Modeling and Managing Sovereign Risk in a Jumpy World

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Summary

Fiscal sustainability requires governments to maintain access to financial markets to fund budget deficits and rollover maturing debt obligations. Policy makers must make decisions in a world with volatile economic, financial, and political conditions where some factors are beyond their control. Barnhill and Kopits (2004) ("BK") develop a Value-at-Risk approach for assessing the probability of Sovereign financial stress. The BK methodology has the advantage of systematically accounting for many sources of risk. However it also has certain limitations including the assumption of constant trends, volatilities and correlations, the requirement to specify analytical models for many government asset and liability accounts, and a simplistic government budget model.

This paper addresses the limitations of BK by: (1) developing a government budget model that incorporates market expectations, (2) greatly simplifying the government balance sheet, and (3) modeling jumps to plausible alternative states of the world with substantially different risk characteristics (i.e. jumps in mean returns, volatilities, correlations, and government policies). Government assets are modeled as the present value of the maximum future primary surpluses that the market expects that the government can and will run if necessary to avoid default on its debt. Government liabilities are modeled as the existing debt portfolio that is specified in substantial detail[3]. Government net worth is modeled to be the difference between the present value of primary surpluses and the value of the debt. Sovereign risk assessments are undertaken with portfolio simulations using a number of correlated stochastic variables (e.g. interest rates, exchange rates, GDP, etc.) as well as the market's expectations regarding maximum acceptable tax rates, minimum acceptable expenditures, and required market risk premium. The average trends, volatilities, and correlations of all of these variables as well as government policy decisions are allowed to change significantly under alternative states of the world.

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[2] We wish to thank the Ministry of Finance in the Government of Brazil for providing publicly available data for the study. This research is based on data from June 2004 and is put forward to illustrate an analytical methodology. The reported results should not be taken as a current, April 2006, risk assessment for Brazil. In a number of ways the actual outcomes for Brazil over the 2004-2006 period have been favorable including a downward trend in the debt to GDP ratio, a significant reduction in interest rate spreads, and a strengthening in the foreign exchange rate. Please do not quote without permission. The author would appreciate comments and suggestions.

[3] This detailed debt portfolio modeling allows for the explicit treatment of crucial risk variables (e.g. exchange rates, interest rates, various other spreads and indices). It is also possible to elaborate the current model to include contingent liabilities and a more complex asset and liability structure along the lines of Barnhill and Kopits (2004).

Initially the paper illustrates a combined scenario and simulation modeling methodology to generate potential distributions of future Sovereign risk indicators (e.g. debt to GDP ratio[4], net worth to GDP ratio, etc.) with attached probabilities under alternative states of the world (i.e. scenarios)[5]. We demonstrate the importance of accounting for: (1) the volatility and correlations of important risk variables, (2) the occurrence of periodic stressful periods in the global economy and financial markets, (3) the market's perceptions regarding the Sovereign's flexibility and commitment to adjust future tax rates and spending levels, if required, to meet debt service obligations, and (4) the detailed structure of the Sovereign debt portfolio. In the combined scenario and simulation analytical framework, decision makers are presumed to make decisions based on multiple objectives (e.g. in the "Optimistic", "Base", and "Stress" scenarios the objective is to reduce the probability of loss of market access to no higher than X%, Y%, and Z% respectively).

With specification of probabilities for alternative states of the world we also demonstrate the capability to implement a jump process[6] which produces one overall risk assessment for sovereign debt sustainability. In this analytical framework decision makers are presumed to make decisions based on a single objective (e.g. reduce the overall probability of loss of market access to no higher than W%). We believe that such adverse outcome probability estimates are useful to better inform fiscal policy decisions and manage the probability of sovereign financial stress. In addition to the advantages previously noted the methodology also has the potential to be incorporated into an integrated sovereign and systemic financial system risk assessment (see Barnhill and Souto (2006)).

Kaminsky, Reinhart, and Vegh (2004), Mendoza and Oviedo (2005), and others note that in volatile environments vulnerable governments frequently follow pro-cyclical fiscal policies. Our model suggests that if debt levels are already high policy makers may have few other alternatives to maintaining market access during stressful periods. We also show that the correlation between the value of the government's debt portfolio and present value of future primary surpluses is a significant risk variable that can be managed in part by the decisions governments make on the structure of the debt.

Our illustrative analysis finds that under current conditions and policies the probability of Brazil facing restricted access to financial markets is low. However should a period of heightened global economic and financial stress arise Brazil could face a significant probability of restricted (or very costly) market access. To reduce such risk and provide policymakers with greater financial flexibility during stress periods, Brazil would appear to need to continue its current policy of running a significant primary surplus in order to reduce its debt to GDP.

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[4] Adrogue (2005), also models distributions of future debt to GDP ratios for Central American countries in a useful and insightful demonstration of the Value-at-Risk methodology applied to fiscal sustainability assessment.

[5] The ValueCalc software packages, copyright FinSoft, Inc. 1996-2005, were used to undertake the risk analyses reported in this study.

[6] Our Jump process has features similar in spirit to both jump diffusion models (see Merton, 1976), and stochastic volatility and correlation models (see Engle, 2002). The probabilities of the alternative states of the world occurring as well as the changing values of the risk variables (e.g. mean growth rates, volatilities, and correlations) under the alternative states of the world may be estimated from historical data and expert opinion.

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I. Introduction

In the past decade, an increasing number of capital-account crises have brought into focus the potential damage inflicted by a vulnerable public sector. Thus, policymakers, academic observers, financial institutions, and credit rating agencies have become increasingly interested in gauging the vulnerability of the public sector particularly in emerging market economies. Recently, in some of these countries, the concern of policymakers has translated into a formal legislative obligation for the government to prepare periodic evaluations of fiscal risk.[7] Scenario analysis and simulation modeling are widely used risk assessment methodologies.[8] This paper represents a blending and extension of these two approaches in an effort to provide a more comprehensive and useful method for modeling and managing fiscal risk, in response to the widespread interest in crisis prevention.

