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Brazilian Public Debt Benchmark: a long-term strategy

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Brazilian Public Debt Benchmark: a long run strategy[1]

1. Introduction

The design of financing strategies for public debt involves choices regarding the balance between expected costs and risks each alternative may root. A key element in this process is the definition of the desired profile for the long-term debt composition. While debt management theory provides some guidance towards general characteristics of the debt portfolio[2], the search for methodologies to determine a benchmark[3] has occupied prominent space on the research agenda of debt managers and other experts in several countries[4], in order to set quantitative guidelines.

The benchmark stands as a long-term reference for the debt portfolio profile used to guide short and medium-term financing strategies[5]. It may be expressed by some set of relevant debt indicators, such as stock composition (e.g. floating versus fixed interest rate; domestic versus foreign currency), duration, maturity profile, etc.

The contribution of multilateral institutions such as the World Bank and the International Monetary Fund to this debate has also been notorious. These two institutions describe the benchmark, in their publication entitled "Guidelines for Public Debt Management" (WB & IMF, 2001), as a powerful tool to represent the debt profile the government want to achieve based on its risk and cost preferences.

In the context of that debate, this work aims to understand the cost-risk tradeoff of alternative debt compositions and contribute to establish a long-term reference for the Brazilian Federal Public Debt (FPD). From the risk management viewpoint, the long-term benchmark represents the direction the manager wants to follow and, in this sense, this research provides a tool to formulate and monitor debt strategies in Brazil.

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[1] Opinions here do not necessarily represent the view of the National Treasury of Brazil. The author is grateful to Pedro Ivo Ferreira de Souza Junior for research assistance.
[2] See, e. g., Calvo (1978), Bohn (1990), Calvo & Guidotti (1990), Missale & Blanchard (1994) and Barro (1997, 1999).
[3] The word "benchmark" in the context of public debt sometimes can refer to the optimal level of indebtedness of a country, as in debt sustainability analysis. Alternatively, it can refer to the optimal debt composition, as regarding to the characteristics of a debt portfolio. This second approach will be used in this work.
[4] Brazil, UK, Canada, Denmark, Portugal and Sweden are well-known examples, as discussed in the third section of this work.
[5] Silva, Cabral & Baghdassarian (2006) discuss the role of a benchmark model in the context of the Brazilian public debt risk management.

Specifically, the objective of this work is to analyze some set of strategies (debt compositions) in order to derive a cost-risk efficient frontier for the FPD. In this sense, a composition is efficient when it has the lowest cost given any chosen risk level. The set of all such compositions defines the efficient frontier, so as to reflect the cost-risk tradeoff faced by the debt manager.

The work is structured in six sections, besides this introduction. The next section discusses the theoretical aspects which justify an active debt management and the importance of associating the debt composition with the way which each type of bond reacts over the macroeconomic shocks. The third section explores the international experience in debt management to define long-term strategies, considering the impacts of shocks that affect the debt cost over time. Next, the fourth section focuses on the Brazilian experience and the recent changes in the FPD.

In the fifth section, a small macroeconomic model along with a yield curve model is specified with the aim of simulating shocks over the main variables that shape the debt cost over time. The application of the model to the Brazilian case is the subject of the sixth section. The analysis of strategies explores the fact that different financing instruments react in different ways depending on the type of macroeconomic shocks affecting the economy. Thus, the debt manager looks for a mix of all available instruments to achieve a balanced long-term portfolio from the cost-risk perspective, as concluded in the seventh section.

2. Debt management theories

The relevance of an appropriate debt portfolio profile is consistent with theories on debt management which arises when assumptions underlying the Ricardian Equivalence Hypothesis[6] are relaxed. Instead of irrelevance of public debt on the economic activity as a consequence of that hypothesis, literature about optimal taxation and time consistence lead to arguments in favor of an active debt management.

The literature about optimal taxation suggests that, if tax rates cause economic deadweight losses, governments should smooth them over time ("tax smoothing"), thus

[6] One consequence of the Ricardian Equivalence hypothesis is the neutrality of the debt on economic activity, once debt and taxes are equivalent from an inter-temporal perspective. However, Ricardian Equivalence depends basically upon the following assumptions: (1) infinite planning horizon; (2) complete markets and (3) non-distortionary taxes. See Barro (1974, 1979, and 1989).

minimizing the distortions due to revenue collection. In this context, the debt profile and its risks are relevant for policy purposes, since fluctuations in the debt costs would lead to changes in tax rates. On its turn, the literature about credibility and time consistence shows situations in which governments could have incentives to manipulate the value of its liabilities leading to time inconsistency problems between optimal monetary and fiscal policies. Accordingly, the choice of an appropriated debt profile could make those policies time consistent.

Togo (2007) remarks that the tax smoothing literature establishes the importance of debt management in supporting the fiscal policy, while the time consistence literature gives more emphasis on the relevance of debt management to monetary policy. Besides that, debt management should be treated as a separate macroeconomic policy, along and in coordination with the fiscal and monetary policies. In the short-term, the possibility of tradeoffs between objectives of the three policies would lessen their credibility and effectiveness. On the other hand, the lessening of these attributes of policy implementation could be prevented by decentralizing those policies, which means to separate debt management from the others.

Of course, a decentralized policy needs to have a specific objective. According to the WB & IMF (2001) the aim of the public debt management is “to ensure that the government’s financing needs and its payment obligations are met at the lowest possible cost over the medium to long-term, consistent with a prudent degree of risk.” In fact, that is the essence of the debt management objective in many countries, including Brazil.

Two aspects deserve attention on this objective. The first is the reference to the time frame the debt manager consider in assessing costs, suggesting the relevance of policies that are more efficient in the medium and long-term, instead of those oriented for the short-term. The second is the explicit introduction of risk considerations in the goal of the debt management. A satisfactory balance of cost and risk should be aimed because often, the lower the cost of a debt portfolio, the greater its risk.

Once the debt management objective is clearly identified, the search for a strategic benchmark is the next step in the long-term planning process. Several approaches can be adopted to define a methodology to help establishing an optimal debt composition (or an efficient set of compositions).

In this context, the Assets and Liabilities Management (ALM) framework is useful as a tool to map and manage risks of the public debt portfolio. It consists of a dynamic analysis of assets and liabilities that takes into consideration the public debt management strategy and the macroeconomic policies enforced by the government. It aims at an optimal structure in terms of debt composition according to currencies, indexing, interest rate risks and duration to protect the net balance from risks stemming from market factors. The basic premise for outstanding debt immunization is the balancing between the characteristics of assets and liabilities.

As one of the advantages of the ALM approach, it provides a tool to examine coordination and coherence between fiscal, monetary and debt policies. Togo (2007) recognizes several interdependences between those policies, albeit the idea of a decentralized debt management. So, it is useful having a framework that permits to integrate risk assessments of the public debt considering the characteristics of other public policies.

The ALM approach is also articulated with the tax smoothing idea. As Togo (2007) pointed out, the principal asset of a government is the present value of the future flows of primary surplus and the financial characteristics of those flows depend on the objective of fiscal policy. For example, if the government adopts a counter-cyclical fiscal policy, by which primary surpluses are produced when the economy is expanding and primary deficits are generated when there is a recession, the ALM framework requires a debt portfolio with debt service payments lower during a recession than when the economy is growing. That policy would be consistent with the optimal taxation theory, although tax smoothing is not usually stated as an explicit fiscal policy objective.

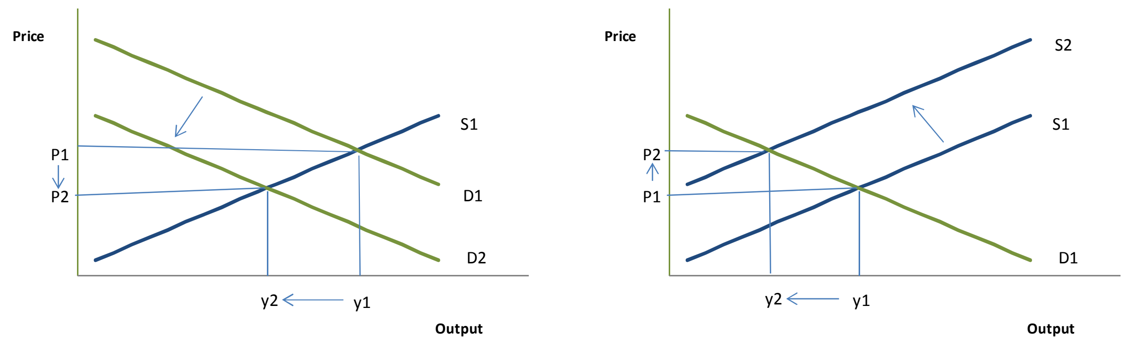
Another way to see the importance of the ALM approach is thinking about what the government would do in case of unexpected shocks that drives the public debt away from its sustainable path. If the debt servicing flows are raised by an unexpected shock, the government should increase the fiscal primary surplus through higher tax revenues or cuts in spending. As Velandia (2002) pointed out, both alternatives could produce high social and economic costs. Governments should avoid variability in the tax rates over time because they would produce social welfare losses, as suggested by the tax smoothing theory. Also, cutting the spending level may lead to welfare costs depending on the services and programs that are affected. Furthermore, the ability to promote budget cuts can be limited especially in the context of economic shocks.

It is clear from this debate that the liability portfolio of the government should be structured to avoid excessive volatility in the debt service flows. Fluctuations in the costs of public debt due to macroeconomic shocks may change the monetary and fiscal policies, leading to high social cost or adding inconsistency to the mix of those policies.

In order to adequately choice the debt profile, it is important to recognize that each debt instrument behaves in a different way depending on the type of shock that hit the economy. For example, inflation-linked debt may hedge the fiscal position in events of aggregate demand shocks, while nominal fixed rate debt may help in protecting the government balance sheet in cases of aggregate supply shocks. These relationships between types of shocks and debt instruments to protect the public finances are well described in Togo (2007), using the diagrams that are following reproduced.

The first diagram (on the left) depicts what happens in the event of a negative aggregate demand shock. The drop in the demand causes contraction in both price and output levels. In this scenario, the government revenues tend to fall, once they are positively correlated with the economy activity level, and the government expenditures tend to rise, for instance, as an effect of counter-cyclical measures. But, at the same time, the service of the inflation linked debt tends to fall, accompanying the price level and hedging the fiscal position.

