Scope and Fundamental Challenges to Public Debt Risk Management – The Brazilian DMO Perspective

Anderson C. D. Silva[1]

World Bank

Rodrigo S. V. Cabral[2]

Brazilian National Treasury

William Baghdassarian[3]

Brazilian National Treasury

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[1] asilva3@worldbank.org (202) – 458-1140
[2] rodrigo.cabral@fazenda.gov.br - (55) 61 3412-3181
[3] william.baghdassarian@fazenda.gov.br - (55) 61 3412-3183

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1 – Introduction

Risk management has long been recognized as a profession in financial markets. Its relevance and sophistication though have substantially increased in the last years especially due to the expansion of derivatives markets, the broader availability of user friendly risk management tools and stricter prudential regulations and risk monitoring from capital market regulators and central banks.

Efforts towards the implementation of modern risk management practices have also ranked high in the agenda of public debt managers. After a series of crises in debt markets in late 90's, a growing set of countries started to explicitly take risk management into account in their formally stated debt management objective, defined by most countries as: "minimizing long-term costs subject to prudent risk levels. [4]"

As a consequence of this process, numerous Debt Management Offices – DMOs around the world have been through significant institutional changes to cope with the demand for improved human and technological capacity[5]. In this respect, the most notorious change has been the focus on strengthening Middle-Office capability, more specifically in the areas of risk management and long-term planning[6]. Public debt risk management became a key attribution across the list of DMO functions.

Accompanying this trend, the Brazilian National Treasury engaged in 2001 in a program with the World Bank to build capacity and develop tools and systems for risk management[7]. Two years later, the resultant Brazilian risk management framework was presented and validated in a seminar attended by experts from several countries and international organizations[8].

A number of studies have, since then, been produced by the risk management team of the Brazilian DMO, presented in academic and professional seminars and published[9]. These studies play an important role in the efforts for continuous improvement of risk management practices in Brazil. However, their highly specialized nature leaves a gap in the understanding of

[4] See Guidelines for Public Debt Management (2001)
[5] This includes, among others, the UK, France, Germany, Brazil and, more recently, Mexico.
6 For a good reference of risk management practices in OECD countries see "OECD Studies in Risk Management" (2006).
[7] Highly regarded consultants, together with a specialized software house were hired to help in the construction of the Brazilian risk management framework.
[8] The "Workshop on Public Debt Management in Brazil" took place in Fundação Getúlio Vargas – FGV, Rio de Janeiro, on March 2003. Participants included senior debt managers from nine countries - Brazil, Czech Republic, Denmark, France, Italy, Portugal, Spain, the United Kingdom and the United States - as well as representatives from the OECD and the World Bank.
[9] References of more technical studies are given to the interested reader throughout the paper.

how these individual pieces of work can be put together to form the complete set of attributions of the public debt risk manager.

The objective of this paper is to describe the scope of activities and the fundamental challenges faced by the public debt risk manager. Its main motivation derives from recurrent demands coming from researchers and countries in earlier stages in capacity building for a consistent map of tools and responsibilities that this profession entails. A good view of the tools that need to be developed and the skills that such position requires may prove to be a useful road map to those intending to improve risk management practices.

The paper is organized as follows: Section 2 presents an overview of the scope of activities of the public debt risk manager, dividing his attributions in what we denominate “macro” and public debt management functions. A discussion on macro functions is conducted in section 3, as well as a simple illustration of the privileged position of the public debt risk manager to run debt sustainability assessments. Section 4 addresses the important role of the risk manager in providing; based on quantitative analysis, a long-term reference (benchmark) to guide short and medium term debt strategies. Section 5, on its turn, presents the main risk management indicators used by a DMO, whereas in section 6 we describe the role that risk managers play in the design and monitoring of debt management strategies. Section 7 consists of our concluding remarks.

2 - Scope and Fundamental Challenges to Public Debt Risk Management

The set of attributions of a public debt risk manager is rather large. This section presents an overview of such attributions and fundamental challenges that public debt risk managers face in addressing demands from different clients (typically senior government officials) and counterparts (debt managers). Most challenges are related to the adaptation of risk management tools already used by academics, investors and market analysts, to the specific needs of a public entity that holds a net liability portfolio.

Although building an exhaustive list of attributions of a risk manager represents a hard task, inevitably subject to disagreements, we attempt to group them in two main categories: macro and public debt management functions. The most common tasks of a risk manager under the first category are debt dynamics exercises and sustainability assessments. Debt management functions include the identification of long term benchmarks (optimal debt composition), the development and regular assessment of risk indicators (for measuring several types of risks) and the design, monitoring and analysis of trade-offs across different refinancing strategies that can be implemented by the debt management office.

The macro functions stated above are not attributions that are exclusive to public debt risk managers. In fact, debt dynamics and sustainability tests are conducted by a wide array of interested parties such as financial analysts and academics. Here the relevance of the risk manager comes from her somewhat private knowledge of the government refinancing strategy and, thus, privileged position in conducting debt dynamics and sustainability exercises.

As a result of incorporating the impact of the actual refinancing strategy one can surely draw more accurate forecasts of debt dynamics and risks. Public debt risk managers can therefore provide senior government officials with valuable information for the formulation and assessment of public policies that affect the level of indebtedness of a country, such as those related to the determination of primary balance targets or to the assumption of contingent liabilities. The expertise of public debt risk managers is a precious resource many times not fully exploited by policy-markers. Section 3 covers these issues, macro functions of a public debt risk manager, in more detail.

The risk analyst is also responsible for providing relevant inputs to the appropriate and prudential conduct of debt management. The list, as mentioned above, is extensive.
Perhaps the best way to illustrate such issues is by treating separately her role in providing general guidelines for the desired debt composition, in producing a broad set of indicators that encompass the different types of risks that need to be monitored, and in elaborating, supervising and assessing the trade-offs of alternative refinancing strategies.

A primary and fundamental question to debt managers is really the one related to what would be the composition and profile of the debt that the government should pursue. The public debt risk manager plays an important role in addressing this question by pointing out pros and cons and possibly quantifying the costs and risks of distinct long-term debt strategies.

Debt management theory provides some guidance towards general characteristics of the debt portfolio. However, in order to provide more specific, quantitative guidelines, several debt specialists have engaged in a debate that has gained increased attention in debt management offices (DMOs) around the world[10], that is: the determination of the benchmark. We discuss these issues in more detail in section 4.

Notwithstanding the importance of a long-term benchmark, another attribution of paramount importance to the debt manager is the identification and design of a comprehensive set of risk indicators. These indicators do not need to be, for the most part, sophisticated. In practice, a

[10] Debt Management Offices in numerous countries have developed research addressing possible methodologies for the determination of benchmarks. Well known examples are Brazil, Canada, Denmark, Portugal and Sweden.

good set of simple indicators such as average maturity life, "refixing-duration[11]" and maturity profile (measured as a percentage of debt maturing in the short-term, for example), can provide useful information regarding refinancing and interest-rate risk of a government debt.

More sophisticated risk measures that usually rely on stochastic simulations are increasingly being used in debt offices to complement traditional indicators. These are in most part adaptations of indicators that have already been developed for the investor's point of view (such as the well known Value at Risk, VaR). The main challenge is to accurately adapt these indicators to the debtor's point of view. Commonly used at-risk indicators are the Cash-Flow-at-Risk, the Cost-at-Risk (or Stock-at-Risk) and the Budget-at-Risk.

Stochastic analysis is also frequently employed by public debt risk managers on an ad hoc basis to provide decision making guidance to specific transactions. Typical examples are debt exchanges and other liability management operations that require trade-off assessments in terms of cost and risk.

The set of risk indicators must also include measures coming from the demand side. That is, the debt manager needs to monitor demand side risk that may cause discontinuity in the programmed debt issuance strategy. This is extremely important in emerging markets where the amount of risk, notably interest-rate risk, which the investor base can bear represent a significant constraint to the smooth implementation of a debt strategy.

The same way market participants measure their interest-rate exposures through indicators such as the PVBP[12] and VaR, the public debt risk manager should also do it in order to identify the pace of risk transferring from the government to the private sector that a refinancing strategy entails. In some circumstances, for example, abnormal VaR levels caused by macro volatility may significantly reduce the demand for fixed-rate securities, forcing debt managers to pay higher costs or even make sudden changes in the composition of debt issues. Demand side risk indicators as well as traditional and stochastic debt management risk indicators are discussed in section 5.

Another important function of a public debt risk manager is his active participation in the design, monitoring and analysis of trade-offs across different refinancing strategies that can be implemented by the debt management office. The process of designing a debt strategy is in

[11] Refixing Duration regards the average time by what the debt is refixed. In the case of Fixed-rate bonds this indicator is equal to average life. Floating-rate bonds (Selic rate) are refixed every day. External Floating –rate Bonds (Libor3) are refixed every 3 months.

[12] PVBP extends for Present Value of a Basis Point and measures the debt sensitivity to changes in interest rates.

many ways a shared responsibility with other areas of a debt office, such as the front-office, for example.

The risk manager identifies possible risks for the implementation of the debt strategy and refines estimates (targets) for the debt composition and profile at specific points in the future (typically one year, as it is common in Annual Borrowing Plans). She also monitors the implementation of a debt strategy to validate and, whenever applicable, suggest correction measures in the conduct of the issuance strategy. The framework used by the Brazilian Debt Management Office to address these attributions is discussed in section 6.

Finally it is important to mention that in order to be able to cope with all these functions there is significant need to invest in building human and technological capacity. The development of risk management systems that allow adequate comparisons of trade-offs in terms of cost and risk across different potential refinancing strategies is a worthwhile initiative that may substantially improve the decision making process in a DMO[13].

3 – "Macro Functions" of a Public Debt Risk Manager

This section discusses the important role that public debt risk managers can play in providing enhanced and more accurate debt dynamics and sustainability exercises. More specifically, we illustrate through a simple example how the debt risk manager, taking into account her developed skills to conduct risk analysis and her privileged access to information regarding the debt refinancing strategy, can aggregate value to commonly used debt sustainability analysis.

Debt sustainability has long been a topic of up most relevance to policy makers, investors and scholars. Although the main variables that drive the debt are well known, conventional assessments of debt sustainability typically based on deterministic forecasts, have shown to be limited in scope. Among their main limitations is their failure in incorporating uncertainty in the model, leading to expected debt ratios that lack a measure of potential dispersion (error).

Efforts towards the development of more sophisticated modeling techniques to the assessment of debt sustainability have been enormous in recent years[14]. Part of this increased interest is certainly related to the higher awareness of risks associated to macroeconomic shocks and to

[13] Brazil has developed a Public Debt Refinancing and Risk Management System (the so-called GERIR). It provides the foundations to Front- and Middle-Office work in the formulation and analysis of debt strategies. Through GERIR, debt analysts simulate several refinancing strategies for the Brazilian public debt and compare their expected results, as measured by several indicators. The system was developed after a comprehensive investigation of international experience and passed by the scrutiny of risk specialists from several countries (Annex 1 provides more details about GERIR)

[14] See Barnhill (2003), Xu & Guezzi (2002), Costa, Silva e Baghdassarian (2004)

the growing use of sustainability analysis by policy makers to define fiscal targets for an effective control over the level of indebtedness of the public sector

The public debt risk manager can aggregate value to the debate towards enhanced modeling for debt sustainability analysis. The tools and models she uses to measure other types of risks (such as the "cost-at-risk" for example[15]) can, with simple adaptations, be employed to generate stochastic debt dynamics paths. As a result, one can complement the typical average expected debt ratios originated from deterministic scenarios with a complete probability distribution of such ratios.

Aggregating uncertainty in debt sustainability analysis may enhance the set of conclusions that one can draw from this type of exercises, but it may not be a sufficient condition to achieve more precise forecasts. Debt sustainability assessments usually cover several periods (most often five to ten years). During this period, the debt composition and profile can change substantially and so can its sensitivity to different scenarios and types of macroeconomic shocks

It becomes relevant, therefore, to make assumptions regarding the refinancing strategy when conducting such assessments. The privileged position of the public debt risk manager in this regard is indisputable. By being an active participant in the process of designing and monitoring the implementation of the debt strategy, she is in the position of an insider when it comes to the running of sustainability tests including the refinancing strategy.

The importance of including a refinancing strategy is even greater in countries that still have unstable debt profiles, that are implementing changes in the debt profile and that have a large share of the debt maturing in the short term. Moreover, this is exactly the set of countries where sustainability tests are more relevant.