Barnhill and Kopits (2004) note that typically, evaluations of fiscal risk are predicated on assessments of fiscal sustainability conducted on the basis of medium- to long-term scenario calculations of public indebtedness—under plausible assumptions about future macroeconomic trends, supplemented with demographic and other environmental prospects. Such assessments of fiscal sustainability rest in the integration of the flow of government balance and the stock of net indebtedness over a multi-period horizon.

A very simplified formula for calculating the net worth of the public sector at t = 0 is:

$$\begin{aligned} W_0 &= PV(Z) - PV(C) - B_0 \\ &= \sum_{t=0}^{\infty} (1+r)^{-t} Z_t - \sum_{t=0}^{\infty} (1+r)^{-t} C_t - \sum_{t=0}^{\infty} (1+r)^{-t} B_t \end{aligned} \tag{1}$$

Z_t is defined as the primary balance in period t. In addition, the flow of expected net unfunded contingent liabilities (C_t) reflects both the distribution of potential liability amounts as well as their probability of occurring in time t. The outstanding debt stock is replaced by the discounted future net amortization schedule (B_t)[9].

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[7] In Brazil, under the Fiscal Responsibility Law of 2000, the authorities are required to prepare, as part of the yearly budgetary guidelines, an evaluation of fiscal risks, including (a) estimates of the quantitative impact (along with its probability) resulting from government decisions and other conditions; (b) estimates of government guarantees and other contingent liabilities; and (c) actuarial, financial and economic assessment of public pensions, unemployment and other workers' insurance schemes, and various contingency funds.

[8] See Barnhill and Kopits (2004), and Adrogue (2005).

[9] The methodologies illustrated in this paper are well suited to deal with important correlated risk variables affecting the market value of a government's debt (i.e. stochastic exchange rates, domestic and foreign interest rates, interest rate spreads, and various indices). These methodologies can also model correlated contingent Sovereign liabilities. Such unfunded contingent liabilities may arises in connection with social security programs, deposit insurance schemes, insurance for natural disasters, or other explicit and implicit government guarantees. The realization of such contingent liabilities is affected by the level of economic activity, demographic trends, effectiveness of bank supervision, adequacy of bank capitalization, the frequency and

The implementation of (1) involves some interesting conceptual questions. First when estimating a Sovereign's net worth will market participants calculate the present value of expected primary surpluses generated by the existing fiscal system, comprised of the tax structure and mandatory spending programs prevailing in $t = 0$, that is, excluding the effect of any unanticipated discretionary action or adjustment in a future period? Alternatively will they make an assessment of the maximum potential primary surpluses that they believe that the Sovereign is capable and willing to generate through changes in tax rates and expenditure levels if required to meet debt service obligations[10]? Current actual primary surplus levels clearly contribute to increases or reductions in Government debt levels in the short-run. However potential primary surplus levels would seem to be the appropriate focus when assessing debt sustainability[11].

If $W \geq 0$, the public sector is deemed to be *solvent* and the intertemporal budget constraint is fully satisfied. In a risk-less environment characterized by certainty of expectations, the government's financial position is sustainable if the present value of the expected future fiscal surpluses is greater than or equal to the current debt outstanding. However, a major shortcoming of this statement is that it refers to expected outcomes, without accounting for downside risk, including the possibility that the government may lose access to financing if lenders' perception of W becoming negative reaches an unacceptable level[12].

Whereas this approach provides some indication of the vulnerability of the public sector, the wide margin of uncertainty inherent in scenario calculations and in summary indicators impairs the usefulness of (1) for evaluating fiscal risk. This is a major shortcoming in the case of emerging market economies that may be particularly vulnerable to sudden shifts in market sentiment in a volatile economic and financial environment. Attempts to correct for this deficiency by performing sensitivity tests with respect to marginal shifts in key risk variables (without accounting for volatilities and correlations), or to arbitrary shocks, are of limited value.[13]

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severity of natural disasters, etc. However for convenience the present application to Brazil will not model government contingent liabilities. For an analysis of correlated sovereign and systemic bank risk see Barnhill and Souto (2006).

[10] Mendoza and Oviedo (2005) deal extensively with the topic of governments attempting to avoid sharp adjustments in payments to private agents despite large fluctuations in public revenues. Their model generates a negative relationship between average public debt and revenue variability. Our risk assessments support this conclusion.

[11] One should of course not overlook the possibility a Government's actual primary surplus (deficit) levels may have a significant impact on the market's assessment of the maximum primary surplus levels that the Government can reasonably be expected to be able to achieve (i.e. in some cases "action speaks louder than words".)

[12] With a bit of algebra it can be shown that the permanent primary surplus needed to maintain the net debt-GDP ratio stable (or non-increasing) over time is given by the following well know equation:

$$z^* \geq \left(\frac{r-g}{1+g}\right) b_{t-1} \tag{2}$$

To address this limitation Barnhill and Kopits (2004) implement a Value-at-risk (VaR)[14] methodology for simulating the future value of the Government of Ecuador's assets, liabilities, and net worth. This approach systematically accounts for the volatility and correlations of many variables (including oil prices and revenues)[15]. It importantly also produces estimates of the probability of sovereign financial stress in a given economic financial, economic, and policy environment. However BK has certain limitations. In particular they require the development of detailed analytical models for valuing many types of Government assets and liabilities, assume constant volatilities and correlations, and have a simplistic budget model.