Figura 1. Aggregate demand and supply shocks



The second diagram (on the right) illustrates the scenario of a negative aggregate supply shock. In this case, price and output levels move in opposite directions due to the supply contraction. While government revenues tend to fall and expenditures tend to rise following the reduction in the economic activity level, inflation linked debt service tends to rise with the price level. In this context, nominal fixed rate bonds would reduce its real debt service, providing protection and predictability to the fiscal balance sheet.

Considering other types of debt, the relationship between floating rate debt and fiscal position also depends on the type of shock. As a consequence of a negative aggregate demand shock, the short-term interest rate tends to be reduced while the fiscal position is less favorable. On the other hand, in case of a negative aggregate supply shock, the short-term interest rate tends to rise in response to the increasing in the price level, while the fiscal position also is less favorable[7]. So, in comparison with fixed rate debt, the floating rate debt tends to behave well in demand but not in supply shocks events. The comparison between inflation-linked and floating rate debt depends on how much the short-term interest rate will change relatively to the inflation rate.

Finally, external shocks affect the exchange rate and, thus, the cost of foreign currency denominated debt. A shock that causes domestic currency devaluation may represent a real depreciation that stimulates the economic activity and, as a result, it increases the government revenues. But, if the domestic currency devaluation causes inflation and makes more expensive the imports, the result may be an economic contraction. The impact of variations in the exchange rate over the fiscal balance also depends on the assets position of the government, especially, the international reserves level.

In this work, a small macroeconomic model is specified in order to evaluate how these different shocks impact the debt dynamic considering several possible debt compositions. Based on stochastic simulations of shocks, a simplified ALM approach is used to analyze the risks mentioned above and help in defining a benchmark debt composition to the Brazilian case. As it will be seen in the next section, the approach used here is in line with experiences of other countries that formulate benchmark models to debt management.

3. International experience in benchmark modeling

Portugal is one of the pioneers to formulate and adopt a desired long-term debt composition to quantify its public debt management objective, as well as to increase the

[7] In fact, in economies with inflation target as in the case of Brazil, if a Taylor Rule is assumed, the result depends on the weight that the Central Bank attributes to inflation vis-à-vis output in its rule. If inflation is more weighted than output, it is expected an increase of the short-term interest rate due to a negative aggregate supply shock.

consistency between daily decisions and long-term objectives. Also, the benchmark is used as a performance measure to evaluate the debt management results. The framework used by Portugal was first released in 1999 by the IGCP[8]. Cost and risk of debt cash-flows are considered as key decision variables but, at the same time, explicit refinancing risk restrictions are introduced in the model. The model provides a set of efficient and robust (regarding changes in the macroeconomic scenarios employed) debt portfolio that are used as inputs in the establishment of its guidelines.

The model assumes a constant nominal debt in steady state, with three basic inputs: (i) stochastic interest rates; (ii) a set of financing strategies; (iii) deterministic scenarios to address other macroeconomic variables. Following, simulations are made in order to evaluate how each debt portfolio evolves over time, to generate a set of efficient and robust portfolios. A restrict subset of the best solutions of the model is presented to the authorities, who choice the acceptable long-term cost and risk, as well as the associated strategy, as the debt benchmark. The final choice also takes into account market restrictions, once the strategy should be feasible to a sovereign issuer.

Denmark is another country that developed benchmark models for public debt[9]. The Danish Government Debt Management establishes targets for the duration of the total debt portfolio as the key strategic benchmark for the interest-rate risk of central government. Duration, in this case, is used as a summary measure to evaluate the cost-risk tradeoff, considering that the shorter the duration of a portfolio, the higher the interest rate risk on the debtor perspective. However, in order to define a target, which is reviewed on an annual basis, a long-term analysis of the expected cost evolution is considered. The interest-rate risk is addressed on an ALM framework, and the main tool used to evaluate that tradeoff is a Cost-at-Risk (CaR)[10] model.

A good experience is provided by the Swedish Government Debt Management[11]. It establishes targets (benchmarks) for the shares of nominal and inflation linked (both in the domestic currency) and foreign currency debt in the aggregated cash-flows of the Central Government Debt. For each one of these types of debt, there are also targets for the average

[8] IGPC: Instituto de Gestão da Tesouraria e do Crédito Público de Portugal. See IGPC (1999): "*Relatório Anual de Gestão da Dívida Pública*".
[9] See Danmarks Nationalbank (2007).
[10] The Cost-at-Risk is defined as the maximum interest costs in a year, for a given probability (e.g. 95 percent), (Danmarks Nationalbank, 2007).
[11] See Bergström & Holmlund (2000), Riksgälden (2008) and Riksgälden (2009).

maturity (duration). The targets for debt shares aim to reduce risks of fluctuations in the cash-flows, while the maturity targets intent to address the interest rate refixing[12] and refunding risk[13].

The Swedish model to evaluate the cost is based on a Running Yield concept (Riksgälden, 2008). A set of equations is used to dynamically simulate yield curves (for the three types of debt), inflation and exchange rates, so that it is possible to calculate the probability distribution of the Running Yield over stochastic scenarios. So, a dispersion measure of that distribution (in fact, the difference between the 95th percentile and the median of the cost distribution) is used as risk indicator. Several portfolios are evaluated to choice the debt maturity target.

Pick &Anthony (2006) presented a model applied to the case of the United Kingdom for simulating the behavior of its public debt. The analysis of financing strategies is done through stochastic scenarios based on the combination of a macroeconomic with a yield curve model. The macroeconomic model describes the evolution of output gap, net primary financing requirement, inflation and short-term interest rate, while the yield curve model specifies the term structure of interest rates, both nominal and real. After the simulations, the cost is measured in terms of cash flows, corresponding to the sum of the nominal coupon payments, and the risk is measured by the dispersion of such payments (measured by its standard deviation or the 95th percentile).

In the work of Pick & Anthony, the debt service payments are treated as a proportion of GDP. The authors present three reasons for that. First, the standardization by GDP provides a better idea of debt burden to public finances. Second, the specification of rules for fiscal policy is usually made with reference to the debt/ GDP ratio. Third, the cost/ GDP provides an indirect way to capture aspects of ALM in debt management in that the main public asset - the future flow of income from taxes - has strong correlation with GDP.

The Canadian model to evaluate public debt strategies also combines macroeconomic with the yield curve model to simulate cost and risk of alternative financing strategies as in Bolder (2008). In the model, the measure of cost is the average annual debt-service charges

[12] “Interest rate refixing risk is the risk that borrowing has to take place at a higher interest rate.” (Riksgälden, 2009)
[13] “Refunding risk is the risk that the state will not succeed in borrowing to cover maturing loans or will have to pay far too high interest rates in order to be able to borrow.” (Riksgälden, 2009)

as a percentage of the total debt stock over a 10-year simulation horizon. However, the author explains the importance of considering the policy objective when defining the appropriate indicator to analyze risk in a debt management model. The risk can be defined in terms of debt costs volatility or, alternatively, by considering its impacts to the public budget. Of course, both approaches are related because the debt service payment is part of the government budget. However, while the first is more focused on the debt management itself, the budgetary approach takes into account the relationship between debt and fiscal policies by introducing, for example, tax smoothing considerations. In turn to integrate more than one objective to the analysis, the model provides a tool to minimize debt-service charges with constraints on other objectives, such as constraints on the average risk of debt-service charges (as measured by the standard deviation) and extreme risk of debt-service charges (as measured by the Cost-at-Risk) or constraints on budgetary risk (as measured by the Budget-at-Risk[14]).

One important aspect of this debate is that there is not a common methodology to define a benchmark for public debt. The definition of strategic targets can be made from traditional indicators for risk assessment, as the duration of the portfolio or from more complex tests, such as those involving stochastic simulations. Another aspect which is not immediate is to specify the measure of risk that, among other factors, should take into account the objectives of debt management, which can be multiple ones. Finally, and most importantly, the models are just one simplified representation of reality and, therefore, with several limitations. Accordingly, as pointed out by Bolder (2008), although modeling can add several benefits to decision-making, the model cannot replace the knowledge and the judgment of the debt manager.

As it is commented in the following section, the National Treasury of Brazil is also advanced in developing an optimal long-term debt composition model. The fact that Brazil is an emerging economy with lots of peculiarities turns the Brazilian case into a rich experience for research.

[14] The absolute budget at risk indicates the maximum amount which the deficit of government (or the minimum surplus) can achieve, for a given probability (for example, 95 percent).

4. The Brazilian experience

The aim of the FPD management is "to minimize long-term borrowing costs, with maintenance of prudent risk levels; at the same time, seek to contribute to smooth operation of the public bond market"[15]. Thus, risk assessment and risk exposure management are fundamental components of strategic public debt planning. In this context, the major risks targeted for constant assessment are the refinancing[16] and the market[17] risks, besides the strategy, operational and legal risks. Refinancing and market risks are at the center of mitigation policies developed by the National Treasury.

The search for a FPD long-term optimal composition is part of the strategic planning process of the Brazilian debt management. As a turn point in this process, an explicit cost-risk analysis was introduced in the Annual Borrowing Plan of 2002[18], which released that the Federal Public Debt management would base its guidelines on an Asset and Liability Management model, considering all items that directly or indirectly affected the debt. Although they have slightly changed its terms, currently[19] the following guidelines are essentially very close to the former ones[20]:

- "Lengthening of average maturities and reduction in the percentage of FPD maturing in 12 months;
- Gradual substitution of floating rate securities by fixed rate or inflation-linked securities;
- Improvement in the External Federal Public Debt - EFPD profile through issuances of benchmark securities, the anticipated buyback program and structured operations;
- Incentives to the development of the forward interest rate structure for federal public securities on the domestic and external markets; and

[15] See the 2009 Annual Borrowing Plan.
[16] Refinancing risk is rooted in the possibility that the National Treasury will have to bear the burden of higher costs to obtain short-term financing or, in an extreme case, will fail to obtain the resources required to honor its debt maturities. This risk is a consequence of the maturity profile once available cash resources are deducted, and of the sensitivity of short-term public debt maturities to shocks in the underlying economic variables.
[17] The market risk is a consequence of alterations in the financing cost of public securities resulting from changes in short-term interest rates, exchange rates, inflation or the interest rate term structure.
[18] Annual Borrowing Plans are released by the National Treasury of Brazil since 2001.
[19] See the 2009 Annual Borrowing Plan.
[20] In 2002, they were: (1) Lengthening of the average term to maturity of securities offered at public auctions; (2) keeping the share of debt due in 12 months at prudent levels; (3) gradual replacement of securities linked to the SELIC rate for fixed-rate securities; (4) further development of the term structure of interest rates (fixed-rate and inflation indexed curves); and (5) seek a smoother public debt maturity profile. The Annual Borrowing Plans released from 2003 to 2007 also mentioned a guideline to reduce the securities linked to exchange rate variation in the total public debt.

