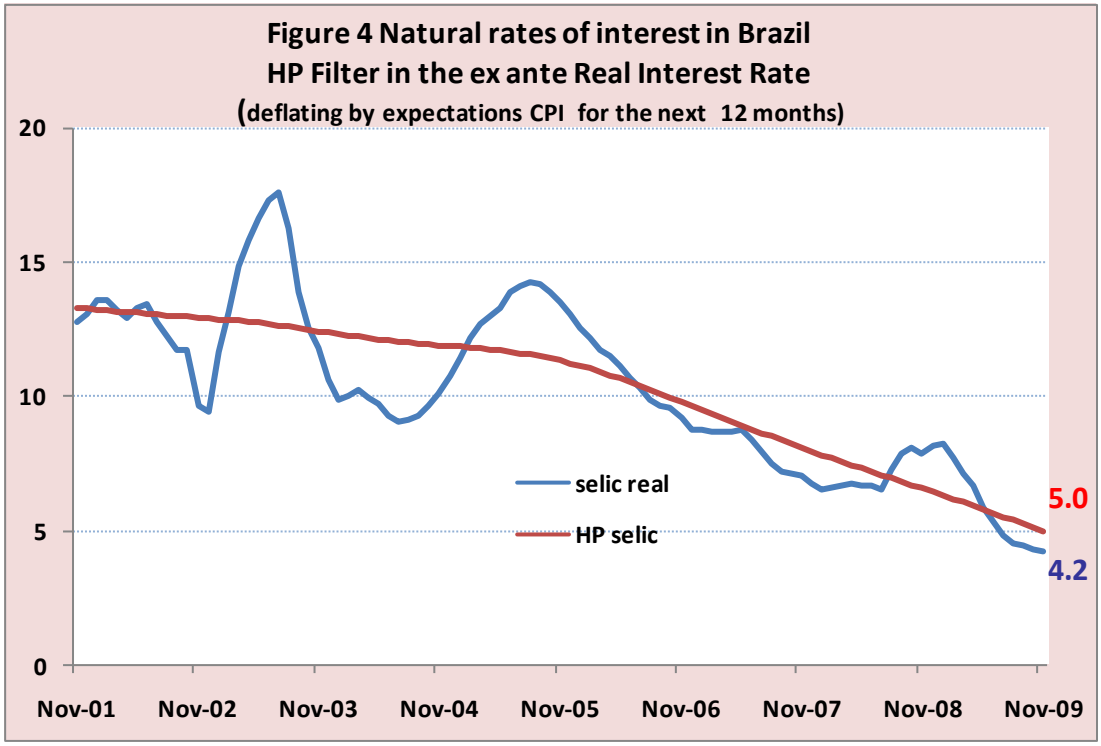


Figure 4 Natural rates of interest in Brazil
HP Filter in the ex ante Real Interest Rate
(deflating by expectations CPI for the next 12 months)
20
15
10
5
0
selic real
HP selic
5.0
4.2
Nov-01
Nov-02
Nov-03
Nov-04
Nov-05
Nov-06
Nov-07
Nov-08
Nov-09