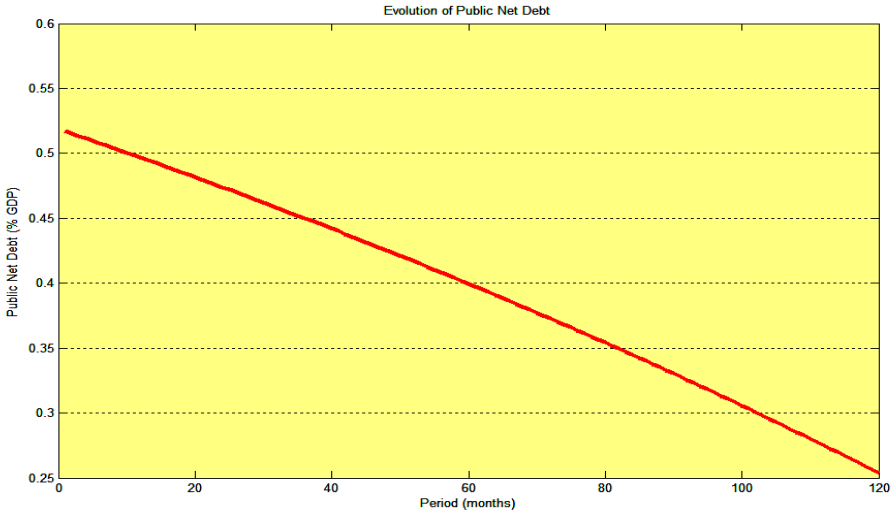
In order to illustrate the advantages of including uncertainty and the refinancing strategy in debt sustainability analysis, we conduct a simple example with hypothetical data. The straightforward nature of this exercise allows us to abstract from the implicit methodological complexities of such analysis and focus on the potential benefits in guiding policy decisions that adding these factors entail.

[15] See section 5, for more details.

3.1 - Enhancing Debt Sustainability Analysis – Including Uncertainty and Refinancing Strategy

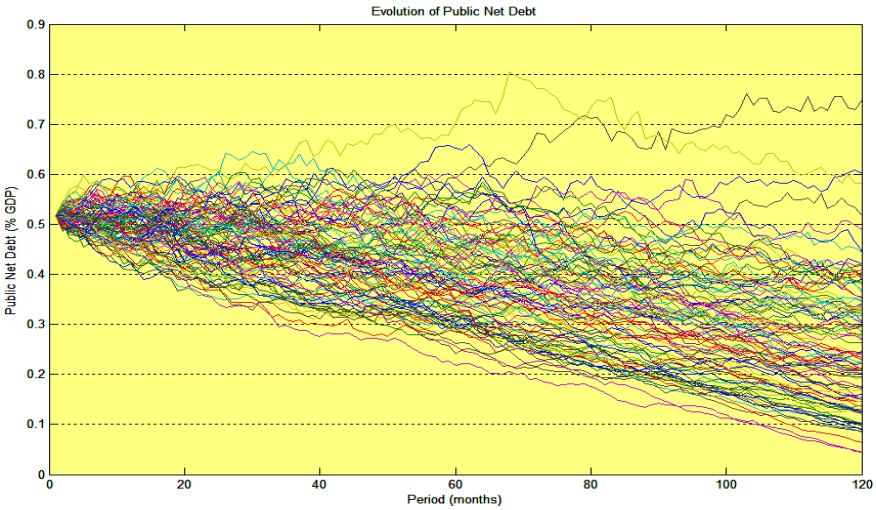
We start with basic assumptions for the main determinants of debt dynamics for a period of ten years, that is: nominal rates, inflation, GDP growth and primary surplus[16]. Then initiating with a debt/GDP ratio of 51.70%, entirely composed of floating-rate instruments, we determine the path of debt/GDP ratio over the ten years. This deterministic scenario allows us to conduct the simplest, but most usual, debt sustainability test. Graph (1) shows the expected trajectory of the debt/GDP ratio.

Graph 1: Expected Trajectory of the Debt/Gdp Ratio



This analysis is supplemented by the generation of stochastic paths for all variables, yielding a distribution of debt/GDP ratios across the different horizons that we carry the tests.

Graph 2: Distribution of Debt/GDP Ratios Across Different Horizons



[16] See Annex 2 for details

Table 1: Deterministic vs. Stochastic Simulation Results (100% floating rate debt)

Period	average DL (determ.)	Average DL (stoch.)	Volat	Relat. Volat
0	51,70%	51,70%	0,00%	0,00%
1	49,94%	49,93%	4,25%	8,50%
2	48,19%	48,09%	6,10%	12,69%
3	46,04%	46,00%	7,68%	16,69%
4	43,87%	43,78%	8,86%	20,25%
5	41,53%	41,46%	9,90%	23,89%
6	39,17%	39,16%	11,03%	28,18%
7	36,91%	36,85%	12,21%	33,12%
8	34,35%	34,26%	13,24%	38,65%
9	31,68%	31,65%	14,23%	44,97%
10	28,68%	28,39%	14,74%	51,91%

*** DL = Debt/GDP**

Note that one can calibrate these models so as to reflect expectations from a base case scenario. Both analysis yield similar average expected debt ratios - as shown in Table (1), but the information set available to policy makers is improved with the incorporation of stochastic scenarios

A policy maker responsible for setting fiscal targets, for example, can better understand the potential margin of error that such targets entails in terms of debt dynamics. In other words, by setting a 4.25% of GDP target for primary balance surplus, this policy maker may expect, based solely on his deterministic scenario that the debt/GDP ratios will fall to 41.53% in 5 years and to 28.68% in 10 years. The stochastic models, on the other hand, may help him assess the risk that these ratios may deviate from their average expected values. This exercise informs the policy maker, for instance, that there is a 95% probability that the debt/GDP ratio will not exceed 57.86% in ten years.

As mentioned above, another important dimension in this sustainability exercises is to include “assumptions” about the debt refinancing strategy. Having in hand the actual debt strategy that the debt office intends to implement is a strong comparative advantage of the public debt risk manager. The graphs below illustrate the results of debt sustainability tests using the same scenarios and stochastic simulations from the previous exercise, but including a refinancing strategy towards long-term fixed-rate debt.

Table 2: Deterministic vs. Stochastic Simulation Results (with refinancing strategy)

Period	average DL (determ.)	Average DL (stoch.)	Volat	Relat. Volat
0	51,70%	51,70%	0,00%	0,00%
1	50,06%	50,11%	1,49%	2,98%
2	48,18%	48,22%	2,13%	4,42%
3	46,39%	46,43%	2,69%	5,80%
4	44,53%	44,58%	3,14%	7,03%
5	42,66%	42,70%	3,52%	8,23%
6	40,51%	40,55%	3,91%	9,64%
7	38,48%	38,52%	4,33%	11,23%
8	36,36%	36,40%	4,69%	12,89%
9	34,34%	34,37%	5,06%	14,73%
10	32,41%	32,44%	5,51%	16,99%

*** DL = Debt/GDP**

Graph 3: Expected Trajectory of the Debt/GDP Ratio Including a Refinancing Strategy towards Long-Term Fixed-Rate Debt

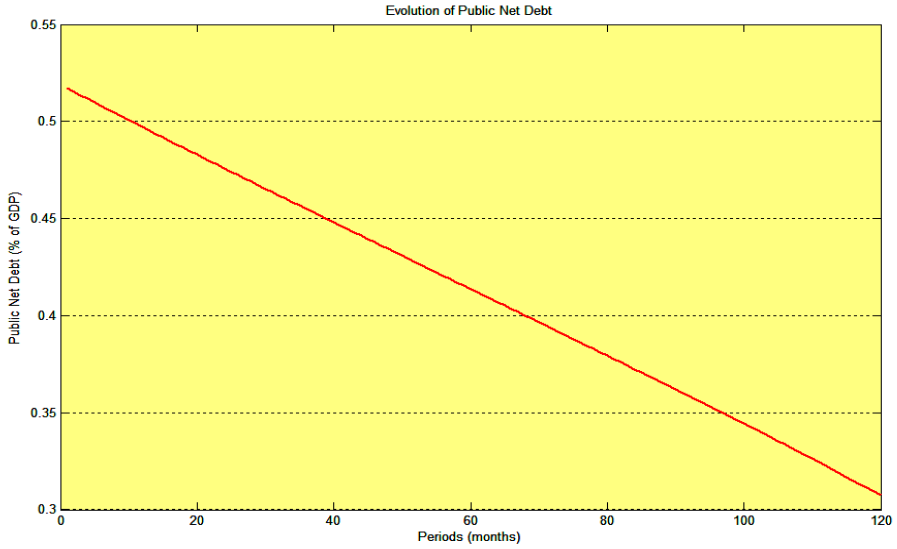


Table 3: Refinancing vs. no-refinancing strategy simulation (deterministic approach)

Period	Without Strategy	With Strategy	Difference
0	51,70%	51,70%	0,00%
1	49,94%	50,06%	-0,11%
2	48,19%	48,18%	0,02%
3	46,04%	46,39%	-0,35%
4	43,87%	44,53%	-0,66%
5	41,53%	42,66%	-1,13%
6	39,17%	40,51%	-1,34%
7	36,91%	38,48%	-1,58%
8	34,35%	36,36%	-2,01%
9	31,68%	34,34%	-2,66%
10	28,68%	32,41%	-3,72%

Graph 4: Distribution of Debt/GDP Ratios across Different Horizons Including o Refinancing Strategy towards Long-Term Fixed-Rate Debt

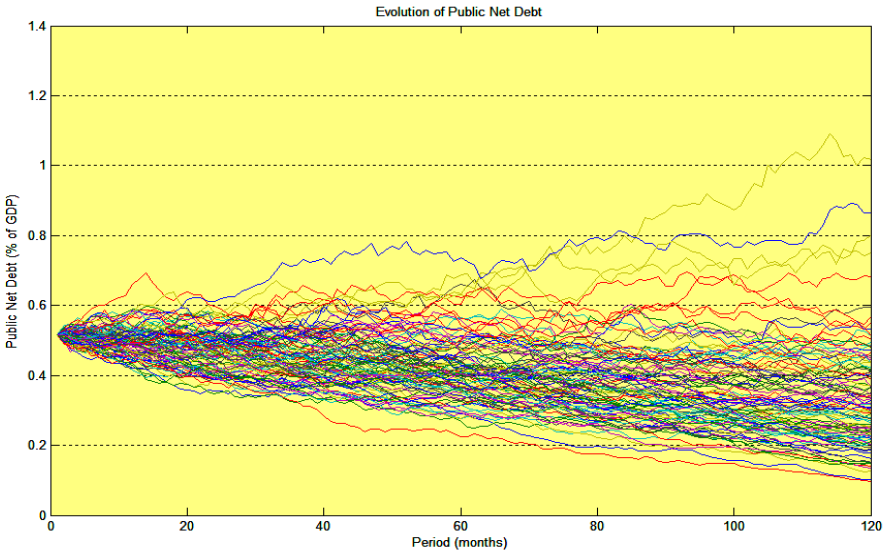


Table 4: Refinancing vs. no-refinancing strategy simulation (stochastic approach)

Period	Without Strategy	With Strategy	Difference
0	51,70%	51,70%	0,00%
1	49,93%	50,11%	-0,18%
2	48,09%	48,22%	-0,13%
3	46,00%	46,43%	-0,44%
4	43,78%	44,58%	-0,80%
5	41,46%	42,70%	-1,24%
6	39,16%	40,55%	-1,40%
7	36,85%	38,52%	-1,67%
8	34,26%	36,40%	-2,13%
9	31,65%	34,37%	-2,72%
10	28,39%	32,44%	-4,05%

Recall that the original debt is comprised of 100% floating-rate instruments (with monthly resettlements). The refinancing strategy with 10 year fixed-rate instruments[17] yields a higher cost coming from an interest rate risk premium charged by debt holders in exchange for a lower debt vulnerability to interest rate movements.

The results above reflect the trade-offs in terms of the costs and risks involved in the strategy. Note that by including the refinancing strategy in the analysis our average expected debt/GDP ratio in ten years raises from 28.68% to 32.41 %. The distribution of expected debt/GDP ratios, however, is much less dispersed as a result of its reduced exposure to shocks. The introduction

[17] The refinancing strategy assumes that 1% of the floating-rate share is redeemed every month in exchange for 10 year fixed-rate instruments. At the end of 10 years we obtain the share of fixed–rate debt of 100%.

of fixed-rate instruments brought the relative dispersion (the ratio between one standard deviation and the mean) from 51.91% down to 16.99%

One may claim that the use of stochastic models to complement simpler exercises based on deterministic scenarios may lead to conclusions that are harder to understand and too dependable on model calibration. She can also argue that the use of a few alternative deterministic scenarios may lead to a more intuitive assessment of debt sensitivity to changes in its main determinants.