- Expansion of the investor base."

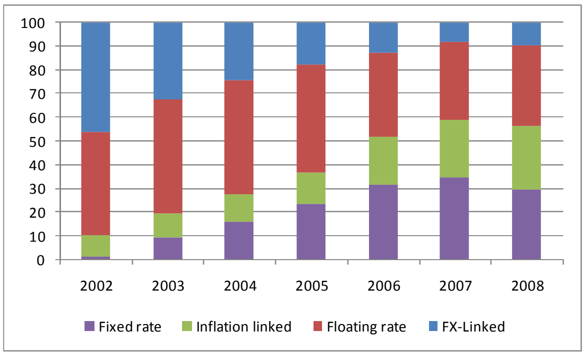
In the 2002 Annual Borrowing Plan, Central Government Assets and Liabilities were grouped in four main risk categories: inflation, exchange rate, floating rate, and fixed rate. The main mismatches in these categories, and consequently the market risk, were associated with net assets linked to inflation and, on the other hand, net liabilities linked to the exchange rates or to the short-term interest rate. Thus, this suggested a strategy based on the issuance of inflation linked and nominal fixed rate bonds in order to gradually improve the cost-risk balance of the debt portfolio. This proposal also took into account the characteristics of the future Central Government surpluses, once they represent the main asset of government.

The Central Government Assets and Liabilities also were grouped according to eight categories of time to maturity[21]. The refinance risk was associated with imbalances in cash-flows and time to maturity structure. At that time, 26% of the total cash-flows were concentrated in the first year. However, it is interesting to note that almost the total of the fixed rate debt had time to maturity up to 12 months. This point addressed the requirement of efforts to develop the term structure of interest rate along with the replacement of floating rate by fixed rate debt, as a way to avoid rising the refinancing risk in that process.

Based on the ALM approach and the guidelines we have mentioned, the National Treasury of Brazil has gradually changed the public debt profile. The relative participation of the debt with remuneration pegged to foreign currency was reduced faster than was that of floating rate debt. These riskier debts were replaced with fixed rate and inflation linked debt. The FPD was composed by 45.8% exchange rate linked, 43.4% floating rate, 9.2% inflation linked and 1.5% fixed rate debt, as a position of December, 2002. Nowadays, the FPD is composed by 9.7% exchange rate linked, 33.8% floating rate, 26.6% inflation linked and 29.9% fixed rate, as of December 2008.

[21] They were: up to 6 months, 6 months to 1 year; 1 year to 2 years; 2 years to 3 years; 3 years to 4 years; 4 year to 5 years; 5 years to 10 years; more than 10 years.

Figura 2. FPD composition



Source: National Treasury of Brazil

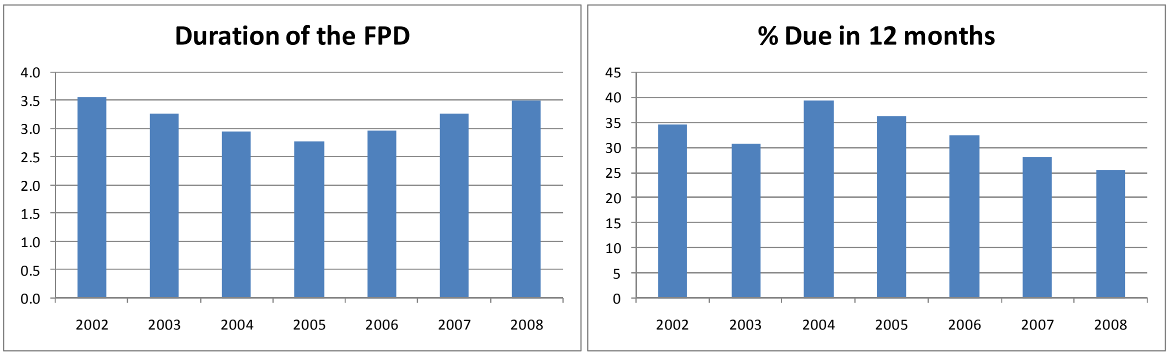
Despite the directive to reduce the relative share of debt with floating rates in the total FPD, it increased in the years 2003 and 2004 and, even back to fall in 2005, the percentage remained higher than in 2002. One reason for this behavior was that reducing the exchange rate linked debt was initially accompanied by increased issue of short-term fixed rates bonds. In order to avoid a possible increase in the refinancing risk, the National Treasury of Brazil continued to issue debt with floating interest rates with average time to maturity greater than that of the fixed rate debt.

After 2005, the share of FX-linked debt continued to reduce, but it can also be seen a pronounced reduction of the participation of debt with floating rates. In contrast, the share of inflation linked debt was enhanced, as well as the share of fixed rate debt. The lengthening of the average time to maturity of these bonds in recent issuances, with the fact that inflation linked bonds are typically long-term instruments, helped in the implementation of that strategy.

Even though the small increase in the participation of floating rate debt observed in 2008 over the previous year, the sum of inflation linked and fixed rate debts currently represent 56.5% of the FPD, which shows a significant change compared to 2002 when that share was of 10.7%. From the ALM viewpoint, this improvement in the debt profile meant a reduction in the market risk of the FPD.

The change in the debt composition initially resulted in shortening the average maturity of the FPD, as well as increasing the proportion of debt due in the short-term (one year). But the consolidation of an environment with greater economic stability in Brazil and the development of the domestic term structure of interest rates make possible, in a second moment, to implement the strategy of lengthening the FPD average maturities and reducing the percentage of debt due in 12 months.

Figura 3. FPD maturity structure



Source: National Treasury of Brazil

The average time to maturity of FPD reduced from 3.6 years in 2002 to 2.8 years in 2005 and then began to increase consistently, reaching 3.5 years in 2008. The other indicator, the percentage due in 12 months was 34.6% in 2002, reaching 39.3% in 2004, but following a downward trajectory to 25.4% in 2008. These indicators show that the National Treasury, in addition to changing the composition of debt has reached the smoothing of their maturities over time. In other words, the adopted strategy combined reduction in both the market and refinancing risks of FPD.

Analysis based on the ALM framework and the use of traditional indicators for risk assessment, like the FPD composition description for the market risk and the percentage maturing within 12 months together with average maturity (duration) for the refinancing risk, support most of these enhancements in the debt profile. More conservative approaches also help in defining the guidelines of the public debt management in Brazil, as discussed in Silva, Cabral & Baghdassarian (2006). That is the case of the stress test, which simulates the negative effects of strong and persistent pressure on real interest rate and real exchange rate on the FPD stock. Another example is the use of stochastic indicators[22], like the Cash-flow-at-risk (CfaR)[23] and the Cost-at-Risk (CaR)[24].

[22] They are indicators of risk based on stochastic simulations that have the advantage of provide probability distributions of the payments flows value or of the stock value. Hence, this kind of risk measurements makes available estimate the losses caused to debt management by negative events aroused in the financial market and, at the same time, it affords to quantify the probability of such events.

[23] The Cash-flow-at-Risk indicates the maximum addition which may occur in FPD outflows for a given period in relation to the expected value of such maturities, considering a given probability (for example, 95% of confidence level).

Recently, the National Treasury of Brazil advanced in developing an optimal long-term debt composition model to the FPD, in order to have more specific guidelines to manage its debt. It is used a framework that search for minimizing the impacts over the public fiscal balance due to shocks that affects the public debt. The idea of the model was released on the 2007 Annual Borrowing Plan[25].

Basically, it is used stochastic[26] models of finance for describing how the relevant macroeconomic variables (interest rates, exchange rates, inflation and GDP) evolve over time. Based on the stochastically simulated scenarios, the debt/GDP ratio evolution is evaluated in order to derive cost and risk measures of a given portfolio. So, after evaluate multiple possible portfolios in terms of cost and risk, an efficient debt portfolio frontier is obtained. Each portfolio in the frontier is different in terms of its composition by indexing factor (fixed and floating rate, inflation-linked and FX-linked) and also in relation to average maturities and the percentage due in 12 months.

One difference of this work in comparison to the model developed by the National Treasury refers to the specification of stochastic scenarios. Instead of using typical financial models to simulate the trajectory of the short-term interest rate, inflation and exchange rate, here it is specified a small macroeconomic model combined with a yield curve model, as it is approached by Pick & Anthony (2006)[27]. This model will evaluate the cost-risk characteristics of different debt portfolios in the presence of specific macroeconomic shocks, distinguishing, for example, the effects of shocks to aggregate demand and supply[28]. This model is described in next section.

[24] In Brazil, the Cost-at-Risk is used to measure uncertainty regarding the debt amount at the end of a period. It indicates the maximum value which the debt can reach, for a given probability. Despite of be defined in terms of stock, instead of interest costs like in the Danish definition, both approaches are directly related once the higher the cost, the higher the debt stock for a given fiscal primary surplus.

[25] See also Silva, Cabral & Baghdassarian (2006).

[26] Stochastic models are those generated from random variables, with probability distribution. Though they are stochastic, these models are correlated in order to ensure macroeconomic consistency.

[27] Cabral & Lopes (2005) also study the cost-risk tradeoff of alternative debt portfolios in the Brazilian case. Two approaches are used to generate stochastic scenarios, one based on finance models and other derived from a macro-structural model. These approaches lead to similar recommendations in terms of guidelines for debt management. In summary, the FPD risk exposure could be reduced by increasing the shares of fixed rate and inflation-linked bonds in the total debt. However, they do not introduce an explicit link between yield curve and the evolution of macroeconomic variables.