This present paper addresses some of the limitations of the BK methodology. The basic approach is to combine a conventional scenario approach, together with simulation modeling of a simplified Government asset and liability structure to assess fiscal sustainability[16]. In essence, this approach allows us to postulate alternative future financial, economic, and policy environments and then simulate plausible distributions of resulting Sovereign risk indicators (e.g. Debt to GDP ratios, Government net worth, etc.). This methodology has several advantages. First it allows for an explicit treatment of periodically observed shifts in the "state of the world" where many factors (e.g. volatilities, correlations, trends) change by substantial amounts[17]. Second the simulation portion of the analysis allows the estimation of the probability of certain adverse outcomes contingent on the state of the world assumed in the analysis. Ultimately, application of such an approach should be helpful for estimating the probabilities of Sovereign financial distress and loss of reasonable market access as well as helping identify the fiscal adjustment and other adjustments needed to compensate for risk and thus ensure fiscal sustainability at a prescribed confidence level in alternative plausible states of the world (i.e. scenarios).

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[13] In the formulation of a standardized framework for assessing external and fiscal sustainability by the Fund staff, it is acknowledged that "assessments of sustainability are probabilistic, since one can normally envisage some states of the world under which a country's debt would be sustainable and others on which it would not. But the proposed framework does not supply these probabilities explicitly; rather, it traces the implications of alternative scenarios and leaves the user to determine the probabilities.... The framework also proposes a set of sensitivity tests, but further work will be necessary to settle on a precise calibration"; see International Monetary Fund (2002, p. 25). The sensitivity analysis suggested for baseline projections of the public debt ratio include: historical averages, as well as one and two standard deviation shocks in the growth rate, rate of interest and primary balance; a 30 percent devaluation; and a 10 percent increase in debt-creating capital inflows.

[14] See Jorion (2001) for a detailed discussion of Value-at-Risk analysis.

[15] In the case of Brazil we have not had the opportunity to develop a more detailed asset and liability structure. This would be a very useful future research topic.

[16] In principal the proposed methodology is well suited to handle much more detailed asset and liability structures.

[17] Alternatives to the current methodology include stochastic volatility and correlation models (see Barnhill and Souto, 2006).

In such an analytical framework risk management decisions could take on the form of adopting policies which produce an adverse outcome (e.g. Debt to GDP ratio above a selected level) less than x% of the time in the base case and less than y% of the time in the stress case.

With the specification of probabilities for jumps to alternative states of the world one overall risk assessment is achievable in which the overall probability of an unfavorable outcome (e.g. negative net worth) can be calculated. This combined scenario / simulation methodology should be most useful for measuring and managing Sovereign risk in emerging market economies.

The paper is organized as follows. Section II discusses the combined scenario / simulation modeling methodology. Section III presents the Brazil scenario/simulation risk assessment. Section IV presents the Brazil jump processrisk assessment. Section V provides some concluding observations and suggestions for future research.

II. Scenario / Simulation Model[18]

In the current illustrative risk assessment applied to Brazil we define three alternative states of the world which will be identified as the "Optimistic", "Base", and "Stress" scenarios[19]. These scenarios make alternative assumptions on the volatility and level of GDP growth and selected other key policy and other variables affecting the assessment of Brazil's debt sustainability. Subsequently for each of these scenarios we undertake simulation analyses for GOB debt risk indicators assuming a one-year time step. Our general approach is to simulate the future financial and economic environment at a pre-set horizon (e.g. 1 year) as a set of correlated stochastic variables (GDP, interest rates, exchange rates, inflation rates, etc.). For each simulated future financial environment and assumed scenario variables we:

- Revalue each debt instrument in the GOB Debt portfolio[20],
- Estimate the GOB primary surplus,
- Estimate the GOB total debt level,
- Calculate the GOB debt to GDP ratio,
- Estimate the present value of the GOB potential future primary surpluses,
- Estimate the GOB net worth,
- Estimate GOB net worth to GDP ratios,

By repeating the simulation a large number of times we develop distributions of simulated values which allow us to estimate the probability of adverse outcomes (e.g. a negative Government net worth).

Alternative Scenarios

Table 1 below gives the assumptions on three alternative states of the world (i.e. scenarios). The "Base" scenario is generally thought to be a reasonable representation of conditions as of March 2006. The "Stress" scenario represent conditions that are similar to those that Brazil has experienced over the period 1997 through 2004 where a number of well know stress events occurred[21]. The optimistic scenario represents estimates of favorable future conditions that are potentially achievable with continued good policy decisions.

Table 1: Alternative Scenarios

[18] See Appendix 1 for a technical discussion of the simulation models used in this study.

[19] We would again like to thanks members of the Government of Brazil Ministry of Finance staff for very helpful discussion on reasonable scenario assumptions as well as data that allowed the estimation of stochastic variable volatilities and correlations.

[20] The data used in this analysis reflect a detailed modeling of the various types and amounts of debt in the GOB debt portfolio as of June, 2004.

[21] These events included the Russian and Asian financial crises, the floating of the foreign exchange rate, the default by Argentina, and the financial turmoil surrounding the previous Presidential election.

Scenario	Optimistic	Base	Stress
Max Tax Rate (% of Nominal GDP)	0.37	0.37	0.37
Target Tax Rate (% of Nominal GDP)	0.36	0.36	0.36
Target Non-Financial Expenditure Rate (% of Nominal GDP)	0.305	0.315	0.325
Target Primary Surplus (% of Nominal GDP)	0.055	0.045	0.035
Maximum Feasible cut in Non-financial Expenditures (% of Nominal GDP)	0.05	0.03	0.01
Expected Nominal GDP Growth Rate (%)	0.1	0.08	0.06
GDP Volatility (%)	0.015	0.025	0.035
Weighting for current GDP Growth rate in calculating the Expected long-run GDP Growth Rate	0.1	0.1	0.1
Assumed Discount rate on Perpetuities (r-g)	0.02	0.04	0.07

In addition the above scenarios we will also look at a variety of scenarios that represent incremental modifications to the Base and Stress scenarios.