Whereas we do not intend to argue against the importance of simpler types of analysis, the illustrative exercise we presented in this section sheds some light on how the public debt risk manager can complement and enhance a policy maker's information set. Despite the relative complexity in the design of risk management models, presenting their results in a user friendly fashion to decision makers is not a difficult task that has already become widely common, notably in the financial sector.

4 – A Long-Run Benchmark

This section highlight one of the fundamental tasks of a public debt risk manager, that is: the establishment of long term goals that may serve as a guideline to the short and medium term debt management strategies.

We start by going back to the first principles of public debt management and recalling that under Ricardian Equivalence assumptions, as defined in Barro (1974), public debt management would be irrelevant[18]. Despite the interesting insights one gets when investigating the Ricardian Equivalence, there is large evidence and it is widely accepted that the strong assumptions behind it do not hold in real life. These assumptions are: (i) agents with infinite planning horizon (complete information); (ii) complete markets; (iii) non-distortive taxes.

The reasonable relaxing of these assumptions turns public debt management relevant, and tax smoothing, market completion, public policy signaling, among others, become part of frequently stated debt management objectives[19].

[18] When talking about long-run goals and debt strategies it is important to point out that theoretically they make sense

[19] The theoretical literature about the relevance of debt management is not restricted to the relaxing of the Ricardian Equivalence assumptions. For our purposes in this work, it suffices to justify its relevance. Lopes (2003) and Bonomo et. al. (2003) offer brief summaries of the literature.

Once public debt management is recognized as a relevant subject, identifying the desirable structure of the debt becomes a fundamental task that needs to be addressed to guide short and medium-term debt operations.

In order to better understand the role this long-term reference plays to a debt manager, one can draw an analogy to the situation of an adventurer in the middle of a forest who holds a compass and knows exactly the direction she must take. Having a compassing and knowing the direction she wants to go is the only way she can be assured that her next steps will lead to the desired location. For the risk manager, the long-term benchmark represents the direction she wants to follow and, her compass, the tool she uses to formulate and monitor her strategy.

Debt management theory provides some guidance towards general characteristics of the debt portfolio. However, in order to provide more specific, quantitative guidelines, several debt specialists have engaged into a debate that has gained increased attention in debt offices around the world. The search for methodologies to the determination of the benchmark has become an important topic in the research agenda of debt managers in numerous countries. Brazil, Canada, Denmark, Portugal and Sweden are well-known examples.

The contribution of multilateral institutions such as the World Bank and the International Monetary Fund to this debate has also been notorious. These two institutions describe the benchmark, in their publication entitled “Guidelines for Public Debt Management” (2001), as a powerful tool to represent the debt profile that the government would like to achieve, based on its risk and expected costs preferences.

Usually, the benchmark is represented by some set of relevant debt indicators, such as composition, duration, debt profile etc. The idea is that it stands as a long-run goal, representing the preferences of society. In mathematical terms, one could face it as an optimization problem, where the government wants to maximize its objective utility function given some restrictions[20].

Some countries may decide upon their benchmarks based on very simple analysis and ad-hoc assumptions. A debt manager could, for example, conclude based on his beliefs regarding diversification benefits that the ideal composition of the debt should be a mix of nominal and inflation-indexed debt. The rationale behind it is the fact that a portfolio with these securities may yield a more stable debt servicing profile under recurrent demand and supply shocks.

[20] As most countries define as their main debt management objective as minimizing long term costs subject to prudent risk levels (see IMF and World Bank, 2001), the general identification of the objective function and the restrictions is straightforward.

Others may go in the desirable route of developing risk indicators[21] and investigating the trade-offs that they produce as outputs. Taking some different hypothetical debt compositions and calculating a set of relevant cost and risk indicators for each one may be a useful way of examining the pros and cons of distinct compositions.

Finally, one could use an even more analytical framework, building a model from which the optimal portfolio would emerge endogenously. It is certainly difficult to think about a supra-rational model that could consider all objectives and restrictions of debt management and, by itself, give the solution to this optimum portfolio question.

Canada, Denmark, Portugal, Sweden and Brazil, as mentioned before, are some examples of countries that use more analytically-intense frameworks for benchmarking. Cabral (2004) describes briefly how some countries deal with this subject[22]

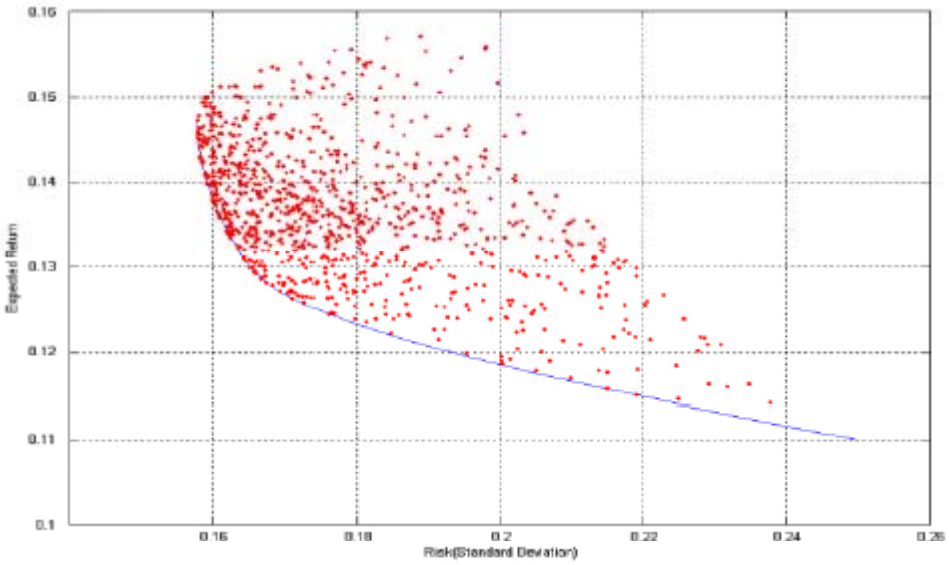
Portugal was one of the first countries to develop such a methodology. Granger (1999) and Matos (2001) illustrate how their model works. Basically, it is a cash-flow simulation model, having as inputs stochastic simulations of interest rates, different financing strategies and deterministic scenarios for other economic variables, resulting in some "efficient" portfolios.

The Swedish model is also based on cash-flows, with autoregressive processes for inflation, GDP, long-term interest rates and exchange rates, as well as a Taylor rule for the short-term interest rates. With some assumptions about the financing needs, a number of different portfolios are evaluated, with nominal and real cost measures. Bergstrom and Holmlung (2000) describe the model in detail.

The Brazilian approach, as described in Cabral and Lopes (2004), is basically an efficient frontier analysis, where costs and risks are measured in terms of the debt/GDP ratios. Steady-state compositions are simulated through a number of different periods based on stochastic scenarios and assumptions about the pricing of assets. With some portfolios evaluated in terms of cost and risk, as well as the correlation matrix, it is possible to draw an efficient frontier.[23] Looking at the efficient frontier, the debt manager would choose, based on its risk appetite, the single point representing the benchmark.

[21] We will cover these subjects in section 5.
[22] Other useful references for international experience are Guidelines for Public Debt Management (2001) and Nars (1997).
[23] It is important to highlight that this is an efficient frontier from the issuer point of view, quite different, therefore, from the one designed by an investor.

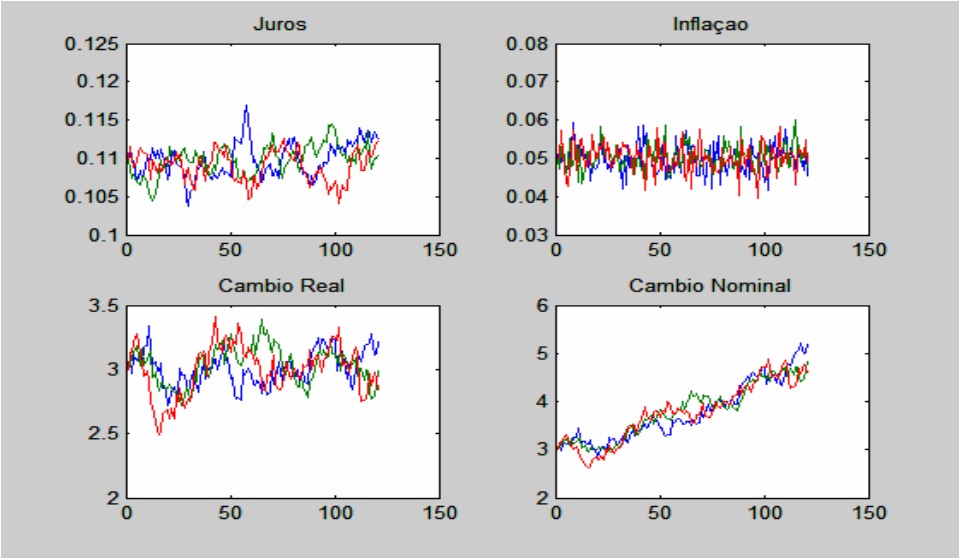
Graph 5: Efficient Frontier



In the Brazilian model, the stochastic scenarios can be generated by two different and somewhat complementary ways. In the first one, some correlated financial stochastic models are employed, namely a Cox, Ingersoll and Ross – CIR - model to the domestic and external interest-rates, a Brownian motion process for the price indexes and a Chan, karolyi, Longstaff and Sanders - CKLS model for the real exchange rates, the residuals being correlated using a Cholesky decomposition. A second methodology uses a macro-structural model to describe the evolution of the main economic variables (an IS and Phillips curve, a Taylor rule and equations for the evolution of exchange rates and risk premium)[24].

[24] Annex 3 provides a short description of these models, whereas Cabral (2004), Cabral and Lopes (2005) and Costa, Silva e Bagdassarian (2003) provide examples of actual implementation of these models.

Graph 6: Stochastic Simulations



Using the models described above, one can run Monte Carlo simulations and reach a distribution of debt/GDP ratios over a specific time horizon for each portfolio (debt composition) considered. Mean and standard deviations are taken from these distributions as cost and risk measures and correlations are calculated based on simulation of portfolios with single and mixed assets, what leads to a design of an efficient frontier.

Having provided an overview of commonly used methodologies to determine the benchmark, we turn to some important issues that deserve attention in the process of conducting these exercises.

First of all, a benchmark model should, ideally, be independent from current market conditions. Although this may sound odd at first glance, one should bear in mind the fact that it is exactly by separating the long run objectives from the circumstantial and tactical restrictions that the benchmark finds its main use.

Second, benchmark modeling should incorporate demand side restrictions to the extent possible, i.e., choosing an optimum composition based solely on supply side objectives, without examining the potential demand for that portfolio, is a myopic decision with reduced chances of success.

Moving to more general aspects, the formalization of the benchmark is a matter of no less importance. Actually, if a benchmark is established, but not formalized, it may be useless[25]. Formalization means some kind of superior approval, by the Cabinet or the Congress, which could delegate to the Debt Management Office – DMO – the power and duty to pursue those

[25] Of course the same argument applies to the medium and long-term strategy.

objectives. Besides that, a formalized benchmark brings transparency for the public debt manager as well as some guarantee of continuity across different governors. Establishing a well-defined governance process is yet another crucial step, although this matter is out of the scope of this paper.

Another critical issue regards the design of a transitional (medium and long-term) strategy to achieve the benchmark. As one could infer, this is not a trivial task, particularly in less developed countries with current portfolios very distant from their desirable ones. Coming back to mathematical thoughts, such transition would involve a complex problem of optimization, in order to find the strategy that optimizes the path between current conditions and long-run goals. On the other hand, regarding these specific countries, it might be the case that the existence of many market restrictions simplifies their choice, since it may eliminate many possible strategies.

As we said before, it would be quite naïve to think that reality could be replicated by means of analytically solving for the optimal debt composition through an extremely complex benchmark model encompassing all objectives and restrictions debt management is subject to. That is certainly not the case. However, modeling can be very useful in at least two aspects. First, it prevents one from the risk of relying only on intuition (science has largely shown intuition may be misleading).

Second, the process of building and extensively discussing some modeling can become a process of capacity building, as relevant concepts and trade offs involved are included in the debate. Instead of being rivals, analytical modeling and the subjective expertise of debt managers should be seen as complements to each other.

Apart from being an important tool in the strategic planning, a benchmark can also provide the risk manager a way to measure performance, comparing the current portfolio with the “optimum” (desirable) one. If these two compositions are very distant one from the other, intermediate benchmark portfolios can be chosen to the comparison. It is important to point out that some degree of freedom might be given to the one in charge of conducting the day-by-day financing strategy, typically the front-office, for small tactical actions as current market conditions might differ from the scenario used to design the medium-term strategy.