[28] Borges (2006) shows cost-risk tradeoffs between the FPD main financing instruments depending on the type of macroeconomic shock, as well as on the effects of different monetary and fiscal policy regimes over the public debt evolution. However, setting a benchmark for the FPD is not object of that work.

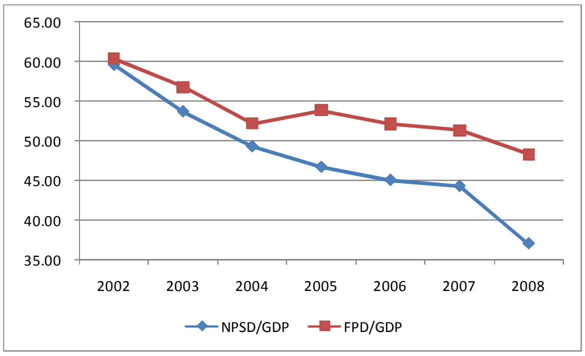
5. A Benchmark Model for the Brazilian Case

5.1. The model in a ALM context

Before describing the model itself, it is important to differentiate the concept of FPD presented in the previous section from the concept of Net Public Sector Debt (NPSD), which is also widely used in Brazil.

There are two main differences in those concepts. First, the FPD is comprised of only liabilities of the Federal Government. On the other hand, the NPSD includes debt of other entities of the public sector[29] in addition to the FPD, e.g. liabilities of the Central Bank and states. In this concept, intra-governmental debts are excluded. Second, as a net concept, the NPSD deduct public sector financial assets (e.g. international reserves) from the liabilities. The next graph shows the recent evolution of FPD and NPSD as a proportion of GDP.

Figura 4. NPSD and FPD as a proportion of GDP



Source: National Treasury of Brazil and Central Bank of Brazil

The NPSD is a relevant concept to evaluate cost and risk based on the idea that all assets and liabilities must be taken into account in order to evaluate the fiscal position of government, as in the ALM framework. Furthermore, as a measure based on the ALM, this concept captures the interactions among fiscal, monetary and debt policies. In fact, NPSD/GDP ratio is an indicator widely used in analyzing debt sustainability[30].

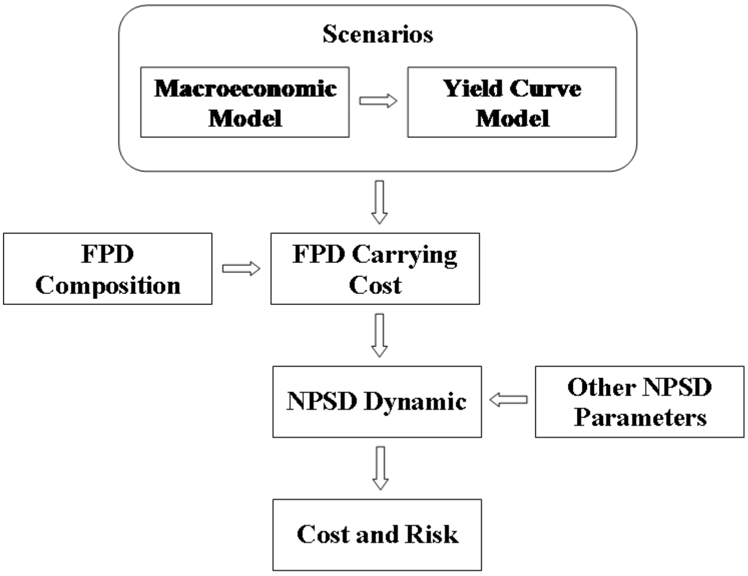
[29] The public sector as used to measure NPSD comprises the nonfinancial public sector plus the Central Bank. Hence, it encompasses direct federal, state and municipal administrations, indirect administrations, the public Social Security system, state-owned nonfinancial federal, state and municipal companies, as well as the Central Bank of Brazil.

[30] Debt sustainability can be understood as the ability of government to meet its inter-temporal budget restriction without disrupting the monetary and fiscal policies (Costa, 2009).

However, the FPD is the aggregate that is under direct control of the National Treasury, so that the composition of the FPD is the key variable for decisions in this work. Therefore, it is useful to supplement the cost-risk evaluation based on the dynamic of the FPD/ GDP ratio over time. This analysis captures aspects of the ALM framework in two ways: directly, once the changes in the monetary base and fiscal primary balance affect the evolution of the FPD; and indirectly, by examining the debt level as a proportion of GDP.

In sum, this work evaluates the cost-risk tradeoff caused by alternative choices of the FPD profile, taking into account the assets and liabilities structure of the public sector as captured by the dynamic of the NSPD. Besides, this analysis is complemented by another based on the dynamic of the FPD. The general idea of the model is depicted in the diagram below.

Figura 5. Structure of the benchmark model



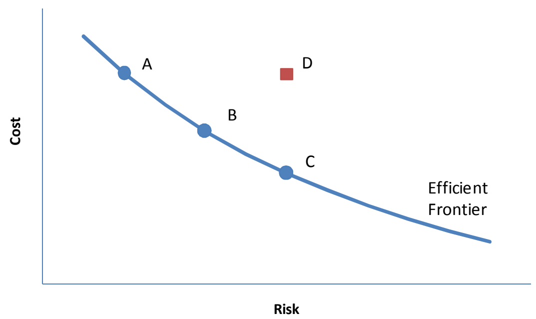
Initially, scenarios of key macroeconomic variables (output, inflation, short-term interest rate and exchange rate) and yield curves are generated with the purpose of simulating the evolution for the main factors that influence the trajectory of public debt over time.

Then, based on these scenarios the FPD carrying cost is computed for a given debt composition, which comprises a mix of securities representing the FPD which differ according to type of return and time to maturity. The model is simulated assuming a situation of steady state, so that each debt composition is implicitly associated with an issuance strategy that keeps constant the characteristics of the debt profile[31].

Next, the evolution of the NPSD (as well as the FPD) is calculated based on the FPD carrying cost, the public sector primary surplus and the evolution of other assets and liabilities that comprise the NPSD[32]. Thus, cost and risk indicators are derived from the analysis of the NPSD (and FPD) according to the effects of stochastic shocks on the dynamic of the debt level.

The model provides a tool to simulate the performance of alternative debt strategies over stochastic scenarios, with the aim of deriving an efficient frontier, like in the following figure, to express the FPD cost-risk tradeoff. Compositions along the frontier (A, B and C) are efficient because the risk necessarily increases when one pursue to reduce the debt cost by switching between these compositions (from A to B; or from B to C). Compositions above and to the right of the frontier are inefficient because they increase the risk given a level of cost (D compared to A), or increase the cost given a level of risk (D compared to C), or increase both the cost and risk compared to an efficient composition (D compared to B).

Figura 6. Efficient frontier



Operationally, the efficient frontier depends on the cost average and standard deviation of each bond, as well as the cost correlation matrix of those bonds. First, simulations based on debt compositions with 100% of a specific bond provide its cost

[31] In a steady state, it is assumed that the share of the financing instrument i (X_i) in the total debt is constant in all periods. It is considered further that a fraction $1/\tau_i$ of the instrument i matures in each period. So, the participation of the instrument i in period t is equal to the fraction remaining in stock $[1-1/\tau_i]X_{i(t-1)}$ plus the new issues of this instrument (B_{it}). That is $X_{it} = [1-1/\tau_i]X_{i(t-1)} + B_{it}$. X_i is constant if, and only if, $B_{it} = B_i = X_i/\tau_i$. More details about the relationship between stock and flow compositions in a steady state can be seen in Togo (2005).
[32] In order to define the evolution of "other assets and liabilities", a set of exogenous information is added to the model in this step.

average and standard deviation. Second, simulations based on portfolios with 50%-50% pair of bonds help in calculating the cost correlation matrix[33,34].

5.2. Macroeconomic Model

Structural models are frequently used to evaluate the impact of aggregate demand and supply shocks on prices and output, as well as the response of the monetary authority to those shocks. Typically, a small model to an open economy comprises four equations, like the following, in order to simulate the evolution of the main macroeconomic variables that affects the debt cost over time.

(1) $$y_{t+1} = \alpha_0 + \alpha_1 y_t + \alpha_2 (i_t(0) - \pi_t) + \varepsilon_{t+1}^{y}$$

(2) $$\pi_{t+1} = \bar{\pi}(1 - \delta_1) + \delta_1 \pi_t + \delta_2 y_t + \delta_3 (e_t - e_{t-1}) + \varepsilon_{t+1}^{\pi}$$

(3) $$e_{t+1} = \theta_0 + \theta_1 e_t + \theta_2 (i_t(0) - \pi_t) + \varepsilon_{t+1}^{e}$$

(4) $$i_{t+1}(0) = \bar{\imath}(1 - \rho) + \rho i_t(0) + \kappa(\pi_{t+1} - \bar{\pi}) + \psi y_{t+1}$$

The first equation is an aggregate demand or IS curve, which captures how the previous period output gap (y_t) and the real interest rate ($i_t(0)$-π_t) affect the output gap (y_{t+1})[35]. In the simulation model, stochastic shocks change the path of the relevant variables. In this sense, the error term in the IS curve, ε_{t+1}^{y}, stands for aggregate demand shocks. A restriction on the value of the parameter α_0 imposes that the output gap averages to zero on its equilibrium value. So, this parameter equates to $\alpha_0 = -\alpha_2(\bar{\imath} - \bar{\pi})$[36].

[33] The correlation between the cost of two bonds is extracted based on the following relation: the cost variance (σ_p^2) of a two-bonds portfolio is $\sigma_p^2 = w_1^2\sigma_1^2 + (1 - w_1)^2\sigma_2^2 + 2\rho_{12}\sigma_1\sigma_2$, where σ_1 is the cost variance of the bond 1; σ_2 is the cost variance of the bond 2; ρ_{12} is the cost correlation between the assets 1 and 2; $0 \leq w_1 \leq 1$ is the weight of the bond 1 in the portfolio.