GOB Debt Portfolio Model

The Government of Brazil has a wide variety of different types of securities outstanding including fixed rate, variable rate, FX indexed, inflation indexed, and foreign currency denominated debt (Global Bonds, Euro Bonds, Yen Bonds, Brady Bonds, IBRD/IDB/IMF loans, floating rate loans, zero coupon fixed rate loans, NTN-C, NTN-B, NTN-D). These instruments also have various maturities ranging from very short-term to 2040. To adequately capture the risk characteristics of this diverse portfolio of debt instruments we modeled 149 separate securities selected to closely replicate the value of the total portfolio.

Also reflecting the anticipated debt refunding strategy for the GOB we assumed that:

- Debt that matures within one year is refunded with fixed interest rate and inflation index securities,
- The amount of fixed rate debt approximately doubles,
- The amount of inflation indexed debt goes up by approximately 67%,
- The amount of floating rate debt goes down by about 5%.

Table 2 below gives a distributional analysis for the various types of debt in GOB Debt Portfolio as of June, 2004.

Financial and Economic Environment Model

In addition to the above scenario assumptions and debt portfolio model we also require volatility and correlation estimates for other financial and economic environment variables used in the simulations. These variables were selected to include all variables required to value each of the different types of debt in the GOB Debt Portfolio. In total we model 20 Correlated stochastic variables including 3 economic indicators (GDP plus two inflation indices), 4 term structures (Selic, US Government, Euro, Japanese Government), 4 exchange rates ($R versus USD, Euro, and Yen), 9 interest rate spreads (GOB vs. USG, GOB Blended rate vs. USG, LIBOR vs. USG, GOB vs. LIBOR, GOB vs. Euro, GOB vs. Japanese Gov, IPCA Index vs. Selic, IPG_M vs. Selic, FX Index vs. Selic. The volatilities and correlations for these variables were estimated from data provided by the GOB Ministry of Finance for the period January 1999 to July 2004. Appendix 2 provides information on the simulated distributions of selected financial and economic variables.

Table 2: GOB Debt Portfolio Composition as of June 2004[22]

Type of Security	Fraction of total
Floating Rate Debt	0.475
Fixed Rate Debt	0.138
Global bonds (USD Denominated)	0.105
Inflation Indexed Debt	0.072
World Bank/IBD/IMF (USD Denominated)	0.059
Exchange Rate Indexed Debt	0.054
Brady Bonds (USD Denominated)	0.050
Inflation Indexed Debt	0.022
Euro Denominated Bonds	0.021
Yen Denominated Bonds	0.004
total	1.000

[22] From June 2004 to April 2006 the Brazilian debt profile has changed significantly. Concerning the domestic debt composition, the share of fixed rate bonds has increased to 28%, and the share of inflation indexed bonds increased to 20%. In the same period, the percentage of FX linked bonds fell to 2%, and floating rate bonds to 47%. Concerning domestic debt maturities, Brazil is now selling bonds maturing in 2045 (NTN-B). The National Treasury is engaged in a program to buyback short-term maturities of external debt (until 2010) and has already bought more than USD 4 billion. Further all Brady bonds will be called on April 15, 2006.

Present Value GOB Primary Surplus

We also wish to model the present value of the Government's Maximum Feasible Primary Surplus (PVMFPS) at the end of the simulation time step (t=1). To do this we will define several models.

First we model the maximum feasible revenue in period t (MFR_t) that the *market expects* that the GOB is capable or willing to generate, if required, in order to avoid defaulting on its debt service obligations[23].

$MFR_t = MTR*GDP_t$ (3)

Where:

MTR = the maximum tax rate that the market expects that the Government can or will sustain if required to service its debt.

GDP_t = the expected GDP in period t.

We assume that at t=0 the market has an expectation regarding the long-term growth rate in GDP (EG_0). We also assume that the market's expected GDP growth rate at t=1 is a weighted average of EG_0 and the realized GDP growth rate between t=0 and t=1 (G_1). Thus:

$EG_1 = EG_0\ (1\text{- lambda}) + G_1\ \text{lambda}$ (4)

Lambda = the assumed weighting of the next period's GDP growth rate for the purpose of calculating the EG at the end of the next period.

Thus at t=1

$MFR_2 = MTR*GDP_1*e^{(EG1)}$ (5)

Further using the model for a perpetual annuity growing at a constant rate the present value of the Government's maximum future revenue stream at t=1 is[24]:

$PVMFR_1 = MTR_2/(R\text{-}EG)$ (6)

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[23] In principal there is no reason that much more detailed models with yearly estimates of multiple revenue streams (i.e. natural resource sales, state owned enterprises, etc, residual GDP) with varying tax rates could not be estimated and implemented. Similarly yearly estimates of multiple expense streams could be extremely useful and are certainly feasible to implement. This is essentially the approach adopted by Barnhill and Kopits (2004).

[24] The use of the growing perpetuity model is strictly for convenience as well as a lack of data to estimate maximum future revenues and minimum future expenses year by year long into the future

For model stability reasons we will assume a constant discount rate (R-EG) for the perpetually growing annuities. This is equivalent to assuming that the market's discount rate (R) has a constant spread over EG (i.e. R-EG=constant).

We also need to model the present value at t=1 of the perpetual annuity of minimum future non-financial payments ($PVMFP_1$) that the *market expects* that the Government will feel required to make even if it is forced to default on its debt obligations.

$$PVMFP_1=GDP_0\exp(EG_0*2)(TNFER-MCNFE)/(R-EG) \quad (7)$$

Where:

TNFER = the target non-financial expenditure rate as a percent of expected GDP for the next period (e.g. one-year).