Although it is out of the scope of this paper, it is worth reminding that it is often the case that a benchmark is designed under an Assets and Liabilities Management – ALM – framework. It makes the whole sense for the government when accessing its risks not to manage its liabilities with no concerns about its assets. Governments’ financial assets may vary significantly across different countries but they generally share something in common: their main asset is the ability, or the right, to collect taxes. This way, the characteristics of the future primary surpluses

become a key-driving factor for the determination of the optimal debt structure, i.e., the benchmark.

5 –Public Debt Risk Indicators

Public debt risk has many dimensions. In this section we illustrate the main indicators commonly used by public debt risk managers. Most of them are of rather simple computation and we call them "traditional indicators." Others use stochastic simulations and usually belong to the "at-risk" family of indicators. Although not too complicated, these indicators represent adaptations to the debtor's point of view of measures of risk that are frequently employed by the private sector, such as the Value at Risk.

It is worth mentioning that, despite the simplicity of the so called "traditional indicators," many countries in fact do not compute them and there appears to have no international methodological consensus across those countries that do. The first problem appears to come from a chronic problem that many debt offices face in terms of back-office systems to compute in an aggregate and accurate fashion even the most simple debt indicator, that is, the debt stock. The second problem, the lack of methodological consensus, is also of important consequences as it makes cross-country comparisons of risk indicators a "risky" task to conduct.

To make the latter point clear, we show a table below with the average maturity of the Brazilian Federal and Domestic Public Debt using two different methodologies. The differences are striking. To the extent that not only debt managers, but also a whole range of investors and rating agencies use these indicators for international comparisons, this exercise raises a flag of caution to those going on that route[26].

Table 5: Differences between Average Life and Average Maturity

December/05	Average Maturity			
	Traditional Methodology		New Methodology	
	Months	Years	Months	Years
Domestic Debt	**27.49**	**2.29**	**40.28**	**3.36**
External Debt	**73.11**	**6.09**	**138.89**	**11.57**
Bonds	75.79	6.32	147.59	12.30
Contractual Debt	61.51	5.13	101.26	8.44
Federal Public Debt	**34.48**	**2.87**	**55.40**	**4.62**

[26] This exercise was actually motivated by the frequent comparisons of financial analysts and rating agencies of the Brazilian debt average maturity with those of peer countries. Annex 4 provides a brief description about the differences in the methodologies to compute the average maturity.

The methodology traditionally used by the National Treasury weights all disbursements (including coupon payments) on calculating the average public debt term. The methodology used by most countries, however, limits itself to the flow of principal payments. Though there is no internationally unified nomenclature in this area, these two forms are frequently cited as average term and average life, respectively. The first methodology gives greater emphasis to the refinancing risk of the debt but - being more conservative - makes it difficult to draw international comparisons. In view of this, the National Treasury decided to announce these statistics according to both methodologies.

In the traditional Treasury methodology, the average term of the internal federal public securities debt was 27.5 months at the end of 2005 while, for purposes of international comparison, this term was 40.3 months. It was noted that this term exceeded that of the debts of various investment grade countries, such as Mexico and Poland.

We present the measures of risk in the following sub-sections by grouping them across important types of risks[27] that the debt is exposed to: Market risk[28], refinancing risk, budget risk and demand side risk.

5.1 - Market Risk

Market risk can be defined as the uncertainty related to the expected costs owing to the volatility in the market indexes or currencies. Although in financial markets this type of risk has a strong relationship with the volatility of asset market prices, in the case of public debt this type of risk regards to changes in the value of the portfolio (the debt stock).

Although an apparently simple concept, one would be intrigued by the degree of discussion that involves the methodology to compute market risk due to divergences on how to reach a relevant measure of stock, which forms the basis for any market risk calculation. Should one evaluate the debt in terms of mark-to-market (MtM) or mark-to-curve (MtC)? Should we express the stock in nominal or in real values?[29] These are frequent questions that arise from this debate that have deserved a lot of attention from Debt Management Offices.

Despite the relevant discussion above, many countries compute measures of market risk. In the category of "traditional" indicators, duration, refixing-duration and convexity are the most

[27] Note that there are other types of risks that are not covered here, such as credit risk and operational risk, We intend to cover those in a future version of this paper.
[28] Although we will call market risk, a more precise approach would be "value" risk since the debt value in Brazil is not evaluated in terms of mark-to-market value, but in terms of a present value with fixed yield (the original internal rate of return in the issue moment).
[29] These and other discussions about the stock are presented in Baghdassarian (2003) and in Bonomo et al. (2002).

common, whereas the so-called “Cost-at-Risk” is the one coming from the stochastic group of indicators. Stress tests are commonly used as complements in market risk analysis to measure the consequences of severe shocks, most often in the interest and exchange-rates.

The concepts of duration and convexity are straightforward and, usually, there are no significant methodological differences on the way to compute them from the point of view of an investor or that of a risk manager[30]. Duration targets are in fact used by many countries such as Denmark and Sweden. For this reason we do not focus too much attention on describing them in this paper[31].

Refixing-duration is still a somewhat less prevalent concept. It measures the average time that it takes for the whole debt to be affected by a sudden change in interest-rates. For fixed-rate bonds this is equivalent to duration. However, for floating-rate bonds, for example, it represents the time span between resettlements in interest rates (in the Libor for example). Due to the fact that the debt stock in most countries usually comprises a mix of nominal and indexed instruments (most often, floating-rate, inflation-indexed, or exchange-rate linked), the use of this indicator has become increasingly common.

$$Duration_{refixing} = \frac{1}{PV} x \sum_{i-1}^{n} PV_i x T_i \qquad (01)$$

Where:

$T_i = 0{,}03$ (one day)	- For Selic linked bonds;
$0 < T_i < 1$	- For TR bonds
$0 < T_i < 3/6$	- For Libor3/6 bonds
$T_i = t_i$	- For others.

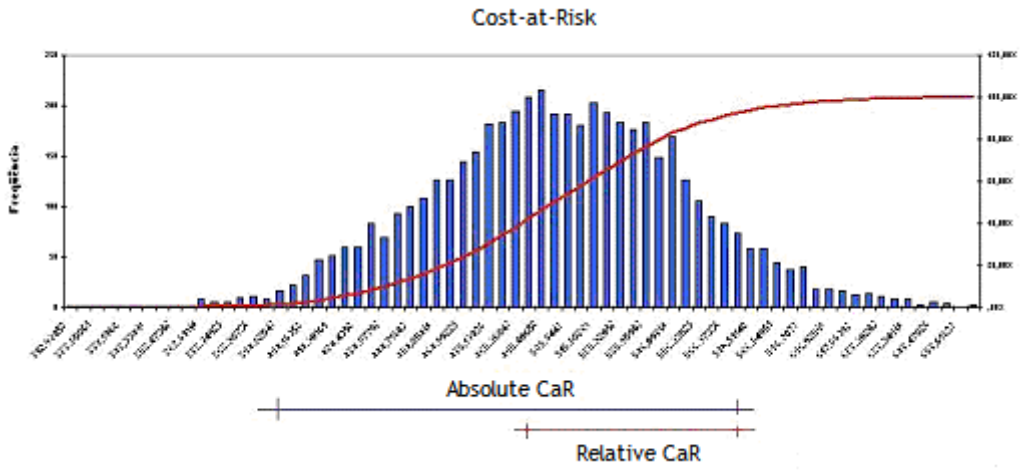
The Cost-at-Risk (CaR)[32] represents the maximum expected value that the debt stock can reach over a determined period, given a certain level of significance. While the market-risk indicators discussed so far are indicators of sensitivity of the debt stock to sudden changes, especially in interest rates, the CaR provides a measure of uncertainty regarding the expected

[30] Although in many cases investors use Mark-to-market values, while debt managers use Mark-to-Curve.
[31] There is an interesting debate on the use of duration as targets. One should be careful on how closely she should follow these targets as it may entail odd policy conclusions from a debtor point of view. Think of an increase in interest rates, causing a reduction in duration. In order to follow the target the debt manager would be induced to issue longer-term debt. But this is exactly the time that interest-rates are high!
[32] Baghdassarian (2003) presents the methodology to calculate CaR, CfaR and BaR.

stock in the future (say over a period of one, five and ten years). This indicator has also the advantage of incorporating the effects of a broad range of risk factors that may affect the debt stock such as changes in interest-rates, inflation, exchange-rate and GDP (when appropriate).

One can calculate absolute and/or relative CaR. The absolute CaR consists of the difference between the future stock, considering some level of significance, and the initial stock of the debt. In turn, the relative CaR measures the difference between the future stock, considering some level of significance and the mean. Graph 7 below illustrates the concept.

Graph 7: Cost-at-Risk (CaR)



The use of CaR has close relation with the discussion we presented in section 3 on the role that debt risk manager can play in debt sustainability assessments. It is indeed an instrument that aggregates uncertainty as well as hypothesis for the refinancing of the debt.

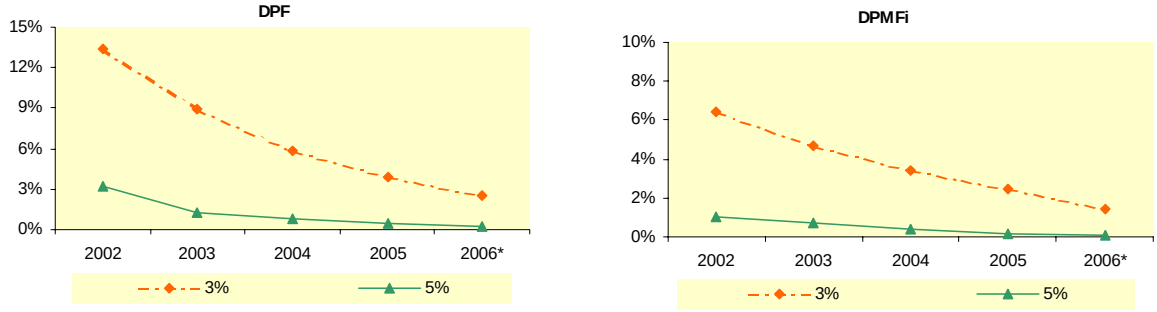
Another important point to consider here, is to emphasize that despite their similar properties, Cost-at-Risk and the better known VaR guard relevant differences. One may tell that CaR is an adaptation of VaR to take into account the specific needs from the point of view of the issuer, concerned about its debt stock value (many times on a MtC basis, as opposed to MtM in VaR) over a much longer period than VaR is usually computed for. These "subtle" differences, including the importance of considering the refinancing strategy, yield significant challenges in the modeling of such tool, especially with regards to the pricing different instruments to be issued in the future and their relative risk premium[33].

Finally, as a complement to the measures above, public debt risk managers conduct stress tests. There are various ways of running these exercises. A normal approach consists of

[33] A discussion about these complexities is out of the scope of this paper. For further discussions see Bonomo, Costa, Rocque & Silva (2003) and Cabral (2004).

applying shocks to key variables such as interest and exchange rates in terms of standard deviations, based on the distribution of their historical values over a period of time. Measuring the sensitivity of such shocks has become an important practice in Brazil, be it to look how it has behaved in the past or to help to measure the future consequences of debt strategy that is being implemented. It has also deserved much attention in the analysis of rating agencies and financial analysts[34]. Graph 08 shows an example of a test conducted for the Brazilian debt and presented in the Annual Borrowing Plan 2006.[35]

Graph 08: Probability of DPF and DPMFi Stock Increases of 3% and 5% of GDP as a Result of Interest and Exchange Rate Shocks.



5.2 -Refinancing Risk

Refinancing risk of a public debt is defined as the risk of adverse changes in the stream of debt payments upon its refinancing. In extreme cases it may even lead to the incapacity of a government to roll-over part or the total amount of the debt coming due at a particular date.

Similarly to the case of market-risk, the indicators of refinance risk can also be divided in simple "traditional" measures and a correspondent "at-risk" measure. The Brazilian Debt Office uses three indicators to evaluate this type of risk: Average life, percentage of the debt maturing in the short term (usually in one year) and the Cash-flow at Risk (CfaR)[36]. As we will show below, each of them measures refinancing risk from a different perspective and their joint use is advised.

The average life measures an equilibrium point of all debt maturities. Since it is a type of mean, an assessment of the evolution of this indicator along the time can help us to prevent a systematic debt life shortening, what could bring problems for debt managers.