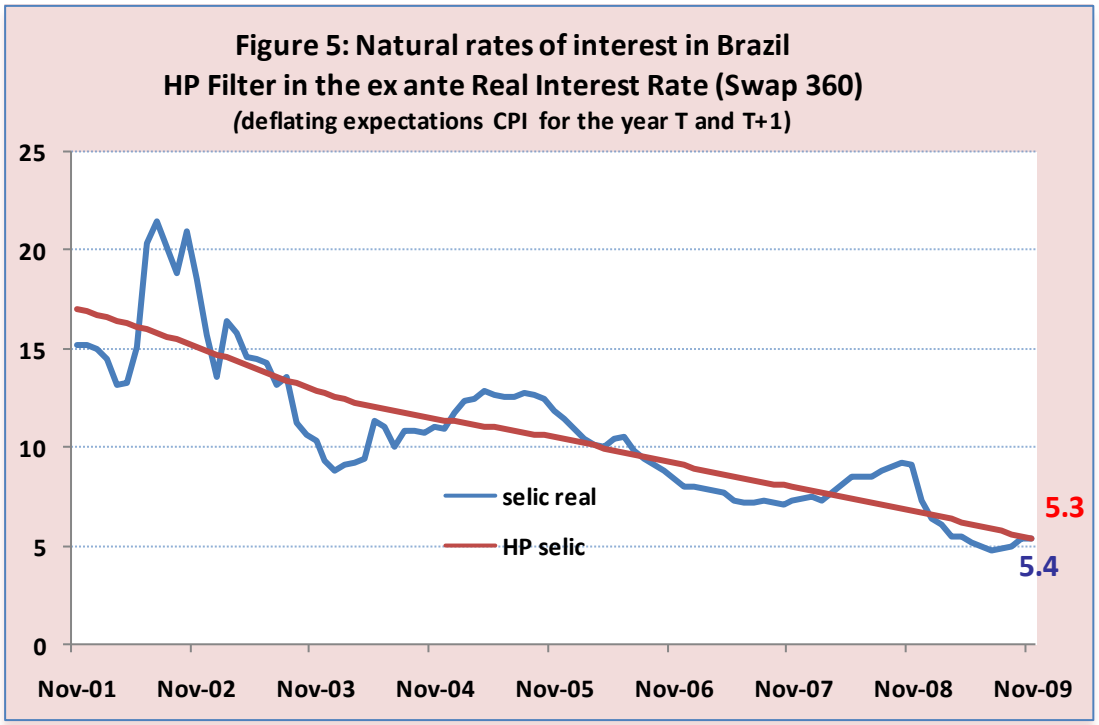


Figure 5: Natural rates of interest in Brazil
HP Filter in the ex ante Real Interest Rate (Swap 360)
(deflating expectations CPI for the year T and T+1)
25
20
15
10
5
0
selic real
HP selic
5.3
5.4
Nov-01
Nov-02
Nov-03
Nov-04
Nov-05
Nov-06
Nov-07
Nov-08
Nov-09

In general, the HP filter results suggest that over the last years, Brazil has experienced a pronounced decline in natural interest rate, regardless of interest rate instruments evaluated. Unsurprisingly, the natural interest rate obtained by the filters showed fluctuations, reflecting different short-term moments and high standard deviations in that period - as high as 12.7% at the 2002 and low as 4.7% in late 2009 -, as a result of the shocks since 1999. The pre-election crisis in 2002, for example, required that the Brazilian monetary authority take a very strong stance in the subsequent year, causing explanatory variables of the natural rate, such as output gap, inflation, and inflation expectations, to have a volatile behavior in that period.

3.2 Dynamic Taylor Rule using the Kalman Filter

The reaction function proposed by Taylor (1993) establishes that monetary policy decisions regarding the behavior of the benchmark interest rate may be well represented by a linear relationship, whose major component is the natural interest rate.

Clarida et al. (1999) mentioned that in a situation in which deviations of inflation from its target and deviations of economic activity from its potential level equal zero, the intercept could be understood as the equilibrium real interest rate.

Leigh (2005), in his turn, emphasized that as the natural rate of interest is affected by dynamic factors (e.g.: productivity growth), taking for granted that the natural rate is properly represented by the constant in the Taylor rule could be extremely restrictive.

In this paper, the equilibrium interest rate is extracted from the reaction function using a dynamic model for the Taylor rule intercept. This procedure allows extracting the real interest rate with which the Brazilian Central Bank worked implicitly throughout the analyzed period, in an attempt to achieve its inflation targets and to help smoothing the short-term business cycles.

So, as the reaction function of the Brazilian Central Bank is the starting point, the real rate derived from this function can be understood as the equilibrium interest rate implicit in monetary policy decisions throughout the analyzed period that tend to eliminate the output gap and the deviations of inflation expectations from inflation targets. This way, the comparison of this rate with the natural rate of interest allows for a qualitative analysis of how conservative are the monetary policy decisions taken by the Brazilian Central Bank.

The estimation of the real interest rate consistent with the reaction function was made using the Kalman filter, with the following structure:

$$i_t = r^*_{et} + \beta_1 i_{t-1} + \beta_2 D_{j,t} + \beta_3 h_{t-2} + \varepsilon_{t,1} \quad (1)$$

$$r^*_{et} = r^*_{et-1} + \varepsilon_{t,2} \quad (2)$$

where r^*_{et} is the real interest rate implicit in the decisions made by the monetary authority; i_t nominal benchmark Selic rate (monthly) deflated by expectations CPI for the year T and T +1 obtained in the market survey conducted by the Brazilian Central

Bank[4]; i_{t-1} real benchmark Selic rate lagged one period; $D_{j,t}$ weighted deviation of expected inflation from the inflation target; h_{t-2} output gap lagged two periods[5].