MCNFE = the maximum feasible cut in non-financial expenditures as a percent of GDP that the market expects that the Government can and will sustain if required to service its debt.

Thus in effect we are assuming that at t=0 the market forms expectations about the present value of minimum future Government payments that do not change during the course of the simulation. Of course in alternative states of the world (i.e. different scenarios) the variables affecting $PVMFP_1$ would change and it would have different values.

The present value of the Government's Maximum Feasible Primary Surplus (PVMFPS) at t=1 is:

$$PVMFPS_1= PVMFR_1 - PVMFP_1 \quad (8)$$

Government "Net Worth"

The government's net worth is a risk indicator of considerable interest however it is unobservable. Thus we define a proxy value.

The Government's "Proxy Net Worth" at t=1 is:

$$\text{Proxy Net Worth} = PVMFPS_1 - \text{Market Value of Debt}_1 \quad (9)$$

Where:

Market Value of Debt_1 = the sum of the market values of all of the various outstanding debt issues at t=1.

Target Primary Surplus

We also utilize the Government's Target Primary Surplus (TPS) in our analysis.

$$TPS = TTR - TNFER \tag{10}$$

Where:
TTR = Target Tax Rate expressed as a percent of GDP
TNFER = Target Non-Financial Expenditure Rate expressed as a percent of GDP

The realized primary surplus will of course directly affect the level of debt that the Government has outstanding at the end of the simulation time step (e.g. t=1). Large primary surpluses may, overtime, reduce a government's debt to GDP ratio and provide policy makers additional financial flexibility. Inadequate primary surpluses (deficits) can result in growing debt levels that over time increase the risk of sovereign financial stress. Of course the Government's realized primary surpluses may also have a significant impact on the formation of the market's expectations regarding the maximum potential surpluses that the Government may be willing and capable of generating in the future if required.

III. Brazil Risk Assessment: Scenario/Simulation Approach

In this section we report the results of the combined scenario and simulation analyses. In the next section we will also combine the Optimistic, Base, and Stress analyses into one overall risk assessment via a jump process. The reader should keep in mind that these are preliminary assessments that are put forward to illustrate a methodology. Also it should be kept in mind that there is no definitive mapping of the level for risk indicators (e.g. Debt to GDP ratio, or Government net worth) with the severity of any loss of market access.

Nominal local currency GDP levels and volatility are important risk drivers in the current risk assessments. Table 3 below provides a distributional analysis for the simulated GDP levels for the various scenarios. In each case we start the simulation at 6/30/2004 and end it at 6/30/2005. These simulations confirm that the simulated GDP levels are consistent with the input assumptions made in the various scenarios. Also only in the stress case is there a significant probability of negative nominal GDP growth when measured in local currency[25,26].

Table 4 gives a distributional analysis for the GOB's simulated Debt to GDP ratio at a one-year time step (i.e. 6/30/05) under the alternative scenarios. These distributions reflect *correlated* fluctuations in the value of the GOB's debt portfolio and GDP levels. Fluctuations in the value of the debt portfolio reflect the initial composition of the GOB's debt portfolio, changes in the value of this portfolio resulting from fluctuations in interest rates, interest rate spreads, FX rates, and the size of the realized primary surplus (deficit). The distributions of Debt to GDP ratios get more dispersed and larger at any given confidence level as one moves from the Optimistic scenario to the Stress scenario. These simulations indicate a moderate probability of a significant increase in the GOB Debt to GDP ratio particularly if a Stress scenario should develop.

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[25] The mean and standard deviation of nominal local currency GDP growth for the Base scenario were estimated by the GOB Ministry of Finance based on surveys of market participant expectations. For the Stress scenario estimates for these variables were calculated from historical data over the 1996 to 2004 period. For the Optimistic scenario estimates for these variables were assumed to be consistent with Brazil's economy continuing on a path leading to sustained and relative stable GDP growth.

[26] Appendix 2 contains the simulated distributions for selected other financial and economic variables (e.g. interest rates, and FX rates) as well as the distribution of one-year cost for the GOB Debt Portfolio (i.e. interest payments plus change in market value). These simulated distributions also match up with the model input assumptions.

Table 3: Simulated Brazil GDP Growth Rates As of 6/30/05

	Optimistic	Base	Stress
Mean	0.100	0.080	0.059
Std	0.015	0.025	0.035
Max	0.141	0.155	0.174
Min	0.059	0.004	-0.056
Percentile Distribution			
0	0.059	0.004	-0.056
0.01	0.066	0.020	-0.024
0.025	0.070	0.030	-0.009
0.05	0.075	0.037	0.000
0.1	0.080	0.048	0.013
0.5	0.100	0.080	0.059
0.9	0.120	0.111	0.106
0.95	0.125	0.122	0.119
0.99	0.134	0.139	0.142
1	0.141	0.155	0.174

The GOB has increasing flexibility in the types of securities it chooses to issue. The last column in Table 4 provides a distributional analysis of future Debt to GDP ratios based on the assumption that the entire debt of the Government of Brazil was financed with one-year fixed rate local currency debt. This distribution can be seen to have a marginally higher mean Debt to GDP ratio (reflecting marginally higher average costs of financing relative to the actual Debt portfolio) but also a lower standard deviation and lower extreme values for the Debt to GDP ratio. This reduced risk profile reflects the fact that if Brazil finances only with one-year fixed rate domestic currency debt then one-year from now the value of the initial GOB debt portfolio will not be affected by fluctuations in exchange rates, domestic or foreign interest rates, interest rate spreads, or other indices[27]. Thus the structure of the Government's debt portfolio can be a significant policy choice when assessing and managing Sovereign risk[28].