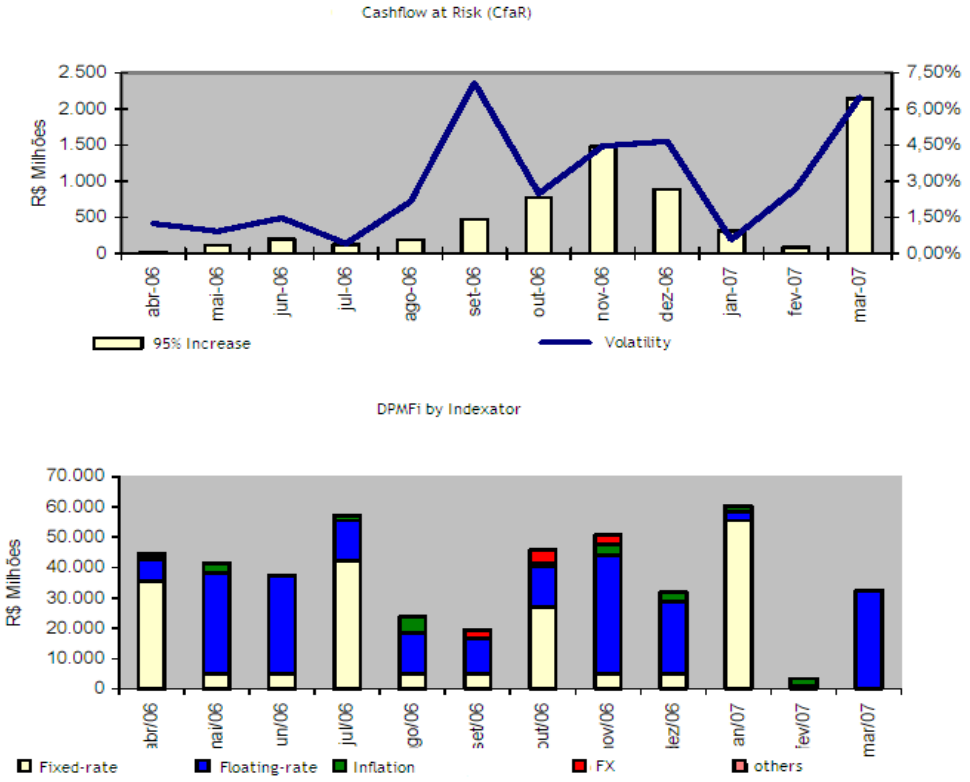
[34] This type of analysis has in practice deserved much attention in discussion of Brazilian authorities with rating agencies such as Fitch, Moody's and Standard & Poors.
[35] For a better description see Brazilian Annual Borrowing Plan (2006).
[36] Baghdassarian (2003) presents the methodology used to calculate the CfaR.

The second indicator is the percentage of the debt maturing in less than one year. It is a complementary measure to the average life and is more focused in the short term. While average life concerns to a systemic reduction in the debt life, the percentage of the debt maturing in less than one year is more focused on the cash requirements to pay the debt maturing in one year. In other words, it is related to liquidity risk.

The last indicator used to measure the exposure to the refinancing process is the Cash-flow at Risk (CfaR), which assesses the uncertainty of future cash-flows. CfaR measures, with some level of significance, the maximum expected cash-flow (payments) at the specific dates or periods in the future. A fixed rate bond, denominated in local currency will not have this type of risk, since there is no risk factor regarding the cash-flows[37]. On other hand, it is difficult to know in advance what will be the cash-flow of a foreign exchange debt, expressed in terms of local currency. The same rationale applies to other types of instruments, such as floating-rate or inflation-indexed securities.

The graphs below show how this indicator has been used in practice by the Brazilian Debt Office. It is taken from the Brazilian Annual Debt Report (2004) and illustrates the trade-offs across instruments in terms of cash-flow risk[38].

Graph 09: Maturity Profile and Cash-flow at Risk



[37] There is an interesting discussion that we do not carry here regarding the cash-flow risk free bond. Many countries consider it to be the nominal bond. However, others concerned with variables in real terms, may argue that inflation-indexed securities are more appropriate candidates deserving such title.
[38] For more details on the methodology to compute CfaR, see Brazilian Annual Debt Report (2004)

As one may observe, the indicators discussed in this sub-section are more complements than substitutes to measure refinancing risk. While the average life and the percentage of the debt maturing in the short term are more focused on a time distribution of debt payments, the cash-flow at risk is concerned to the volume, and its sensitivity to shocks, of payments that the debt manager will face in particular dates in the future.

5.3 -Budget Risk

The concept of Budget-at-Risk (BaR), as it is used in Brazil, relates to the risk that the debt service within a fiscal year (the official Budget period) surpasses the amount originally approved by Congress.

Since the debt service in the budget is measured on a cash basis, BaR is in a sense very similar to CfaR: both regard to the uncertainty of cash-flows. The most important difference between both indicators is that while the Budget-at-Risk is focused in one year, the CfaR is more flexible and can be computed for any specific date or period. Moreover, Budget-at-Risk has an exogenous reference value which is approved by the Congress and thus has as an output the probability of exceeding that value. CfaR on the other hand provides, for a given level of significance (risk), the maximum expected value that the cash-flow is expected to reach.

The close monitoring of budget risk in Brazil, and in other countries in general, is an important task of the public debt risk manager. Observing the probability of exceeding the budget debt managers can anticipate a potential arduous and time consuming mission of presenting to Congress a proposal for a supplemental budget to pay the debt. Although one would expect that there is minimal risk of not getting this type of proposal approved, exposing a country's debt to this type of process may be a sensitive issue that justify close monitoring.

5.4 –Demand Side Risk

We define demand side risk as the risk of sudden shifts in the demand for government bonds. Although this may occur due to several different factors, the most common driver of abrupt short term variations in the demand for government bonds are the interest rates.

Investors due to stricter prudential regulations, or just as an internal investment policy, have increasingly relied in measures of interest-rate exposure to monitor their risk of losses. In fixed-income markets, some of the most commonly used measures are the PVBP (or DV01) and the Value-at-Risk (VaR).

PVBP[39] expresses how much the portfolio value will change given a 1 basis point variation in interest rates. It is similar to the duration concept with the advantage of being also a function of the total volume of the portfolio.

$$PVBP = P(i) - P(i + 0{,}01\%) \tag{02}$$

Where:

i	- Yield
P(i)	- Bond Price

Value-at-Risk (VaR) complements the PVBP by incorporating price volatility. While the PVBP provides us with a measure of absolute sensitivity to changes in interest rates, VaR enhances our set of information by incorporating the probability of such changes[40].

$$\sigma_p^2 = w' \sum w \tag{03}$$

Where:

σ^2_p	- Yield
w	- Vector of weights for the various securities in the portfolio
Σ	- Variance/covariance matrix of R returns on securities in the portfolio

$$VaR = P_0 . \sigma_p . 1{,}95 \tag{04}$$

Where:

P_0	- Initial Price
1,95	- equivalent to a degree of 95% of confidence

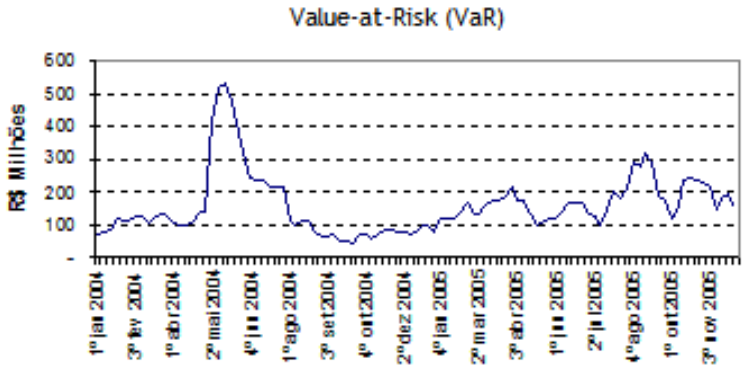
A large share of the demand for government securities, especially in Brazil, observes limits with regards to the amount of interest-rate risk it can be exposed to. This behavior imposes constraints to the debt manager on the transfer of interest rate risk to the market. During moments of volatility, to make matters even worse, VaR can reach high levels and lead to stop-loss operations from investors.

[39] also known as dollar value of 1 basis points – DV01

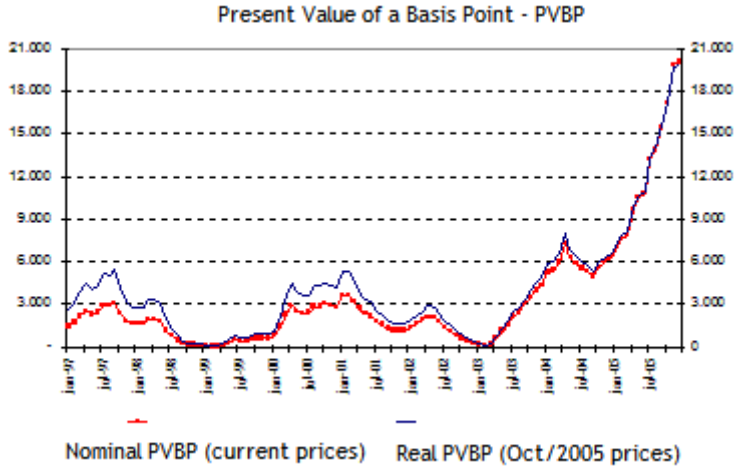
[40] It is perfectly possible in an economy that is gradually becoming more stable, for example, to increase the PVBP, either by augmenting duration or the volume of fixed-rate bonds (or both), and at the same time observe a reduction in VaR (due to a drop in volatility)

The effect of such changes in demand can be disastrous to the implementation of a debt strategy. It is therefore the role of a public debt risk manager to monitor this risk. While conducting this exercise it is important to monitor not only the risk of the current portfolio, but also the implicit pace of risk transfer that the debt strategy entails for the future. This is especially relevant to those countries that are in the process of lengthening the maturity of their bonds and increasing the share of fixed-rate instruments. Brazil fits this profile and the debt office monitors both indicators, as shown in the graphs (09) and (10) below.

Graph 09: Evolution of the Value-at-Risk (VaR)



Graph 10: Evolution of PVBP



6 –The Risk Manager and the Strategy Planning Design

An important responsibility of a risk manager is to take part in the debt planning, which involves designing, monitoring and analyzing the trade-offs among different refinancing strategies that can be implemented by the debt management office. The process of designing a debt strategy

is a responsibility that is shared among different areas in a debt office, such as the front-office, for example.

One of the risk manager's attributions is to identify possible risks concerning the debt strategy implementation and to define desirable targets for debt indicators such as stock, average life and others. Usually, these targets are set for the end of the year[41] (short-term planning) and for some years in the future (long-term planning).

Another attribution is to monitor the implementation of the strategy to validate it and, whenever necessary, suggest corrective measures. Although to avoid inconsistencies between strategy planning and its implementation, sometimes changes are necessary due most often to significant unpredictable variations in market conditions. Relevant modifications in scenarios usually affect cost and risks of different potential strategies, possibly turning the original strategy into a suboptimal.

In section 4, we presented some ways to determine long-term objectives for public debt. In this section,[42] we will discuss the means to achieve those objectives. In other words, we will be dealing with the transitional debt strategy design and its monitoring. We will present the steps behind the process of building it.

A general transitional strategy should consider not only the long-term objectives, but also short-term restrictions. Silva (2005) divides its process of design, implementation and monitoring in 8 stages as follows:

1. Definition of long-term objectives and guidelines;
2. Development of Macroeconomic Scenarios;
3. Preliminary discussions of scenarios and restrictions;
4. (Transitional) Strategy design and preliminary risk assessment;
5. Definition of targets: Expected results
6. Analysis of opportunities and challenges in the following years[43];
7. Tactical debt planning and execution (short-term) and
8. Monitoring the implementation of the transitional strategy (Annual Borrowing Plan).

[41] Brazilian National Treasury publishes its Annual Borrowing Plan with the current debt composition and desirable targets for the end of the year.
[42] This section will be based on Silva (2005) e Baghdassarian (2003)
[43] In a sense, steps 4 to 6 take part in the same process regarding the design of a transitional strategy and its expression in terms of indicative targets.

Although the risk manager participates in all of these eight steps, her role is especially active in three of them: defining the long-term objectives, designing the transitional strategy (including the definition of targets for debt indicators) and monitoring the strategy implementation.

The definition of long-term objectives is one of the most important risk manager's attributions. It establishes the long term goals that should be achieved by the whole debt management office. Without these objectives, debt management could be focused on short-term opportunities and restrictions, which could lead the debt to a suboptimal structure (more costs and risks). Since we have covered this theme with some degree of detail in section 4, we focus our discussion below in the other two topics.

6.1 The risk manager's role in Transitional Strategy design

The risk manager's participation in the transitional strategy design is central and most of the quantitative work[44] is done in this step. At this stage she uses her skills to select and adequately employ the tools that were developed to measure the various types of risks across different potential strategies.

The first task in this process regards to the analysis of initial conditions to build the transitional strategy. This includes a thorough assessment of: the current debt profile and its relation with the long-term benchmark, the expected cash requirements over the period of the strategy, the expected stream of revenues (other than from debt issuance) for the payment of debt service; and demand side opportunities and restrictions for the issuance of distinct types of securities.