This set of equations can be represented in the state-space form[6] as:

$$[i_t] = [\beta_1 \beta_2 \beta_3 r^*_{et}] \begin{bmatrix} i_{t-1} \\ h_{t-2} \\ D_{j,t} \\ 1 \end{bmatrix} + [\varepsilon_{t,1}] \qquad \text{(Measurement Equation)}$$

$$\begin{bmatrix} \beta_{1,t} \\ \beta_{2,t} \\ \beta_{3t} \\ r^*_{et} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_{1,t-1} \\ \beta_{2,t-1} \\ \beta_{3,t-1} \\ r^*_{et-1} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \varepsilon_{t,2} \end{bmatrix} \qquad \text{(State or Transition Equation)}$$

This framework for the Brazilian reaction function is in agreement with studies that estimated the Taylor rule for Brazil, such as the one conducted by Minella et al. (2002) and Portugal & Barcellos (2009), whose major difference is in the intercept dynamics, which in the present case varies over time, following a random walk in the state equation.

Figure 6 summarizes the results for Brazilian natural interest rate by dynamic Taylor rule using the Kalman Filter. The implementation of this process allowed obtaining a

[4] As demonstrated, this variable was constructed following the suggestion made in Minella et al (2002).

[5] IBGE's output series with seasonal adjustment was used, and the series obtained from the Hodrick-Prescott filter was used as potential output.

[6] See Portugal & Barcellos (2009)

path for the natural rate of interest in Brazil during the inflation targeting regime. As shown in this figure, the mean and median for the natural rate for all analyzed period were 9.9% and 10.6%, respectively, and the standard deviation was close to that found in previous estimates (2.7%). For late 2009, the Taylor exercise shows the natural interest rate closed to 5.0%.

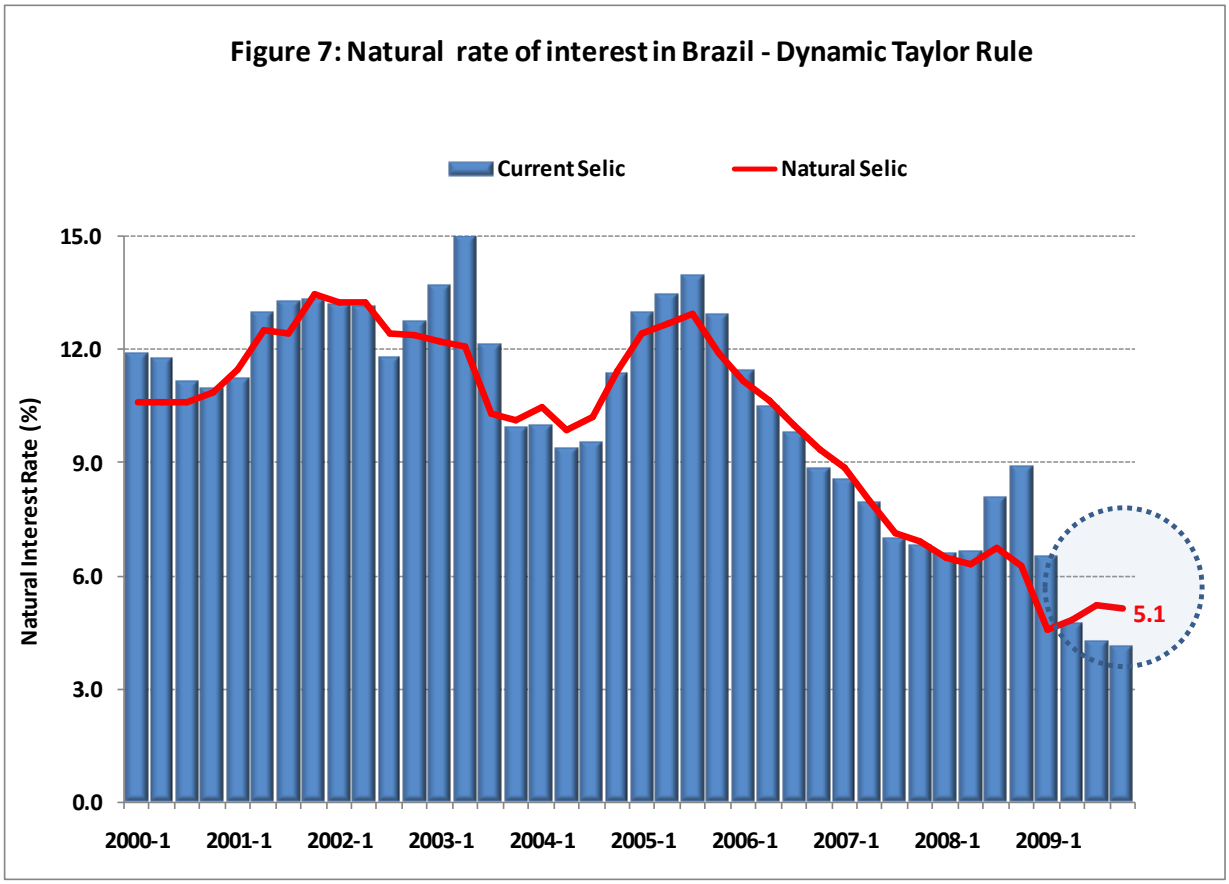
The results for all period was similar to that obtained with the HP filter in the *ex post* and *ex ante* real interest rate series, showing convergence between the most commonly used interests and the natural rate. These results show that the Brazilian monetary authority worked explicitly or implicitly with a real interest rate close to the filters results.