[34] The correlation matrix must be positive semi-definite. If that is not the case, it is used a spectral decomposition to obtain a positive semi-definite matrix which approximates to the original. This decomposition takes into account that when a matrix is not positive semi-definite, it has at least one negative eigenvalue. The decomposition procedures use the positive eigenvalues of the original matrix and substitute the negative ones by zero, in order to recompose the matrix. This method provides reasonable approximations to the correlation matrix. The steps to put into practice the spectral decomposition are presented in Jäckel (2002).

[35] The initial idea was to include the real exchange rate (e_t) in this equation. However, this variable was not statistically significant to explain the evolution of the output gap over the sample period considered to estimate the coefficients of the equation.

[36] That restriction makes the equation (1) equivalent to one in which the relation between the output gap and the real interest rate is establishing by considering all variables as deviations from theirs average values.

The second is a Phillips curve, which shows the influence of the previous period output gap and exchange rate variations[37] (e_t-e_{t-1}) over the inflation rate (π_{t+1}). Exchange rate variations are important because they change prices of imported goods (pass through). Also, the inflation rate depends on its previous value and the Central Bank inflation target ($\bar{\pi}$). As in Pick & Anthony (2006), that target is credible and the expected inflation is consistent with it. The error term in the Phillips curve, ε_{t+1}^{π}, corresponds to aggregate supply shocks.

The third equation, as in Ball (1998), expresses the link between exchange rate and real interest, once higher interest rate in a country relatively to the rest of the world tends to make more attractive the domestic assets, causing an appreciation of the domestic currency. The lag of the exchange rate in the equation captures some degree of persistency in the adjustment of this variable in the presence of shocks which deviate it from its long-term average. As in Ball, the last term in the third equation, ε_{t+1}^{e}, represents shocks in other variables that have effects on the exchange rate, e.g. expectations, investor confidence and foreign interest rate. A restriction on the value of the parameter θ_0 imposes that the real exchange rate deviations average to zero on its equilibrium value. So, this parameter equals to $\theta_0 = -\theta_2(\bar{\imath} - \bar{\pi})$[38].

The last equation describes the monetary policy rule. The Central Bank chooses the short-term interest rate ($i_{t+1}(0)$) according to its objective of minimize deviations of the inflation from its target. That choice also takes into account output gaps. The interest rate of the previous period in the equation represents a monetary policy smoothing term. That indicates the Central Bank adjusts the interest rates in a gradual fashion, avoiding large interest rate changes to its desired target.

5.3.Yield Curve Model for Nominal Bonds

The cost of the public debt depends on the issuance interest rates of the public bonds. Although that cost relies on the short-term interest rate, each debt instrument has particularities regarding, for example, time to maturity and liquidity degree. In this sense, a nominal yield curve model is necessary to obtain the cost of each financing option.

[37] In this work, the real exchange rate is considered as the natural logarithm of an index number of the real exchange rate. Once the index number is normalized to be equal to 1 on its equilibrium value, the natural logarithm of the real exchange rate equals to zero on its equilibrium value.
[38] Besides to assure that the expected real exchange rate equals to zero, that restriction makes the equation (3) equivalent to one in which the relation between the real exchange rate and the real interest rate is established by considering all variables as deviations from theirs average values.

The yield curve here follows as a Nelson-Siegel (NS) function with two latent factors, which represent the yield curve level and slope factors[39] , as in the next equation:

(5) $$i_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-\exp(-\tau/\lambda)}{\tau/\lambda}\right)$$

Some theoretical concerns about the Nelson-Siegel function link the latent factors and to the short-term interest rate[40] from the macroeconomic model. Considering the NS function, changes in the first factor (β_{1t}) parallel shifts all yields regardless of the time to maturity, as it has unitary loading. Hence, this factor represents the yield curve level. It is also a long-term factor since $i_t(\tau)^{\tau\to\infty} = \beta_{1t}$.

Therefore, in this work the long-term level of the yield curve equates to $E[i_t(0)] + \xi$. If the (ad hoc) parameter ξ is positive, it means that a long-term nominal fixed rate bond pays a risk premium compared to the expected short-term interest rate.

The second latent factor (β_{2t}) stands for the yield curve slope. Considering the NS function, $i_t(\tau)^{\tau\to 0} = \beta_{1t} + \beta_{2t}$, and $i_t(\tau)^{\tau\to\infty} - i_t(\tau)^{\tau\to 0} = -\beta_{2t}$. However, from the macroeconomic model, the monetary authority defines the short-term interest rate, i(0), according to the Taylor Rule. Like in Pick & Anthony (2006), the assumption that $\beta_{1t} + \beta_{2t} = i_t(0)$ adds consistency to the model. Hence, the latent factor β_{2t} evolves as in the following equation:

(6) $$\beta_{2t} = i_t(0) - (E[i_t(0)] + \xi)$$

The slope factor in this case averages to $-\xi$, i.e. the negative value of the risk premium on a long-term nominal fixed rate bond.

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[39] Usually, the Nelson-siegel function comprises three factors, as in the following equation from Dielbold & Li (2006):

$$i_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-\exp(-\tau/\lambda)}{\tau/\lambda}\right) + \beta_{3t}\left(\frac{1-\exp(-\tau/\lambda)}{\tau/\lambda} - \exp(-\tau/\lambda)\right)$$

The third latent factor (c_t) corresponds to the yield curve curvature because medium-term yields are more affected by changes in c_t than the short- and long-term yields. The interpretation of the latent factors as level, slope and curvature is in line with the traditional interpretation introduced by Litterman & Scheinkman (1991). While it is a parsimonious model, several works report that usually three factors are enough to explain far more than 90% of the yield curve variations.

[40] This approach is similar to that of Pick & Anthony (2006). However, they use a Nelson-Siegel function with three factors. Due to a lack of theoretical considerations to calibrate the curvature factor, they specified a linear relation between the curvature factor and the output gap, short-term interest rate and inflation rate, based on empirical considerations.

5.4. Yield Curve Model for Inflation- and FX-Linked Bonds

In addition to nominal bonds, the National Treasury of Brazil issues inflation-linked and, in the case of external debt, foreign currency denominated bonds (FX-linked). Thus, defining the coupon rate of these bonds requires specific yield curves. They also take into account the NS function but, in each case, with particular parameters according to theoretical concerns.

The real yield curve, which gives the coupon of the inflation-linked bonds, has the following NS function:

(7) $$r_t(\tau) = \beta_{1t}^r + \beta_{2t}^r \left(\frac{1-\exp\left(-\tau/\lambda^{\mathrm{r}}\right)}{\tau/\lambda^{\mathrm{r}}}\right)$$

The choice of parameters for this equation assume that $\beta_{1t}^r = E[i_t(0) - \pi_t] + \xi^r$. It means that the coupon rate of a long-term inflation-linked bond equals to the expected real interest rate plus a risk premium (which can be different from the nominal bond risk premium). So, the cost difference between a nominal bond and a same maturity inflation-linked bond averages to the differences between theirs risk premium ($\xi - \xi^r$).

In the case of nominal bonds, the short-term interest rate corresponds to the sum of the level and slope factors. Applying the same idea to the short-term real interest rate, $\beta_{1t}^r + \beta_{2t}^r = i_t(0) - \pi_t$. Thus, the latent factor β_{2t}^r evolves like in the following equation:

(8) $$\beta_{2t}^r = (i_t(0) - \pi_t) - (E[i_t(0) - \pi_t] + \xi^r)$$

In a similar fashion, the yield curve for FX-linked bonds follows the next NS function:

(9) $$i_t^*(\tau) = \beta_{1t}^* + \beta_{2t}^* \left(\frac{1-\exp\left(-\tau/\lambda^*\right)}{\tau/\lambda^*}\right)$$

However, to calculate the FX-linked interest rate level it is necessary to take into account the expected variation of the nominal exchange rate. It corresponds to the variation of the real exchange rate plus the internal inflation minus the foreign inflation, according to the Relative Purchasing Power Parity Theory[41].

[41] The nominal exchange rate (η_t) corresponds to $\eta_t = e\frac{P}{P^*}$, where P and P* represent the domestic and foreign price levels, respectively.

Once $\Delta\eta_t$ is defined, the choice of parameters to the previous NS function assumes that $\beta_{1t}^* = E[i_t(0) - \Delta\eta_t] + \xi^*$. It means that the interest rate of FX-linked bonds equates to the expected domestic interest rate minus the expected nominal exchange rate variation plus a long-term risk premium. So, the cost difference between a nominal bond (domestic currency) and a foreign currency denominated bond depends on the comparison of their risk premiums $(\xi - \xi^*)$.

In order to define the slope factor, the foreign short-term interest rate corresponds to the sum of the level and slope factors, i.e. $\beta_{1t}^* + \beta_{2t}^* = i_t(0) - \Delta\eta_t$. Thus, the latent factor β_{2t}^* evolves like in the following equation:

(10) $$\beta_{2t}^* = (i_t(0) - \Delta\eta_t) - (E[i_t(0) - \Delta\eta_t] + \xi^*)$$

By now, the model provides the evolution of the main macroeconomic variables (the interest, inflation and exchange rates, as well as the output gap) and the yield curves to define the cost of each bond. The next step of the simulations uses these scenarios to calculate the carrying cost for any debt composition, which is a crucial element to define the debt dynamic[42].

5.5.Debt Dynamics

In this work, the following accounting identity is the starting point to derive the dynamics of the NPSD/ GDP ratio ($npsd_t$) and its relation with the FPD/ GDP ratio (fpd_t). In this identity, the NPSD assets and liabilities encompass four main categories: FPD, monetary base (m_t), international reserves (f_t), and other net liabilities of the public sector[43](l_t).

(11) $$npsd_t = fpd_t + m_t - f_t + l_t$$

The stock of the FPD in period t is equal to its stock in the previous period plus its carrying cost (c_t), less the primary fiscal surplus (s_t), less the change in the monetary base. The monetary base assumes a constant proportion of GDP over time ($m_t = m$). International reserves, as well as other liabilities of the public sector, start from an initial amount and

[42] See appendix B for details about the carrying cost of each FPD financing instrument in this work.