An appropriate speed of convergence to the long-term benchmark will critically depend on these assessments. Note that this is a comprehensive assignment with most tools presented in the previous section being used at some stage. A cash-flow at risk (CfaR) analysis, for example, is important at this point, since it provides a measure of uncertainty regarding the stream of payments[45]. Another good example refers to the use the demand side indicators that we have discussed (PVBP/DV01, and VaR).

Based on the examinations above, the front-office designs a detailed debt issuance strategy, usually for the period of one year.[46] This is a cyclical process that requires active participation of

[44] The Brazilian Debt Management Office has developed a strategic planning and risk management system (GERIR) that helps the process of development and analysis of several strategies. In Appendix 1 we will present some features of GERIR system.

[45] CfaR is also used afterwards to assess the expected consequences in terms of cash-flow risk of alternative financing strategies.

[46] Longer-term strategies are also designed but with less degree of detail. Among other things, the one year strategy includes exact expected amounts of each security to be issued in all auctions throughout the year.

the public debt risk manager. It demands a series of simulations, analysis of expected results and revisions. A strategy is prepared, their results analyzed and revised until one reaches satisfactory (and realistic) outcomes.

In Brazil, the debt strategies are simulated based on four different scenarios (base-case, optimistic, pessimistic and stress). Issuance strategies are usually not the same for these different scenarios as they often entail distinct opportunities and restrictions to the pursuit of long-term objectives. In other words, the speed of convergence to the benchmark is state dependent.

One year targets (ranges) are established and published as a result of this one year strategy (see table 6). Besides her responsibility in providing accurate assessments regarding the feasibility of such targets, the risk manager becomes at this stage the "guardian" for the adequate implementation of the strategy to ensure these targets are met.

Table 6: Results and Targets for DPF and DPMFi

Results and Targets for the Federal Public Debt - DPF

Indicators	Dec-04	PAF-2005	
		Minimum	Maximum
Stock of DPF held by the public (R$ billion)	1013.9	1160	1240
Average maturity of DPF (months)	35.3	36	41
% Maturing in 12 months	39.3	34	40
Share of DPF (%)			
Fixed rate	16.1	16	25
Floating rate	45.7	39	47
Exchange rate	24.2	12	16
Price Index	11.9	18	23
Others	2.1	1	3

Source: National Treasury

Results and Targets for the Federal Domestic Public Debt - DPMFi

Indicators	Dec-04	PAF-2005	
		Minimum	Maximum
Stock of DPMFi held by the public (R$ billion)	810.3	940	1000
Average maturity of DPMFi (months)	28.1	28	34
% Maturing in 12 months	46.1	40	45
Share of DPMFi (%)			
Fixed rate	20.1	20	30
Floating rate	57.1	47	57
Exchange rate	5.2	3	5
Price Index	14.9	15	20
Others	2.7	2	4

Source: National Treasury

6.2 The risk manager's role in monitoring the Transitional Strategy's implementation

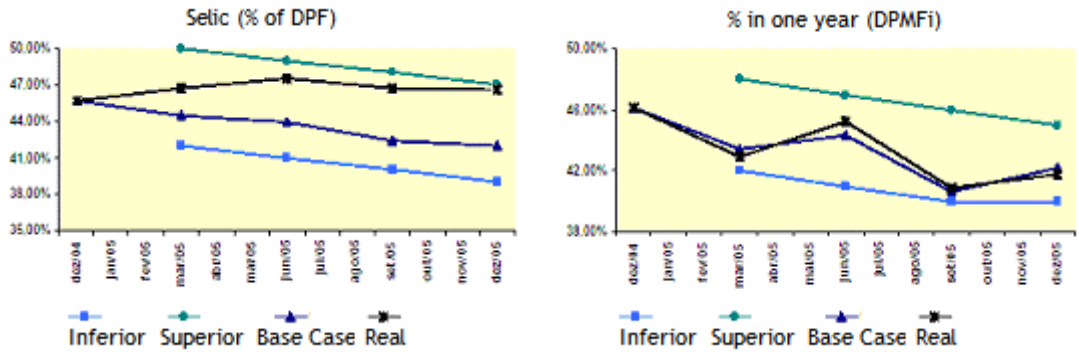
An effective monitoring of the execution of a debt strategy depends on: the preparation of adequate "tracking-error" measures, a careful assessment of the determinants of potential

deviations from pre-established targets, and the existence of adequate forums where the results of these assessments can be discussed and corrective measures taken if necessary.

In Brazil, the likelihood of meeting each of the targets shown in the Graph (11) is checked on a monthly basis. The risk manager tests throughout the year whether the current observed debt indicators are consistent with what had been originally expected by the time of the strategy elaboration. She also verifies whether the strategy planned for the remaining month(s) of that year will meet the targets pre-established in the Annual Borrowing Plan.

Graph (11) presents some examples of tracking-error exercises of two indicators: the participation of floating-rate bonds in the Federal Public Debt DPF; and the percentage of Federal Domestic Public Debt - DPMFi maturing in one year.

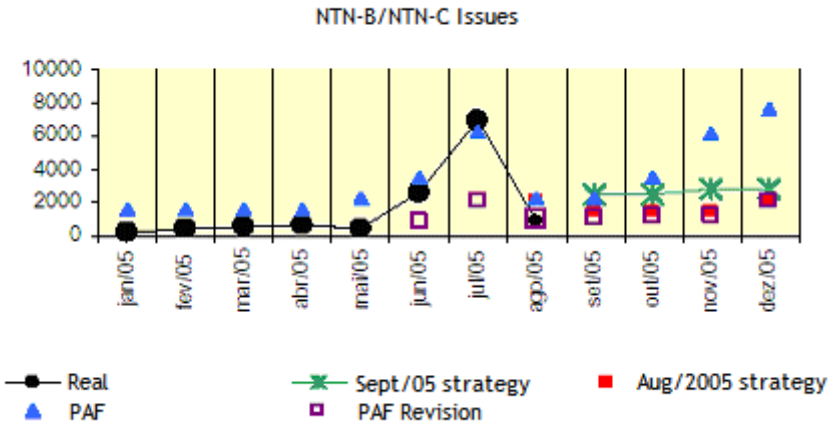
Graph 11: Monitoring Debt Targets



Note that despite the fact that targets are only published for end of year values the public debt risk manager determines caps and floors for these indicators on a quarterly basis. The two lines within these bands represent a comparison between the results programmed in the original strategy and the ones that have been reached (or expected) given the strategy that has been actually implemented.

Another indicator that is closely checked, despite not being part of the published targets, is the amount of issuance of each type of bond. Detecting in advance main deviations in the programmed debt strategy enhance the awareness of debt managers about their impact and help in the design of corrective measures. The graph (12) illustrates this analysis for inflation-linked bonds in Brazil.

Graph 12: NTN-B/NTN-C Issuances/2005



As mentioned above, upon the identification of deviations from pre-established targets it is also the role of the risk manager to assess the determinants of such deviations. Usually these deviations come from two sources: differences between the original scenario of macro variables (interest-rates, inflation, exchange-rate etc); or unexpected opportunities or constraints to strategy implementation. The result from such analysis has to be made clear to debt managers.

Finally, all this analysis would be useless if there were no appropriate forum to present it. In the Brazilian debt office the most important forum in which the strategy is monitored are the monthly meetings of the Public Debt Committee.

This committee is formed by high-level managers of back, middle and front-offices, and the Treasury Deputy-Secretary responsible for the debt management area. The macroeconomic/financial environment and the very short-term (one month) strategy are discussed in the committee, as well as their impact over the risk indicators for the next months and the end of the year. These results are compared to the original ranges and in case of deviation the strategy is to the extent possible adjusted in order to reduce the divergence[47].

Another important forum represents the quarterly meeting of the Public Debt Committee to discuss a broader long-term strategy. Subjects like benchmark, market development and others are discussed in this forum.

47 The risk management division also produces a risk management monthly report where other risk assessments are done.

7 – Concluding Remarks

The aim of this paper was to draw attention to the role of the public debt risk manager, describing her main attributions and tools, i.e., to give a comprehensive view of her principal concerns and macro-functions. In doing so, we tried to provide a general overview, instead of covering specific issues in detail, such as risk modeling techniques.

We acknowledge that the mapping out of all attributions of the public debt risk manager is an ambitious task, subject to several gaps and criticisms. One may always think of relevant topics that she thinks were left out. Indeed, in a future version we intend to include some issues such as tools to deal with the risk of contingent liabilities and different approaches to credit risk[48].

Despite these limitations, we believe that the paper can serve as a useful guide to those who want to get more familiar with the profession of the public debt risk manager. In an environment that debt offices around the world have been paying significant efforts to modernize their risk management practices this can serve as a starting point to get a good grasp of the activities involved. It also serves to enhance the awareness of Brazilian policy makers on how to best explore the skills and outputs that can be provided by public debt risk managers.

48 The contingent claims approach that has been explored by the IMF, see Gapen & Gray (2005) and Barnhill (2003) are potential methodologies to be explored.

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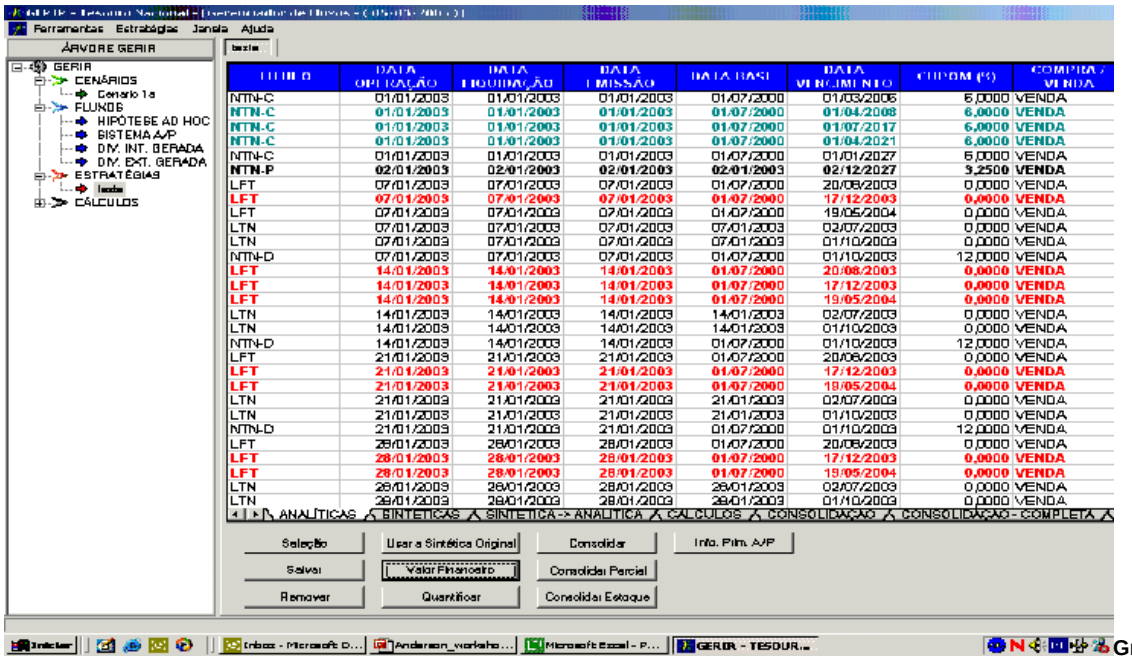
Annex 1 – The GERIR system

Geris system is an important tool for debt managers since allows them to evaluate different strategies and their trade-offs. Besides, give them some sophisticated tools to assess the debt in a probabilistic environment (CaR, CfaR, BaR and VaR).