Table 4: Simulated GOB Debt[29] To GDP Ratio As of 6/30/05

[27] Of course while the market value of short-term debt fluctuates much less than longer-term fixed rate debt its year to year interest expense is volatile.

[28] Adrogue (2005) undertakes an extensive VaR analysis of debt to GDP ratios for Central American countries. The analysis presented in this paper support Adrogue's conclusion that GDP growth, inflation, primary fiscal spending, the domestic short-term interest rate, the exchange rate, foreign interest rates, the sovereign spread, and the structure of the government's debt portfolio are important risk variables affecting the distribution of future debt to GDP ratios.

	Optimistic	Base	Stress	Base with GOB Debt Portfolio consisting entirely of one-year fixed rate local currency securities
Initial (6/30/2004)	0.569	0.569	0.569	0.569
Mean	0.539	0.561	0.585	0.570
Std	0.022	0.026	0.032	0.015
Max	0.613	0.654	0.714	0.633
Min	0.473	0.478	0.490	0.513
Percentile Distribution				
0%	0.473	0.478	0.490	0.513
1%	0.490	0.504	0.509	0.537
2%	0.492	0.509	0.523	0.540
3%	0.497	0.515	0.529	0.543
4%	0.499	0.518	0.533	0.545
5%	0.501	0.520	0.538	0.546
6%	0.504	0.521	0.541	0.547
7%	0.507	0.523	0.542	0.548
8%	0.508	0.524	0.544	0.549
9%	0.508	0.526	0.545	0.550
10%	0.510	0.527	0.546	0.551
20%	0.520	0.540	0.559	0.558
30%	0.526	0.548	0.568	0.563
40%	0.533	0.554	0.577	0.567
50%	0.538	0.561	0.583	0.570
60%	0.545	0.567	0.590	0.573
70%	0.551	0.574	0.598	0.578
80%	0.557	0.581	0.609	0.582
90%	0.566	0.594	0.625	0.590
95%	0.576	0.606	0.639	0.595
99%	0.594	0.621	0.668	0.605
100%	0.613	0.654	0.714	0.633

Table 5 below gives a distributional analysis of average simulated GOB Debt to GDP ratios versus other simulated variables[30] for the Base case. The reported numbers reflect the average values for the variables by deciles (that is the 1'st decile is the lowest 10% of observations for Debt to GDP, …, the 10'th decile is the highest 10% of observations of Debt to GDP). As one might expect intuitively the highest simulated debt to GDP ratios occur when:

[29] In the analysis we are working with the Gross Debt of the GOB. That is we have not deducted financial assets to arrive at net debt.

[30] Goldfajn and Guardia (2005) argue that debt sustainability exercises should be based on medium and long-run scenarios. Interestingly they correctly anticipated that in Brazil the real exchange rate was undervalued (e.g. likely to strengthen) and that real interest rates were high and likely to decline.

- simulated GDP growth is relatively low,
- the FX rate is weakening (causing the local currency value of foreign currency denominated debt to rise), and
- domestic interest rates are declining (causing the value of fixed rate debt to rise).

Table 5: Average Debt to GDP Ratios and Other Variables by Deciles of Simulated Observations

Deciles	GOB Debt to GDP Ratio	Percent Change in GDP	Percent Change in R$ versus USD Exchange Rate	Percent Change in Selic	Inflation Rate
1	0.517	10.6%	2.2%	1.4%	5.9%
2	0.535	9.4%	6.7%	0.8%	6.1%
3	0.545	8.9%	12.0%	0.5%	5.7%
4	0.551	8.5%	11.7%	0.0%	5.9%
5	0.557	7.9%	14.9%	0.6%	6.3%
6	0.564	7.9%	15.2%	-0.3%	6.1%
7	0.570	7.3%	17.0%	-0.5%	5.9%
8	0.577	7.4%	23.8%	-0.3%	6.3%
9	0.586	6.4%	22.8%	-1.3%	6.0%
10	0.608	5.4%	28.1%	-2.1%	6.1%

Adrogue (2005), Pattillo, Poirson, and Ricci (2002), and Reinhart, Rogoff, and Savastano (2003) indicate that while debt to GDP is a widely used risk indicator it is a poor predictor of solvency. We will therefore also focus attention on a government's net worth to GDP ratio.

Table 6 gives a distributional analysis for the GOB's simulated Net Worth to GDP ratio under the Optimistic, Base, and Stress scenarios. Also given is a modified base case scenario where the maximum feasible primary surplus is set equal to the current target of approximately 4.5% of GDP, and a modified Stress scenario where the maximum feasible primary surplus is increased from an assumed 5.5% to 7.5%.

In these simulations Net Worth was calculated to be the difference between the present value of the maximum potential primary surpluses and the value of the Debt Portfolio both simulated at a one-year time step (i.e. 6/30/2005). These distributions reflect variations in all of the variables affecting the present value of Brazil's maximum potential primary surplus, the initial composition of the GOB's debt portfolio, and changes in the value of this portfolio resulting from fluctuations in interest rates, interest rate spreads, FX rates, and the size of the realized primary surplus (deficit). They also reflect the correlations between changes in the present value of the primary surplus and the value of the debt portfolio.

As one moves from the Optimistic to the Base and Stress scenarios the distributions shift so that at any probability level one has a smaller Net Worth to GDP ratio. In the Stress scenario there is a significant probability (on the order of 13%) of achieving a negative net worth.

In the Base with MFPS = 4.5% scenario the entire distribution of Net Worth to GDP ratios shifts downward relative to the Base scenario (where the MFPS is approximately 8.5%). For example the mean Net Worth to GDP ratios decline from approximately 1.74 to 0.65. At the 99% confidence level the Net Worth to GDP ratio declines from approximately 1.2 to 0.04. At something less than a 1% probability the simulated Net Worth to GDP ratio in the modified Base case is negative. The only difference between these two scenarios is the market's expectations regarding the maximum future primary surpluses that the Government is willing and capable to generate if required to service its debt obligations. Clearly market expectations regarding a Government's willingness and capacity to generate future surpluses is an important variable to be considered in assessing and managing Sovereign risk.