[43] All variables in this equation are expressed as proportion of GDP. The group of other liabilities of the public sector is subdivided in liabilities linked to floating interest rates, linked to inflation, and linked to exchange rate variation. The appendix C describes variables with values in level, instead of ratio of GDP, and more details regarding the dynamic of the NPSD assets and liabilities.

evolve according to their nominal remuneration rates ($c_t^{Reserves}$ and c_t^l, respectively). Hence, the FPD, international reserves (f_t) and other liabilities of the public sector (l_t) evolve over time as a proportion of GDP like in the following equations[44]:

(12) $$fpd_t = fpd_{t-1}\frac{(1+c_t)}{(1+\gamma_t)} - s_t - \frac{\gamma_t m}{(1+\gamma_t)}$$

(13) $$f_t = f_{t-1}\frac{\left(1+c_t^{Reserves}\right)}{(1+\gamma_t)}$$

(14) $$l_t = l_{t-1}\frac{\left(1+c_t^l\right)}{(1+\gamma_t)}$$

Finally, the next equation describes a relation between the primary fiscal balance as a proportion of GDP and the output gap.

(15) $$s_{t+1} = \nu_0 + \nu_1 \mathrm{y_t}$$

In addition to linking the fiscal block of equations to the macroeconomic model, this equation aims to capture effects of anti-cyclical fiscal policies over the debt evolution.

5.6.Cost and Risk Measures

The average carrying cost (c_t) is an immediate cost measure to compare different financing alternatives. At the same time, the carrying cost standard deviation constitutes a risk measure of each type of bond over stochastic shocks. However, the exam of the carrying cost does not address itself an ALM perspective.

In order to capture the assets and liabilities characteristics in the analysis, this work uses the NPSD/ GDP (or FPD/ GDP) ratio at the end of a 10-year period as a cost indicator. Although that is an indirect way to measure cost, the NPSD/ GDP ratio in the model is different over time between alternative debt compositions due exclusively to the carrying cost of each strategy, once the stochastic scenarios for the macroeconomic and fiscal variables are the same for all debt portfolios.

Related to the cost indicator, the standard deviation of the NPSD/ GDP (or FPD/ GDP) ratio at the end of a 10-year period measures the risk associated with each financing strategy. In an ALM perspective, the dispersion degree of the debt ratio probability

[44] See appendix C.

distribution is different between alternative debt compositions due not only to a divergent behavior of each bond over stochastic shocks, but also to mismatches between the characteristics of assets and liabilities which composes the NPSD (including the future flows of primary fiscal surpluses).

By now, it is important to emphasize that the model addresses several points regarding the debt cost-risk tradeoff. However, besides to use the NPSD/ GDP to evaluate cost and risk, it does not intent to forecast the evolution of the debt stock over time, but to evaluate the cost-risk of alternative strategies to finance a given initial stock. In this sense, addressing the question of the optimal level of the Brazilian indebtedness, like in a debt sustainability analysis, is beyond the scope of this paper.

6. Applying the Model to the Brazilian Case: A illustration[45]

6.1. Macroeconomic and Yield Curve Models

The definition of parameters to implement the model to the Brazilian case is based on a mix of estimation and calibration. One difficulty to employ only statistical estimation is that samples covering several past periods are characterized by structural breakpoints in the Brazilian economy. Another reason to use calibration is that the model pursuit to evaluate the debt in the long-term. However, the level of some economic variables on a longer time horizon is not adequately captured by simple extrapolation of their recent behavior.

The following table summarizes the parameterized equations of the macroeconomic model[46].

Table 1. Parameterized equations of output gap, inflation and exchange rate

$$y_{t+1} = 0.0079 + 0.54y_t - 0.70(i_t(0) - \pi_t) + \varepsilon_{t+1}^{y}$$

$$\pi_{t+1} = 0.00875(1 - 0.5) + 0.5\pi_t + 0.07y_t + 0.035(e_t - e_{t-1}) + \varepsilon_{t+1}^{\pi}$$

$$e_{t+1} = 0.03 + 0.9e_t - 2.67(i_t(0) - \pi_t) + \varepsilon_{t+1}^{e}$$

[45] The exercise of this section aims to apply the model to the Brazilian case. However, the following parameters were set up just for the sake of illustration and should be carefully revised in order to define a specific quantitative benchmark for the FPD.
[46] The parameters were adjusted in order to make simulations in a quarterly basis.

The parameters corresponding to the constant term in the equations are calibrated so that the output gap averages zero[47], the expected inflation rate is equivalent to 3.5% per annum (which represents a target lower than the current 4.5% per annum set by the Central Bank of Brazil), and the logarithm of exchange rate equilibrium is zero.

The other parameters are estimated[48] with the technique of seemingly unrelated regression (SUR). The SUR is a method of linear regression for multiple equations which allows correlated contemporary errors between equations[49]. Thus, compared to the ordinary least square (OLS) procedure, the SUR provides a more efficient estimation of the residuals covariance matrix of the model, which constitutes an input to simulate the stochastic shocks[50].

Regarding the short-term interest rate equation, two set of parameters are considered. The following table summarizes them.

Table 2. Parameterized equations of the monetary rules

Rule 1 − $i_{t+1}(0) = 0.02(1 - 0.8) + 0.8i_t(0) + 0.5(\pi_{t+1} - \bar{\pi}) + 0.5y_{t+1}$
Rule 2 − $i_{t+1}(0) = 0.02(1 - 0.8) + 0.8i_t(0) + 0.8(\pi_{t+1} - \bar{\pi}) + 0.2y_{t+1}$

First, the model assumes a Taylor Rule that gives equal weights to the inflation and output gap (Rule 1). In this case, $\kappa = 0.5$ and $\psi = 0.5$. The other case considers a monetary policy with higher weight to the inflation than to the output gap (Rule 2), by setting up $\kappa = 0.8$ and $\psi = 0.2$. In both rules, the smoothing term is estimated by means the OLS procedure, and the constant term is calibrated so that the short-term interest rate averages to 8.0% per annum[51].

The dynamic of the debt depends also on the primary fiscal balance, which is linked to the output gap. The model assumes a linear relationship between the two variables, where the slope parameter is set to be -0.25, according to an OLS estimative. The negative coefficient indicates some degree of anti-cyclical fiscal policy. The constant term of the relation is calibrated so that the primary fiscal balance averages to zero over time. Given the

[47] It is assumed that the potential output growth is 4.5% per annum.
[48] The estimation uses quarterly data over the period 2000-2008.
[49] See Greene (1998).
[50] The appendix A presents the covariance structure of the macroeconomic shocks in the simulations.
[51] The parameters were adjusted in order to make simulations in a quarterly basis.

parameter of the simulations, a zero primary fiscal balance keeps the NPSD/ GDP ratio roughly constant on average over time.

As pointed out before, this work does not aim to project the evolution of the debt over time, but to evaluate the cost-risk tradeoff of alternative instruments to finance the debt. Thus, it is more important to express a relation between fiscal policy and macroeconomic shocks than to forecast the level of primary fiscal balance, which is beyond the scope of this research. In the simulations, however, given the overall set of parameters in the model, all trajectories of the public debt are consistent with a sustainable indebtedness level.

The next table shows the resultant base case scenario (mean), as well as the 5th and 95th percentiles of the simulated scenarios for the main macroeconomic variables, based on the parameters described above.

Table 3. Simulated scenarios for the macroeconomic variables – Percentage per annum[52]

	inflation rate	short-term interest rate	variation of nominal exchange rate	gdp growth	fiscal primary surplus
mean	3.5%	8.0%	1.5%	4.5%	0.0%
5th	1.3%	2.0%	-26.5%	0.6%	-1.4%
95th	5.7%	14.0%	29.5%	8.4%	1.4%

Regarding the yield curve model, the chosen parameters imply rates equivalent to 8.8%, 5.1% and 7.0% respectively for the level factors (the long-term bonds yields) of fixed rate, inflation-linked and FX-linked bonds. The slope factors on average are equivalent to the risk premiums of each yield curve, i.e., the difference between the level factor and the short-term yield. They are set to be 0.8%, 0.6% and 0.5% respectively for fixed rate, inflation-linked and FX-linked bonds.

The previous section shows that besides the evolution of the interest, inflation and exchange rates (macroeconomic model), the debt carrying cost is determined by the issuance cost of each bond (yield curve model), as well as the debt composition. In the Brazilian case, there are four different types of instruments available to finance the debt: fixed rate bonds, floating rate bonds (indexed to the overnight interest rate), inflation-linked bonds, and FX-

[52] These scenarios correspond to the values at the 40th period (10th year) of the simulations.

linked bonds. Also, the instruments vary in terms of time to maturity. In this sense, the model assumes the following basket of short, medium and long-term representative bonds:

- Fixed rate: 1, 3, 5 and 10 years[53];
- Floating rate: 4 years[54]
- Inflation-linked: 5, 10 and 30 years[55]
- FX-linked: 10 and 30 years[56]

The following table shows the base case scenario (mean), as well as the 5th and 95th percentiles of the simulated interest rates' coupon (i.e., the yield curves) and total cost of the above financing instruments.

Table 4. Simulated scenarios for yield and debt cost – Percentage per annum[57]

	Yield (interest rate's coupon)			Cost (Yield + Index Variation)		
	Mean	5th	95th	Mean	5th	95th
Floating	0.0%	0.0%	0.0%	8.0%	2.0%	11.0%
Fix01y	8.3%	4.3%	10.3%	8.3%	4.3%	10.3%
Fix03y	8.5%	6.4%	9.6%	8.5%	6.4%	9.6%
Fix05y	8.6%	7.3%	9.3%	8.6%	7.3%	9.3%
Fix10y	8.7%	8.0%	9.0%	8.7%	8.0%	9.0%
IL05y	4.9%	3.2%	5.8%	8.5%	6.3%	10.6%
IL10y	5.0%	4.1%	5.5%	8.5%	6.3%	10.8%
IL30y	5.1%	4.8%	5.2%	8.6%	6.3%	10.9%
FX10y	6.9%	1.8%	9.5%	8.4%	-18.4%	35.3%
FX30y	7.0%	5.3%	7.8%	8.6%	-18.4%	35.5%

6.2.Dynamic of debt and the efficient frontier

The next procedure in the simulations is to calculate the dynamic of the FPD/ GDP and NPSD/ GDP ratios over time. This step requires setting up initial values to define the weights of the assets and liabilities included in the NPSD, like is summarized in the table below.