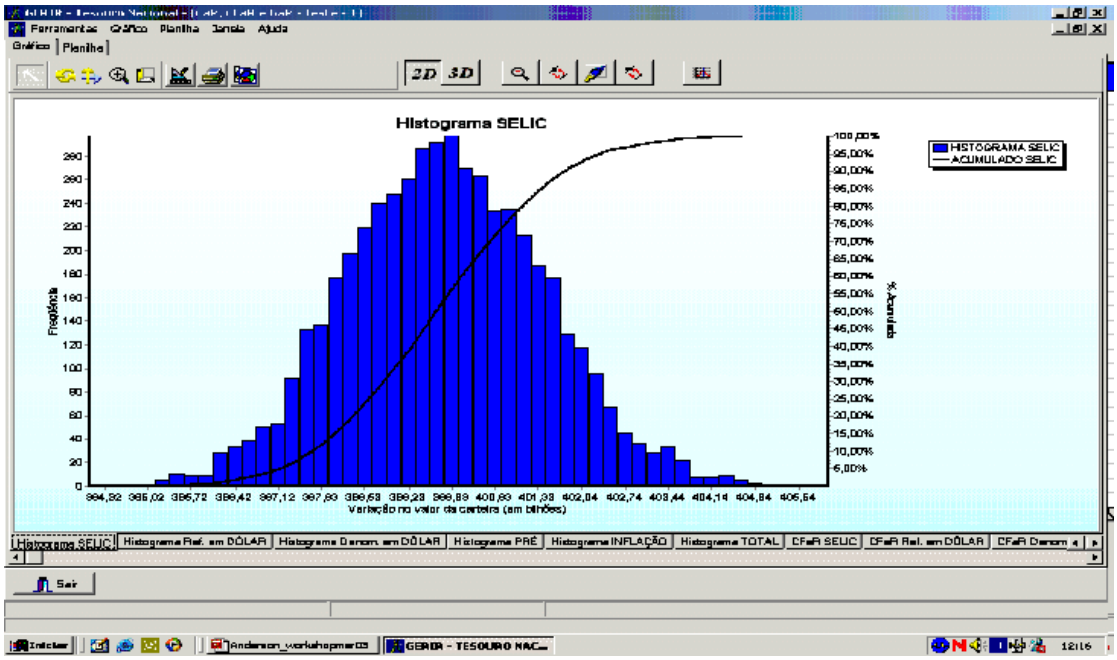
It is a very flexible system in terms of strategy implementation (issuances, buy-backs, exchanges, etc), as well as very powerful in terms of generated indicators (stock, stock average life, new issues average life, percentage maturing in one year, duration, composition, maturity profile, etc).

Its development started in 2001 and finished in 2003, when it becomes very important part of the strategy design process. In 2004, it was started an improvement and probably by the end of may (2006) will be finished. The idea of this improvement was to make it easier to deal by users. Graphs 13 to 16 present some screens to give us an idea about the system appearance.

Graph 13: GERIR System – Strategy Module



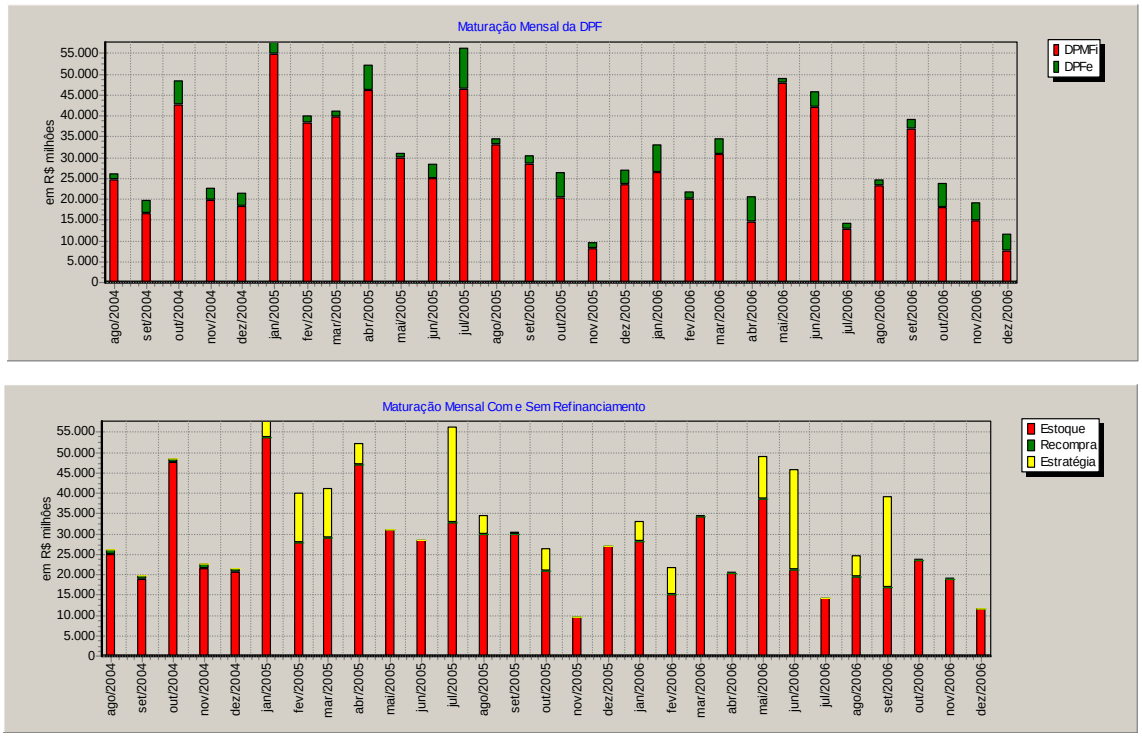
Graph 14: GERIR System – CaR Module



Graph 15: GERIR System – New Strategy Design Module

MÓDULO GERENCIAL DE ESTRATÉGIA DE FINANCIAMENTO												
Título			LTN	LTN	LTN	LTN	LTN	LFT	LFT	LFT	LFT	GLOBAL
Cupom			0%	0%	0%	0%	0%	0%	0%	0%	0%	8%
Data Base			01/07/00	01/07/00	01/07/00	01/07/00	01/07/00	01/07/00	01/07/00	01/07/00	01/07/00	01/01/05
Vencimento			01/01/08	01/04/08	01/07/08	01/10/08	01/01/06	01/01/12	01/01/14	01/01/16	01/01/18	01/05/30
Mat. do indexador no mês (R$ milhões)			1.452	2.906	1.565	1.000	15.377	1.128	6.327	32.892		
Mat. total no vencimento (R$ milhões)			1.452	2.906	1.565	1.000	15.377	1.128	6.327	32.892	-	-
FINANCEIRO (R$ milhõ	Emissão	Resgate										
01/08/2005	11.500	2.395	-		1.000			500		10.000		
01/09/2005	1.631	2.435		1.000		631						
01/10/2005	11.000	2.470		1.000								10.000
01/11/2005	15.000	2.506					15.000					
01/12/2005	2.471	2.540							2.471			
01/01/2006	-	17.952										
01/02/2006	-	2.614										
01/03/2006	-	2.644										
01/04/2006	-	2.680										
01/05/2006	-	2.710										
01/06/2006	-	2.749										
01/07/2006	-	2.782										
01/08/2006	-	2.819										
01/09/2006	-	2.857										
01/10/2006	-	2.889										
01/11/2006	-	476										
01/12/2006	-	5.923										
Emissão Total	41.602	61.442	-	2.000	1.000	631	15.000	500	2.471	10.000	-	-

Graph 16: GERIR System – Maturity Profile with and Without Strategy



Annex 2 – Public Debt Simulations – Section 3 exercise

Usually, sustainability exercises consider deterministic macroeconomic scenarios and some hypothesis about primary surplus and signori age. However, there are, at least, two aspects that should be also considered in order to get more accurate estimates. The first regards to the uncertainty about the scenarios and the last is about the public debt composition.

Although we presented some discussions regarding these issues in section 3, we did not discuss technical aspects regarding the model. Therefore, the aim of this section is to present the general methodology, the hypothesis and the parameters used in the section 3 to illustrate the consequences of uncertainty and of refinancing process over debt sustainability exercises.

Basically, there were four simulations. The first can be considered the "base-case" since there is no uncertainty or refinancing strategy. We inserted the uncertainty in the second exercise, although there is not refinancing strategy yet. We used a deterministic scenario to evaluate refinancing strategy in the third exercise. Finally, the last exercise considers not only, the refinancing strategy but also the uncertainty.

Before detailing the exercises, it is important to present the general framework used to include the uncertainty in the simulations. Basically, we performed Monte Carlo simulations in order to generate thousands of macroeconomic scenarios. CIR (Cox-Ingersoll-Ross) and Geometric Brownian Motion[49] were the models used to generate the scenarios for interest rates and GDP/inflation respectivelly. Equations 07 and 08 below illustrate those models.

$$\delta s = a.\left(b - s(t)\right).\delta t + \sigma.s(t)^{\frac{1}{2}}.\delta z \qquad (07)$$

Where:

s	- interest-rates
a	- mean-reverting speed
b	- long-term interest rate
σ	- volatility
dz	- Wiener process

$$\frac{\delta s}{s} = \mu.\delta t + \sigma.\varepsilon.\sqrt{\delta t} \qquad (08)$$

[49] See Baghdassarian (2006) for more details about these models.

Where:

s	- inflation
μ	- inflation growth

Besides those models, we used traditional Blanchard model to simulate debt evolution. Equation 09 shows this model.

$$d_t = \left(\frac{1+r}{1+n}\right).d_{t-1} - (t_t - g_t) - \frac{M_t - M_{t-1}}{p_t * PIB} \quad (09)$$

where:

d_t	: Net Debt in t, as a proportion of GDP
d_{t-1}	: Net Debt in t-1, as a proportion of GDP
r	: real interest rate
n	: GDP growth rate
t_t	: Taxes, as a proportion of GDP
g_t	: Expenditures as a proportion of GDP
M_t	: Monetary base in t
M_{t-1}	: Monetary base in t-1
p_t	: Current prices level
PIB	: GDP

One interesting point is that all macroeconomic models were calibrated to simulate long-term expectations and not current level ones. The idea is that if we had adopted a current level approach, a lot of other discussions could have taken place.

The first exercise considers a deterministic approach to scenarios generation and maintains 100% of the debt linked to floating rate index. Besides, it considers a primary surplus of 4,25% of GDP, annual inflation about 3%, nominal interest rates around 11% a year and initial net debt of 51,7% of GDP. With these parameters we have drawn the evolution of the net public debt for the next ten years. An important hypothesis is that all floating rate debt is entirely refinanced with floating rate debt.

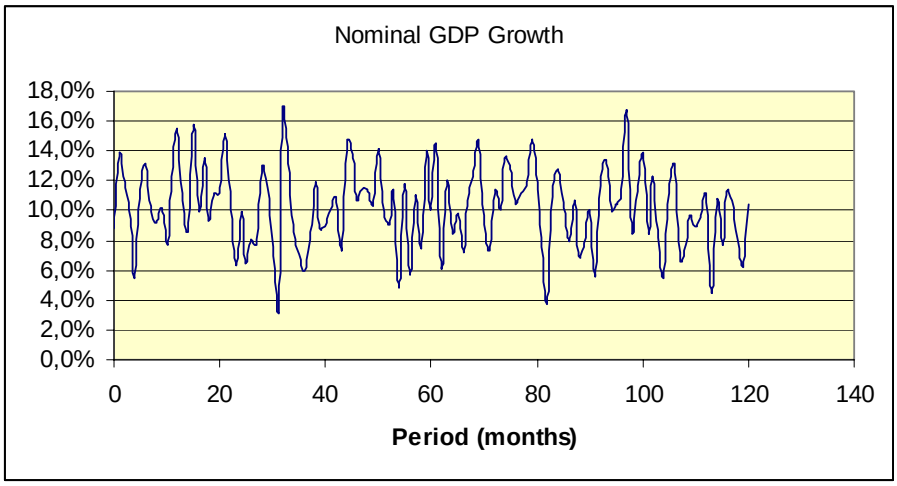
The second exercise is very similar to the first, but instead of a deterministic scenario, we used a thousand of different scenarios to analyse the uncertainty regarding the first exercise conclusions. Table (07) presents the evolution of the net public debt both in the determinist environment and in the stochastic one. It is important to remember that in the case of stochastic simulations we used the mean and the standard deviation to express the results.