In the Stress with MFPS = 7.5% scenario the entire distribution of Net Worth to GDP ratios shifts up relative to the Stress scenario (where the MFPS is approximately 5.5%). For example the mean Net Worth to GDP ratios increases from approximately 0.24 to 0.56. At the 99% confidence level the Net Worth to GDP ratio increases from approximately -0.28 to 0.09. The probability of a negative net worth goes down from approximately 13% to less than 1%. The only difference between these two scenarios is the market's expectations regarding the maximum future primary surpluses that the Government is willing and capable of generating if required to service its debt obligations. In times of market stress vulnerable Governments may find it necessary to adopt pro-cyclical fiscal policies so as to influence market expectations and maintain market access.

Table 6: Simulated GOB "Net Worth" To GDP Ratio As of 6/30/05

	Optimistic	Base (MFPS=8.5%)	Base with MFPS =4.5%	Stress (MFPS = 5.5%)	Stress with MFPS = 7.5%
Mean	5.81301105	1.73628833	0.648731609	0.243281851	0.567512378
Std	0.24546749	0.234120016	0.261308293	0.213586973	0.202248754
Max	6.57478174	2.47927254	1.373022861	0.887856993	1.185908161
Min	5.02289215	0.951699417	-0.124274703	-0.484179862	-0.128652826
Percentile Distribution					
0%	5.02289215	0.951699417	-0.124274703	-0.484179862	-0.128652826
1%	5.24567334	1.197169114	0.04450687	-0.275054387	0.089405205
2%	5.31790875	1.239128401	0.104870737	-0.221017059	0.158698004
3%	5.3639898	1.281827422	0.165247742	-0.184032036	0.191043166
4%	5.39388242	1.324626204	0.184478177	-0.16427854	0.209827039
5%	5.41447111	1.348245594	0.204532914	-0.135538459	0.222955477
6%	5.43070502	1.365924391	0.224505688	-0.103759178	0.238636901
7%	5.4464041	1.381300796	0.255008846	-0.080856589	0.253080102
8%	5.46631047	1.395151073	0.270350463	-0.066247097	0.270327642
9%	5.48217315	1.40824816	0.283565596	-0.051608053	0.280118986
10%	5.49029528	1.423385181	0.29895297	-0.033694574	0.287354834
11%	5.49918694	1.432977153	0.31971072	-0.026089494	0.296947943
12%	5.51152267	1.443571819	0.330267991	-0.013234256	0.309580998
13%	5.52257321	1.454935684	0.347767856	-0.000846388	0.32427083
14%	5.53024348	1.463389671	0.361425093	0.012140421	0.334531978
15%	5.54269411	1.470806033	0.368783395	0.024265728	0.343432162
25%	5.63080014	1.568406497	0.474682603	0.108189199	0.425766265
50%	5.79842327	1.722439381	0.655829658	0.229843936	0.55656848
75%	5.97728437	1.880534382	0.829441734	0.369135153	0.692972838
90%	6.12066083	2.03516428	0.991111737	0.505896195	0.831659427
95%	6.19595092	2.112352084	1.073971393	0.599057136	0.896083167
99%	6.35188568	2.257031685	1.224113165	0.727726076	1.021879966
100%	6.57478174	2.47927254	1.373022861	0.887856993	1.185908161

Table 7 below gives distributional analyses for GOB Net Worth to GDP ratios for various incremental modifications to the Base scenario. A 1% increase in assumed GDP volatility results in a much broader distribution of net worth to GDP ratios. A reduction in expected GDP growth by 1% shifts the entire distribution of Net Worth to GDP ratios to a lower level. A reduction of 1% in the target primary surplus also shifts the entire distribution of Net Worth to GDP ratios lower. Finally a 1% increase in the discount rate the market applies to projected future primary surpluses also shifts the entire distribution of Net Worth to GDP ratios lower. Unfortunately in a Stress period GDP volatility goes up, GDP growth goes down, and market risk premiums go up. All of these factors are largely beyond policy-maker's control. In such a situation an increase in the target primary surplus may be one of just a few potentially effective short-term policy option open to decisions makers[31]. As discussed above vulnerable governments may thus be forced into pro-cyclical fiscal decisions in order to maintain reasonable market access during stressful economic conditions.

Table 7: Simulated GOB "Net Worth" To GDP Ratio As of 6/30/05

	Base	Base with GDP Volatility 1% Higher	Base with Expected GDP Growth 1% lower and Minimum Expenditures held constant	Base with Target Primary Surplus 1% Lower	Base with Perpetuity Discount Rate 1% Higher
Mean	1.73628833	1.730239949	1.552599812	1.455036925	1.27613335
Std	0.234120016	0.33605807	0.237579944	0.247709734	0.19565346
Max	2.47927254	2.770521751	2.394280827	2.264615284	2.02126572
Min	0.951699417	0.606792037	0.640992143	0.599850825	0.47239411

[31] See Kaminsky, Reinhart, and Vegh (2004).

IV. Brazil Risk Assessment: Jump Process Approach

A significant limitation of the above scenario/simulation methodology is that it does not give one overall estimate of the probability of an adverse outcome which could lead to restricted market access. In this section we will endeavor to rectify that shortcoming. In particular we put forward the following Jump process model.

In our Jump process we will assume that:

1. there are a finite number of states of the world,
2. each of these states of the world may have substantially different risk characteristics (i.e. risk variable trends, volatilities, correlations, and government policies) and can be described in enough detail that we can implement a simulation risk assessment such as that described in section III above, and
3. based on an analysis of historical data and expert opinion we can estimate the probabilities of each state of the world occurring over the period of our risk assessment.