[53] In the tables and figures, they are labeled Fix01y, Fix03y, Fix05y and Fix10y, respectively.
[54] In the model, the floating rate bond is sold at par. So, the chosen bond is representative of a basket of bonds with average time to maturity of 4 years, e. g. a basket with 50%-50% of 3-years and 5-years time to maturity.
[55] In the tables and figures, they are labeled IL05y, IL10y and IL30y, respectively.
[56] In the tables and figures, they are labeled FX10y and FX30y, respectively.
[57] These scenarios correspond to the values at the 40th period (10th year) of the simulations.

Table 5. Initial values for the NPSD components – Percentage of GDP

Federal Public Debt	**fpd**	**48.0%**
Monetary Base	m	5.0%
International Reserves	f	15.0%
Other public sector's net liabilities		
Floating rate	$l^{floating}$	-1.0%
Inflation-linked	$l^{\pi\text{-linked}}$	1.0%
FX-linked	$l^{fx\text{-linked}}$	-1.0%
Net Public Sector Debt	**npsd**	**37.0%**

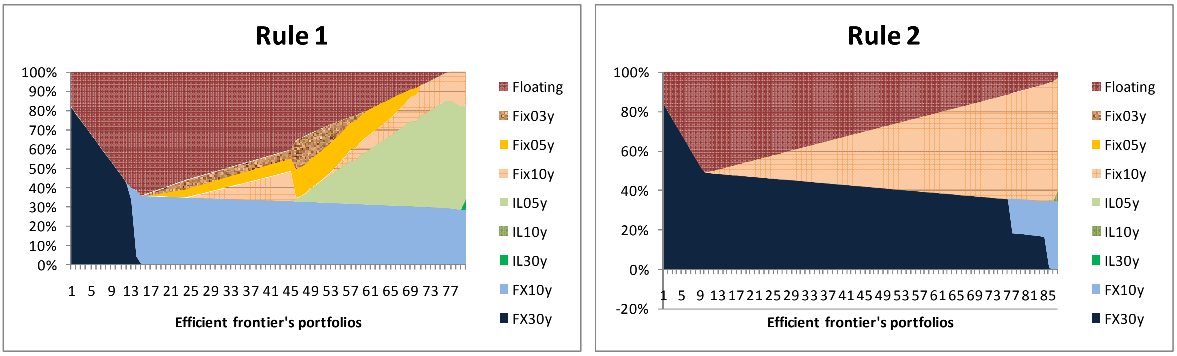
These initial values represent an approximation to the current figure of the NPSD, as of December, 2008. In fact, the NPSD, FPD, monetary base and international reserves corresponded to 37.0%, 48.4%, 5.1% and 16.7%, respectively, as a ratio of GDP at the end of 2008[58]. Regarding the other net liabilities of the public sector, they are based on their 2002-2008 averages and slightly adjusted to assure that the sum of the NPSD components equates to 37.0% of GDP.

Based on the scenarios and initial values, the dynamic of the NPSD/ GDP is calculated for some FPD compositions in order to obtain the cost average and standard deviation for each financing instrument and, also, the cost correlation matrix. Hence, an optimization procedure leads to the efficient frontier. This procedure consists of selecting the debt composition that minimizes the standard deviation of the NPSD/ GDP, for a given cost as indicated by the average of NPSD/ GDP, at the end of a 10-year period[59].

The following figure shows the FPD compositions associated with two efficient frontiers. The first (on left) assumes a Taylor Rule which gives equal weights to the inflation and output gap (Rule 1). The other frontier (on right) corresponds to a Taylor Rule with higher weight to the inflation than to the output gap (Rule 2). The simulations show that the monetary policy regime affects the debt cost-risk tradeoff.

[58] Source: National Treasury of Brazil and Central Bank of Brazil.
[59] The model is simulated over 40 quarters (10 years).

Figura 7. FPD efficient compositions based on the NPSD/ GDP ratio



Portfolios become riskier from the left to the right

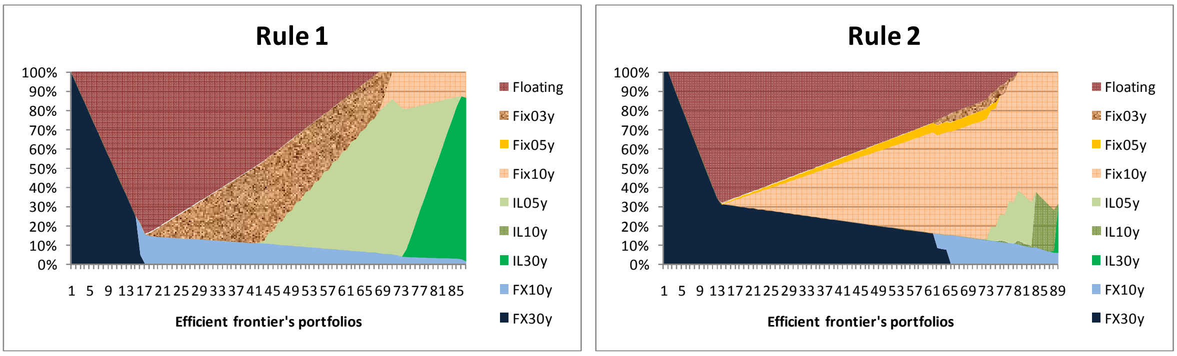
In both cases, the less expensive (and riskier) FPD portfolios are those composed largely by the floating rate and FX-linked bonds. Portfolios with intermediate cost and risk also include fixed rate bonds to that mix. The inflation linked bonds make the portfolio less risky (although, more expensive) depending on the monetary rule. A monetary policy which attributes similar weights to inflation and output gap leads to a high share of inflation-linked bonds in the efficient portfolios. However, a Central Bank policy which gives more emphasis to the inflation excludes these bonds from the efficient frontier.

The simulations illustrate too that long-term bonds are associated with less risky debt profiles. Considering the Rule 1, the efficient frontier indicates that is possible to switch from strategies with high risk toward those with low risk by replacing FX-linked with floating rate bonds. Regarding portfolios with intermediate risk, floating rate bonds give place to short-term fixed rate bonds. And, finally, the less risky debt compositions consist of higher shares of long-term fixed rate bonds. Over the Rule 2, the efficient frontier does not include short-term bonds. The figures with the efficient frontiers are in the Appendix D.

The investigation at this point considered the dynamic of the NPSD/ GDP ratio. Despite of being a more complete evaluation from the ALM perspective, the NPSD includes some assets and liabilities which are not under the direct control of the National Treasury of Brazil. For example, international reserves provide hedge to the FPD's exposition to the exchange rate fluctuations in the case of external debt. However, these assets are also important, for example, to the Balance of Payments management. Thus, it is useful to supplement the analysis by taking into consideration only the dynamic of the FPD/ GPD ratio.

The following figure illustrates the efficient FPD compositions derived from the evolution of the FDP/ GDP ratio for the two monetary rules. Like in the previous frontiers, the less expensive and riskier debt portfolios comprise big shares of floating rate and FX-linked bonds. The main peculiarity now is that the share of FX-linked reduces sharply in the lower risk efficient compositions, in comparison with the NPSD analysis. Essentially, they are substituted by fixed rate bonds for the intermediate risk and, by inflation-linked bonds for the lower risk portfolios. The efficient frontiers figures are in the Appendix D.

Figura 8. FPD efficient compositions based on the FPD/ GDP ratio



Portfolios become riskier from the left to the right

These simulations suggest that even though FX-linked financing instruments are very risky, there is room for foreign currency denominated debt in a long-term strategy for the FPD, from the financial cost-risk viewpoint. The adequate share of this debt needs to take into account the hedge provided by the international reserves. However, by considering that often debt management is not an exclusive objective of international reserves management, the amount of this type of debt in lower risk portfolios tends to be lower than the amount of the international assets owned by the country.

The share of floating rate bonds in the long-term portfolio depends on the desired cost-risk balance which the debt manager chooses. Ex-ante, floating rate debt provides lower costs, although at a higher risk. Ex-post, this financing alternative potentially becomes more expensive due to macroeconomic shocks. This result does not depend on whether the evaluation is based on the dynamic of the NPSD/ GDP or FPD/ GDP ratios.

Regarding the shares of fixed rate and inflation-linked bonds, the choice depends on how the Central Bank reacts to each type of macroeconomic shocks (except for higher risk portfolios). The simulations suggest that both of these two categories of financing instruments need to integrate the long-term debt portfolio. A supplementary exercise provides additional evidence to address this question.

The exercise consists of constraining the efficient frontier to exclude floating rate and FX-linked bonds. As a result, the Rule 1 implies that the efficient debt composition include only short-term fixed rate and inflation-linked bonds. In this case, the higher the share of inflation-linked bonds, the lower the risk of a strategy. Under the Rule 2, however, a large share of medium-term fixed rate bonds composes the intermediate and lower risk efficient portfolios, replacing part of the inflation-linked bonds which integrate them under the Rule 1.

The results of this exercise change when no more than one type of macroeconomic shock affects the path of the variables over time. If only aggregate demand shocks hit the economy, under the Rule 1, the fixed rate bonds integrate only the higher risk portfolios. Under the Rule 2, the short-term fixed rate bonds replace the medium-term fixed rate bonds in the efficient portfolios.

On the other hand, if only aggregate supply shocks occur, fixed rate replace inflation-linked bonds in the intermediate and lower risk efficient portfolios compared to the case of only demand shocks. Under the Rule 1, only a small proportion of the inflation-linked debt is replaced. But, under the Rule 2, the substitution is almost complete.

The conclusion of this exercise is that both the fixed rate and inflation-linked bonds are important to reduce the risks of the FPD. Hence, a mix of those bonds provides hedge against the aggregate demand and supply shocks and, also, behaves well in case of changes in the monetary policy regime, once in the practice is difficult to precise the weights which the Central Bank attributes to the inflation and output gap in the monetary rule.

7. Final Considerations

The search for an optimal portfolio is a critical element in the process of strategic planning of debt in Brazil. In this sense, the design of a long-term strategy for the FPD requires to investigate how each financing instrument reacts over different types of economic shocks. In this work, a macroeconomic and a yield curve model help in understanding the effects of shocks on the main variables that shape the FPD cost over time. In an ALM framework, a mix of all available instruments is desired for the purpose of having a balanced debt portfolio from the cost-risk viewpoint.