Figure 6: Natural interest rate summary - Dynamic Taylor Rule

	ex antes real selic[1]
Mean	**9.9**
Median	**10.6**
Maximum	**13.5**
Minimum	**4.5**
Standard deviation	**2.7**
Period:	**Jan/00 a Dec/09**

(1) deflating the benchmark Selic rate according to the expectations CPI for the year T and T +1

Figure 7 shows Taylor results for Brazilian natural interest rate from 2000 to 2009.



In general, the Taylor rule results confirm the previous exercise. Since the implementation of inflation targeting regime in 1999, Brazil has experienced a pronounced decline in natural interest rate. It's also observed fluctuations reflecting different short-term moments. The pre-election crisis in 2002, for example, required that the Brazilian monetary authority took a very strong stance in the subsequent year, causing explanatory variables of the natural rate, such as output gap, inflation, and inflation expectations, had a volatile behavior in that period.

Recently, in face of the global financial crisis, the Central Bank has cut the Selic rate aggressively. It's interesting to note that Taylor rule results suggest that the Brazilian

Central Bank has brought current real interest rates below the 'neutral' level – and correctly so. As result, Brazil's economic recovery has been more stronger than in its trading partners, as shown by GDP performance in the last quarter of 2009 (some 8,0% growth in annualized terms) and the stronger domestic demand has provided the background to the recent of inflation.

3.3 Interest rate GAP

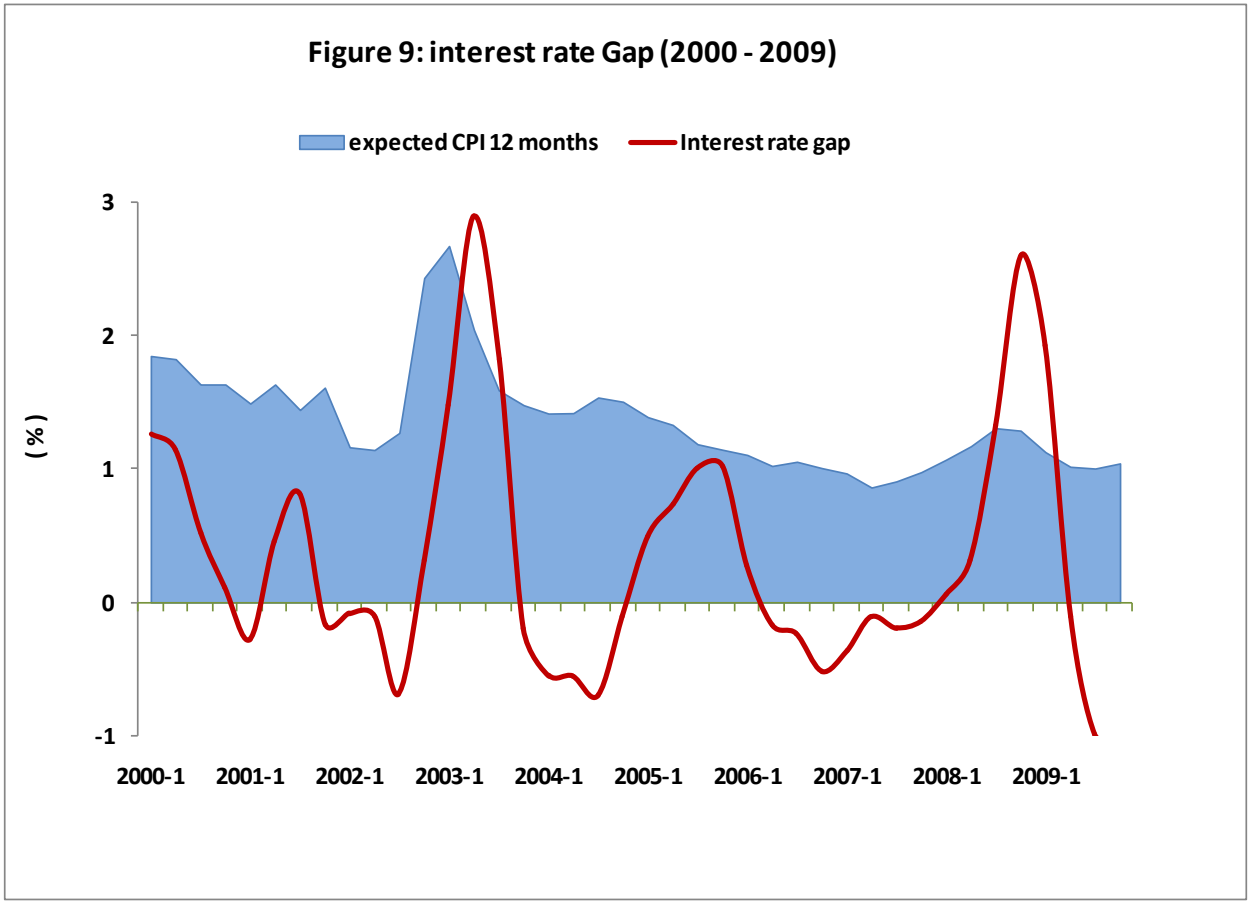
As the monetary authority tries to assume a policy-neutral stance on the determination of the benchmark interest rate, the comparison of the natural rate with this estimate should yield results that are very close to zero, since persistently positive output gaps indicate conservative monetary policy, while persistently negative ones indicate an expansive policy.