Table 7: Deterministic x Stochastic scenarios generation (without refinancing strategy)

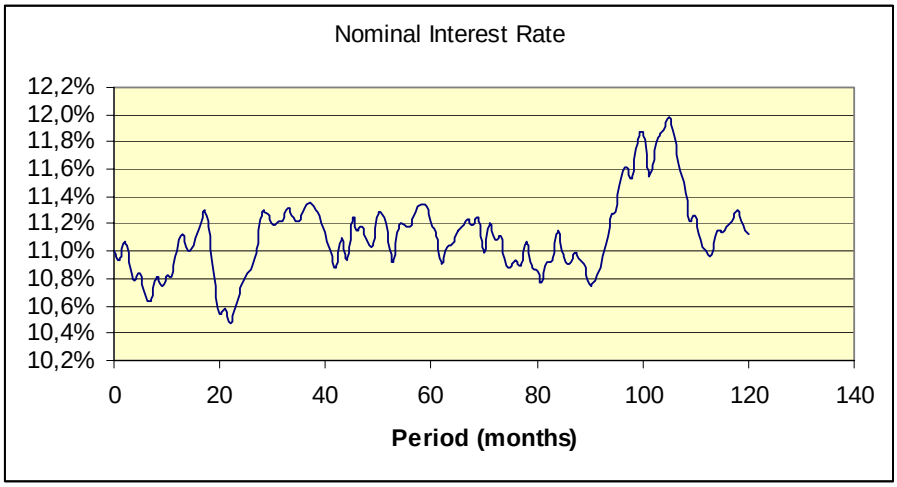
Period	average DL (determ.)	Average DL (stoch.)	Volat	Relat. Volat
0	51,70%	51,70%	0,00%	0,00%
1	49,94%	49,93%	4,25%	8,50%
2	48,19%	48,09%	6,10%	12,69%
3	46,04%	46,00%	7,68%	16,69%
4	43,87%	43,78%	8,86%	20,25%
5	41,53%	41,46%	9,90%	23,89%
6	39,17%	39,16%	11,03%	28,18%
7	36,91%	36,85%	12,21%	33,12%
8	34,35%	34,26%	13,24%	38,65%
9	31,68%	31,65%	14,23%	44,97%
10	28,68%	28,39%	14,74%	51,91%

Besides Table 7, Graphs (15) and (16) show the nominal GDP growth and the nominal interest rate evolution (monthly) in the stochastic environment (in mean terms).

Graph 15: Nominal GDP growth



Graph 16: Nominal Interest Rates



The third and the fourth exercises, instead of using a fixed portfolio (100% floating rate bonds), they consider refinancing's effect over the net debt evolution. Again, there is a deterministic and a stochastic approach to evaluate the results.

We used the same macroeconomic hypothesis to the first exercise but adopted a strategy of changing the debt composition in order to increase the fixed-rate share by 1% a month. Table (8) shows the net public debt evolution.

Table 8: Deterministic x Stochastic scenarios generation (with refinancing strategy)

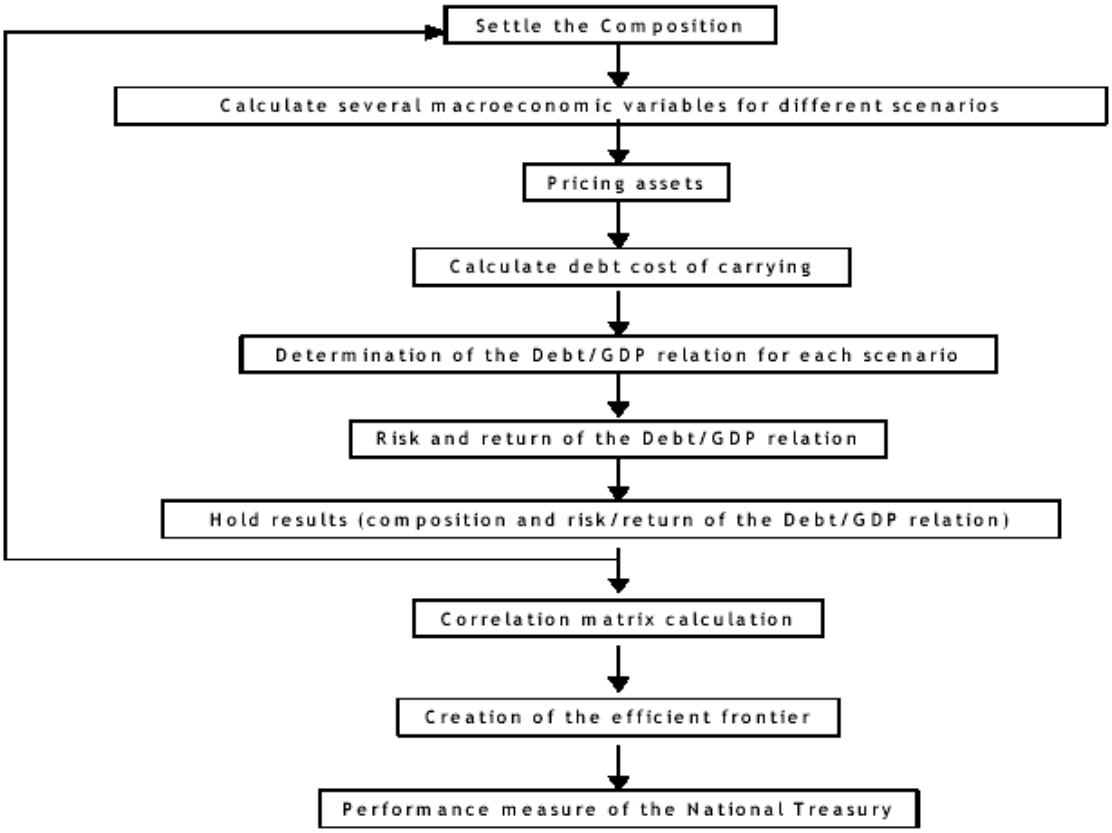
Period	average DL (determ.)	Average DL (stoch.)	Volat	Relat. Volat
0	51,70%	51,70%	0,00%	0,00%
1	50,06%	50,11%	1,49%	2,98%
2	48,18%	48,22%	2,13%	4,42%
3	46,39%	46,43%	2,69%	5,80%
4	44,53%	44,58%	3,14%	7,03%
5	42,66%	42,70%	3,52%	8,23%
6	40,51%	40,55%	3,91%	9,64%
7	38,48%	38,52%	4,33%	11,23%
8	36,36%	36,40%	4,69%	12,89%
9	34,34%	34,37%	5,06%	14,73%
10	32,41%	32,44%	5,51%	16,99%

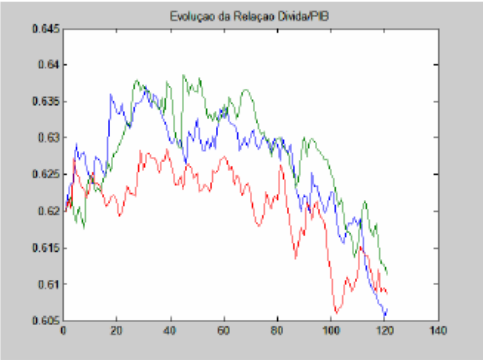
As expected, the floating rate strategy is less costly than the fixed-rate one. However, its uncertainty is much higher. If we consider 2 standard deviations, while the fixed-rate strategy could lead the net public debt to values around 43.5% of GDP, in the case of floating-rate strategy these values are around 57,9%. Therefore, depending of the risk aversion, 2.8% is not expensive to protect against an increase of almost 14% of GDP.

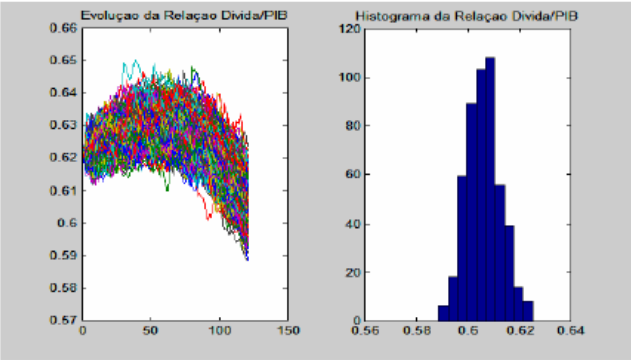
Annex 3 – the long-term Benchmark

Brazil has done some benchmark studies using two alternative approaches to model the economic environment. One using correlated stochastic processes (interest rate models, Brownian motions, etc.); and the other using a macro-structural model (similar to models used by Central Banks for monetary policy purposes).

The diagram and charts below illustrate the different stages of the simulation process.







The first chart above (on the left-hand side) illustrates just a few Monte Carlo simulations of the path of debt to GDP ratios, given one debt financing strategy, over an horizon of 120 months. Looking at any single path one can observe how the volatility of such ratio can be captured by

the stochastic processes of the macro variables that affect the debt dynamics. In practice, however, these simulations are run several (hundreds or thousands) times for each debt financing strategy (see charts on the right-hand side). Through this procedure one is able to build a distribution of expected debt to GDP ratios and find the corresponding average expected cost and risk of each strategy. The results of several strategies can, therefore, be plotted so as to reflect their tradeoffs and an efficient frontier drawn.

In the first approach, we use some stochastic financial models to describe the evolution of the economic indicators. We assume that nominal interest rates follow a CIR model:

$$dJ_t = \alpha(J^* - J_t)dt + \sigma_1\sqrt{J_t}dz_t^1$$

where: J_t: nominal interest rate at time t

α : mean reversion velocity

J^* : long-term mean of interest rates

σ_1 : interest rates volatility

dz_t^1 : Wiener process

From the model, we know that the price of a nominal bond will be given by:

$$P(t,T) = A(t,T)e^{-B(t,T)J}$$

where: $$B(t,T) = \frac{2\left(e^{\gamma(T-t)} - 1\right)}{(\gamma+\alpha)\left(e^{\gamma(T-t)} - 1\right) + 2\gamma}$$

$$A(t,T) = \left[\frac{2\gamma e^{(\alpha+\gamma)(T-t)/2}}{(\gamma+\alpha)\left(e^{\gamma(T-t)} - 1\right) + 2\gamma}\right]^{2\alpha J^*{}_1/\sigma_1^2}$$

$$\gamma = \sqrt{\alpha^2 + 2\sigma_1^2}$$

For the real exchange rates, a CKLS model is assumed to hold:

$$dC_t = \beta(C^* - C_t)dt + \sigma_2 C_t dz_t^2$$

where: C_t: real exchange rate at time t

β : mean reversion velocity of the real exchange rate

C^* : long-term real exchange rate

σ_2 : exchange rate volatility

dz_t^2 : Wiener process

The domestic price index follows a geometric Brownian motion:

$$dI_t = \mu I_t dt + \sigma_3 I_t dz_t^3$$

where: I_t : price index at time t

μ : mean growth rate of price index

σ_3 : price index volatility

dz_t^3 : Wiener process

The external price index is deterministic and the processes are correlated by the Cholesky method. The floating rate bond is assumed to be sold at par value and the coupon of the inflation linked and the exchange-linked bonds are given by:

Inflation coupon = nominal rate – inflation expectation

where inflation expectation is given by $E\left[\frac{dI_t}{I_t}\right] = \mu dt$

exchange rate coupon = nominal rate – exchange rate expectation

where exchange rate expectation is given by $E\left[\frac{dN_t}{N_t}\right] = \left[\beta\left(\frac{I_t}{I_t^e}\frac{C^*}{N_t} - 1\right) + \mu - \mu^e\right]dt$

In a complementary approach, we use a small macroeconomic structural model to describe the evolution of the economic indicators:

$$y = \beta r_{-1} + \delta e_{-1} + \lambda y_{-1} + \varepsilon$$

$$\pi = \zeta \pi_{-1} + \alpha y_{-2} + \gamma(e_{-1} - e_{-2}) + \eta$$

$$e = \chi Embi + \nu$$

$$Embi = \kappa Embi_{-1} + \varpi(Divida / PIB)_{-1} + \nu$$

$$r = \rho r_{-1} + \psi(\pi_{-1} - \pi^*) + \phi y_{-1}$$

where: y: GDP

r: interest rate

π: inflation

e: exchange-rate

Divida/PIB: Debt/GDP

Currently, the Brazilian middle-office is improving both models. In the first approach, new processes are being modeled for the external interest rates in different currencies to incorporate external debt into the model. Also, we are working with different maturities for each type of bond and making the primary surplus endogenous. In the macro-structural approach, we are working in a more robust model, which turns to be a real challenge for an emerging economy

Annex 4 – Methodological Differences between Average Life and Average Maturity

Although there is a consensus in the Brazilian Debt Office that the most correct way to evaluate the average life is by a formula very similar to a Macaulay Duration, recently, we had to incorporate the average maturity to make Brazilian debt comparable to other countries.

Equation (10) below expresses the methodology used by the Brazilian Debt Office to assess the public debt average life. As mentioned before, this formula is very similar to a Macaulay Duration and to our "Refixing" Duration. The most important differences are the interest rate used to discount cash-flows (Macaulay Duration) and the weight factor - T_i (Refixing Duration).

$$AL = \frac{1}{PV} x \sum_{i-1}^{n} PV_i x T_i \qquad (10)$$

In the case of the Brazilian Average Life, we use the original issue bond yield. Besides, the variable T_i is always the time between now and each of the cash-flows (coupons and principal).

On other hand, the average maturity only considers the principal payments for each bond. Because of it, this methodology usually has a bigger value than the average life. However, as mentioned before, since this methodology does not consider the intermediate coupons, it is not adequate to assess refinancing risk and should be only used to make the Brazilian Debt comparable to other countries.

$$AM = \frac{1}{PV} x \sum_{i-1}^{n} PV_i x M_i \qquad (11)$$

Where, M_i corresponds to the time between now and the bond maturity.