The simulations illustrate that the less expensive, but the riskiest debt portfolios are those with high shares of FX-linked financing instruments. However, a small share of foreign currency denominated debt in a long-term strategy for the FPD is consistent with a prudent level of risk, taking into account that the international reserves provide hedge to this type of debt. The floating rate bonds are also risky, but their risk is lower than that of FX-linked bonds, and they have the advantage of lowering the debt costs.

The fixed rate and inflation-linked bonds also need to integrate the long-term debt portfolio. The fixed rate bonds provide hedge especially over aggregate supply shocks, while the inflation-linked bonds give more protection over aggregate demand shocks. Nevertheless, if the Central Bank tends to be more concerned with inflation in the monetary policy, as it is often the case, the long-term strategy requires relatively more fixed rate than inflation-linked bonds to reduce the risk exposure of the public debt.

Although this study provides a tool to investigate the cost-risk tradeoff of alternative strategies, defining a specific benchmark for the FPD is beyond the aim of this work. The model here is focused on the financial characteristics of that tradeoff, as well as the influence of macroeconomic variables over the debt cost. The choice of the optimal debt composition, however, must consider other aspects of debt management. For example, besides reducing costs subject to prudent risk levels, the government may have objectives like "contributing to the well functioning of the public bond market", as in the case of Brazil[60].

This point and some other peculiarities may change the efficient frontier by adding constraints to the choice of the debt manager. It is important to recognize the strong influence the government may have over bond prices, once it is a big participant in the debt market. The debt manager also addresses the smoothing of the government cash-flows and reducing of the impacts of the public debt over the fiscal budget. Finally, the ability of the government to implement a debt strategy depends on the degree of development of the debt market, as well as the size of the investor base. Thus, changes in the debt profile often occur in a gradual fashion, according to the market conditions.

[60] See the 2008 Annual Borrowing Plan.

In this sense, the recent changing in the FPD composition, lengthening of its average time to maturity and smoothing of its payment flows reflect the application of sound practices in the debt management, as well as the development of the bonds market in Brazil. As a result, the FPD gradually changes in direction to a more efficient profile.

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9. Appendix

9.1. Stochastic Shocks

The debt carrying cost is determined by the evolution of the interest, inflation and exchange rates. In the macroeconomic model, the dynamic of these variables is affected by three types of shocks: demand shocks (ε_{t+1}^{y}), supply shocks (ε_{t+1}^{π}); and external shocks (ε_{t+1}^{e}). It is assumed that these shocks follow a multivariate normal distribution, with a zero mean vector and a contemporary covariance structure according to the next standard deviation and correlation matrix.

Table 6. Standard deviation and correlation matrix of the stochastic shocks

	Standard deviation (quarterly)
ε^{y}	0.18%
ε^{π}	0.32%
ε^{e}	5.53%

Correlation Matrix	ε^{y}	ε^{π}	ε^{e}
ε^{y}	1.00	0.28	0.40
ε^{π}	0.28	1.00	0.08
ε^{e}	0.40	0.08	1.00

The shocks are introduced in the model by extracting random number from a multivariate standard normal distribution[61]. The Cholesky decomposition method is applied to create random number with the previous covariance structure.

9.2. Cost of Carrying Debt

The FPD carrying cost depends on the issuance cost of each type of public securities plus its index variation, where appropriate. This study considers four types of financial instruments used by the National Treasury of Brazil to finance the FPD: floating rate, fixed rate, inflation-linked (domestic debt) and FX-linked (external debt). This section describes how to calculate the cost of each of these instruments in the model.

[61] The random numbers are extracted and, then, normalized with the aim of achieving zero mean and unitary variance (moment matching) for each sequence of shocks.

In the case of the floating rate bonds, their carrying cost ($c_t^{floating}$) is simply the short-term interest rate over the period:

(16) $$c_t^{floating} = i_t(0)$$

The carrying cost of fixed rate bonds in each period is a weighted average of the issuance costs of all bonds that are still in the stock. It is computed as:

(17) $$c_t^{fixed} = \sum_{s=0}^{\tau-1} \omega_{t-s} i_{t-s}(\tau)$$

In which ω_{t-s} is the percentage in t of fixed-rate debt issued in t-s; and $i_{t-s}(\tau)$ is the issuance cost of the bond in t-s.

For the inflation-linked bonds, the carrying cost is composed by the inflation rate and by the evolution of the real interest coupon weighted average for bonds in stock. It is calculated as:

(18) $$c_t^{\pi-linked} = (1 + \sum_{s=0}^{\tau-1} \omega_{t-s}^{\pi} r_{t-s}(\tau))(1 + \pi_t) - 1$$

Where ω_{t-s}^{π} stands for the percentage at *t* of inflation-linked debt issued at t-s; and $r_{t-s}(\tau)$ is the real interest coupon issued at t-s.

Finally, the carrying cost of FX-linked bonds is calculated in a similar fashion to the inflation-linked bonds:

(19) $$c_t^{FX-linked} = (1 + \sum_{s=0}^{\tau-1} \omega_{t-s}^{FX} r_{t-s}^{*}(\tau))(1 + \Delta\eta_t) - 1$$

Where ω_{t-s}^{*} stands for the percentage at t of fx-linked debt issued at t-s; and $r_t^{*}(\tau)$ is the real interest coupon issued at t-s.

Thus, for a given debt composition, its carrying cost is given by the weighted average of each bond type carrying cost as above derived. In other words:

(20) $$c_t = \omega^i c_t^{floating} + \omega c_t^{Fixed} + \omega^{\pi} c_t^{\pi-linked} + \omega^{FX} c_t^{FX-linked}$$

In which c_t is the cost of carrying a debt portfolio; and ω^i, ω, ω^{π}, and ω^{FX} represent the share of each bond type (floating rate, fixed rate, inflation linked and FX-linked, respectively) in the chosen debt composition $\left(\omega^i + \omega + \omega^{\pi} + \omega^{FX} = 1\right)$.

9.3.Debt Dynamics

The following accounting identity is used as a starting point to derive the dynamics of the Net Public Sector Debt (NPSD) and its relationship with the FPD.

(21) $$D_t = X_t + M_t - F_t + L_t^{floating} + L_t^{\pi-linked} + L_t^{FX-linked}$$

In this identity, assets and liabilities that comprise NPSD (D) are grouped into four categories: FPD (X), monetary base (M), international reserves (F) and other net liabilities of the public sector [linked to floating interest rates ($L^{floating}$), linked to inflation ($L_t^{\pi-linked}$), and linked to exchange rate variation ($L^{FX\text{-}linked}$)].

The FPD stock in period t is equal to its stock in the previous period plus its carrying cost (c_t), less the primary fiscal surplus, less the change in the monetary base. So, the FPD evolves according to the following equation:

(22) $$X_t = X_{t-1}(1 + c_t) - S_t + \Delta M_t$$

Regarding to other components of NPSD, the monetary base is kept constant as a proportion of GDP over time, international reserves (equation 23)[62], as well as other obligations of the public sector (equations 24-26), departing from an initial amount and evolve according to their nominal remuneration.

(23) $$F_t = F_{t-1}(1 + r_t^{Reserves})(1 + \Delta\eta)$$

(24) $$L_t^{floating} = L_t^{floating}(1 + i_t(0))$$

(25) $$L_t^{\pi-linked} = L_t^{\pi-linked}\left(1 + c_t^{\pi-linked}\right)$$

(26) $$L_t^{FX-linked} = L_t^{FX-linked}\left(1 + c_t^{FX-linked}\right)$$

Substituting (22) - (26) in (21) and dividing the new equation by the GPD, some algebraic manipulations give the following relationship to describe the evolution of NPSD / GDP ratio:

(27) $$d_t = x_{t-1}\frac{(1+c_t)}{(1+\gamma_t)} - s_t + \frac{m}{(1+\gamma_t)} - f_{t-1}\frac{\left(1+c_t^{Reserves}\right)}{(1+\gamma_t)} + l_{t-1}\frac{(1+c_t^{l})}{(1+\gamma_t)}$$

---

[62] The return rate of the international reserves ($r_t^{Reserves}$) can be different from the average cost of foreign debt.

where:

$$c_t^{Reserves} = (1 + r_t^{Reserves})(1 + \Delta\eta) - 1$$

$$l_t = l_t^{floating} + l_t^{\pi-linked} + l_t^{FX-linked}$$

$$c_t^l = \left(i_t(0) l_t^{floating} + c_t^{\pi-linked} l_t^{\pi-linked} + c_t^{FX-linked} l_t^{FX-linked}\right)/l_t$$

9.4. Efficient frontiers

Figura 9. FPD efficient frontiers based on the NPSD/ GDP ratio

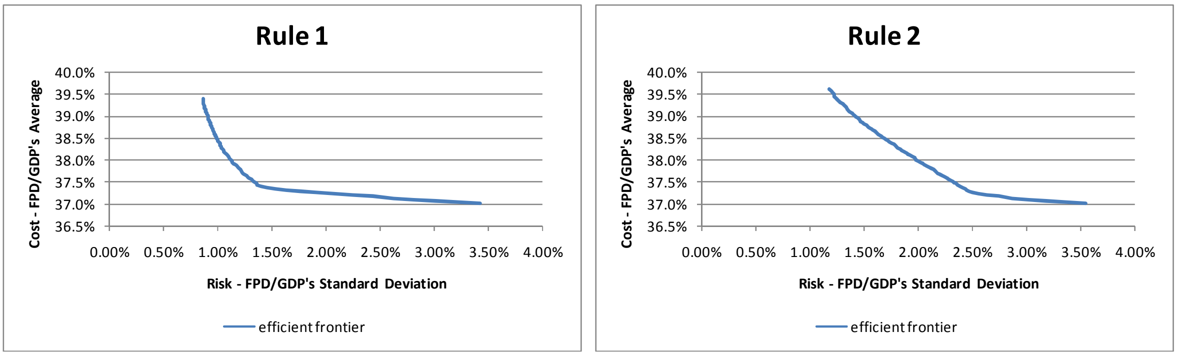

Figura 10. FPD efficient frontiers based on the FPD/ GDP ratio