It's observed in figure 8 that the average Selic rate is higher than the average natural interest rate from 2000.1 to 2005.3, whereas in the following period the average selic rate is lower than the average natural rates. As a result, the average interest rate GAP is positive in the first period and negative in the second. In another words, there is evidence of a more conservative monetary policy until 2005.3 and more expansive since then.

Figure 8: current and natural interest rate - average (%)

	Current Selic	Natural Selic
2000.1 to 2002.2	12.3	11.9
2002.3 to 2003.2	13.3	12.3
2003.3 to 2004.3	10.2	10.2
2004.4 to 2005.3	12.9	12.4
2005.4 to 2009.1	8.6	8.2
2009.2 to 2009.4	4.3	5.0

Figure 9 shows the interest rate gap concerning the expected inflation. In this exercise, the gap is obtained by difference between *ex ante* real interest and the natural rate implicit in the Taylor rule. Particularly from 2005.4 to 2009.4, there are negative/neutral values in interest rate GAP, indicating an expansionary monetary policy while inflation has accelerated.



The interest rate gap estimates show that monetary policy decisions in the inflation targeting regime were actually forward-looking because the evaluation between the interest rate implicit in the reaction function with *ex ante* real interest rate revealed similar results most of the time.

However, its evolution revealed non-negligible differences in regard to what would be understood as a neutral behavior. In 2003, 2005 and 2008 for instance, there was a tendency towards a positive gap, which is compatible with the acknowledged effort of the Central Bank to fight inflation inertia caused by economic shocks in 2002 and the stronger domestic demand provided the background to inflation.

4. CONCLUSIONS

The aim of the study was estimating the natural rate of interest in Brazil defined as the nominal interest rate that keeps inflation steady. Despite the difficulties involved in the precise determination of equilibrium real interest rates, the real interest rate gap can thus serve as a leading indicator of future inflationary or deflationary pressures in the economy.

First, statistical filters was used for the *ex ante* and *ex post* real interest rate series. Then, an estimation of a dynamic Taylor rule was performed using the Kalman Filter. According the results, Brazil has experienced a pronounced decline in natural interest rate over the years, peaked at levels close to 13.5% in 2001, then narrowing all the way to current 5.1% in late 2009. Results are also compatible with those works that had a similar aim; even they used different estimation methods.

These results suggest that the Brazilian Central Bank has brought current real interest rates below the 'neutral' level – and correctly so. As result, Brazil's economic recovery has been stronger than in its trading partners, and the stronger domestic demand has provided the background to the recent of inflation.

The results obtained are also not consistent with the arguments that Brazilian monetary policy has been extremely rigid about the determination of the benchmark interest rate in order to achieve the predefined inflation targets. The monetary authority kept the

Selic rate close to the natural rate the greater part of the period, although its evolution revealed non-negligible differences in regard to what would be understood as a neutral behavior.

Structural convergence to sustainable lower real interest rates will likely prove to be a gradual process over time, still dependent on further structural progress, such as the increase in the total factor productivity, changes in the sensitivity of inflation to the expectations of economic agents, instead of a lenient monetary policy.

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