As a demonstration of the Jump process methodology we assume that there is a 10% probability for the previously described Optimistic, and Stress scenarios and an 80% probability for the Base scenario over the next year[32]. Under those assumptions our Jump process risk analysis works as follows. First we randomly select a state of the world based on the established probabilities. Second we run the simulation risk analysis based on the selected state of the world (i.e. mean returns, volatilities, correlations, government policies, etc.). Third we repeat the previous two steps many times and then analyze the distribution of simulated government risk indicators. Table 8 below provides such an illustrative overall Jump process risk assessment for the GOB and compares that to the Base Scenario above.

[32] While such probability estimates could be generated from an analysis of historical data and expert opinion about future economic, financial, and political prospects, in this illustrative analysis we have simply assumed the probabilities of the various states of the world.

Table 8: Simulated GOB "Net Worth" To GDP Ratio As of 6/30/05

	Jump Process	Base Scenario
Mean	1.9945	1.7363
Std	1.3699	0.2341
Max	6.3892	2.4793
Min	-0.4389	0.9517
Percentile Distribution		
0%	-0.4389	0.9517
1%	-0.0091	1.1972
2%	0.1357	1.2391
3%	0.1701	1.2818
4%	0.2221	1.3246
5%	0.2560	1.3482
6%	0.3120	1.3659
7%	0.3412	1.3813
8%	0.4021	1.3952
9%	0.5208	1.4082
10%	1.1642	1.4234
25%	1.5149	1.5684
50%	1.7432	1.7224
75%	1.9644	1.8805
90%	5.2348	2.0352
95%	5.8260	2.1124
99%	6.1697	2.2570
100%	6.3892	2.4793

Analyzing the information in Table 8 one sees that for the median (i.e. 50'th percentile) net worth to GDP ratio for the Jump process is approximately equal to that of the Base scenario (1.74 vs 1.72). However the standard deviation for the Jump process (1.37) is much larger than the Base case (0.23). Further the extreme values of Net Worth to GDP for the Jump process are much bigger on the upside and smaller on the downside than the base case. This "fat tailed" distribution of simulated net worth to GDP ratios comes about due to the fact that 80 percent of the time the Jump process is modeling the Base case, and 10 percent of the time it is modeling the Optimistic and Stress cases. Overall the illustrative Jump process analysis indicates something between a 1% and 2% probability of a negative net worth for the GOB over a one-year time step.

V. CONCLUDING COMMENTS

The previous analyses demonstrate that the mean or expected levels for sovereign risk indicators provides no more than a first approximation of the vulnerability of the public sector to financial crises. Some mechanism is required to assess the probability of adverse outcomes that threaten continued market access on reasonable terms. We believe that the proposed scenario/simulation and jump process methodologies have the potential to provide useful forward looking information for policy makers.

Given information of the distributions of future risk indicators, if policy makers conclude that the probabilities of achieving unacceptably high Debt to GDP ratios, or unacceptably low Net Worth of GDP ratios (that might affect reasonable market access) are excessive, then they can look at alternative policy actions for reducing those probabilities. In general those alternatives would include:

- reducing the GOB Debt to GDP ratio over time by running larger primary surpluses,
- altering the types of securities used to finance the Government's operations and refund maturing debt obligations so as to alter the volatility and correlations between changes in the present value of the primary surplus and the value of the debt portfolio,
- pursuing monetary and fiscal polices which result in more stable long-term economic growth (i.e. less volatile GDP, inflation, interest rates, FX rates, etc.), and
- continually working to assure the market that the Government has the capacity and willingness to make difficult decisions if necessary to service its debt.

It is widely noted that policy makers in vulnerable economies frequently follow a pro-cyclical fiscal policy[33]. Such behavior is consistent with an effort to maintain market access both by reducing current borrowing requirements and reassuring markets of the Government's willingness and capacity to make painful choices to service its debt over the long-term. If the Government has a high existing debt level there may be few short-term alternatives to increasing the target primary surplus if a stressful period develops. A principal mechanism to avoid being forced into a pro-cyclical fiscal policy is to anticipate that Stress periods will occur occasionally and over time work toward achieving debt levels that are clearly sustainable even during such periods[34].

A central implication of this finding is that emerging market economies with attributes comparable to those of Brazil, namely, which are highly indebted, should implement a strategy aimed at reducing the country risk premium. Brazil has made significant progress in this direction in recent years by consistently running significant primary surpluses and following a monetary policy that has reduce inflation rates. Clearly, a significant fall in the risk premium is conducive to a virtuous cycle of macroeconomic stability and growth. Such a strategy would also benefit from the identification and implementation of policies which encourage economic efficiency and GDP growth. This is important from the standpoint of

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[33] See Kaminsky, G., Reinhart, C., and Vegh, C. (2004) and Mendoza and Oviedo (2005).

[34] See Mendoza and Oviedo (2005).

expanding the tax base. Second is building a robust institutional framework, and within that framework, continuing to exercise considerable prudence in formulating fiscal policy.

More generally as also noted by Barnhill and Kopits (2004), emerging market economies should avoid accumulating beyond a certain stock of government liabilities—especially in the form of short-term debt contracted in foreign currency, at variable interest rates, resulting in maturity and currency mismatches that can give rise to debt servicing problems. As part of prudent public debt management, they should resist the proliferation of guarantees and the accumulation of unfunded contingent liabilities.

Overall, this experimental application of the scenario / simulation and jump process methodologies for Sovereign risk assessment represent promising avenues for more realistic assessments of fiscal sustainability under uncertainty. In comparison to the conventional approach to sustainability analysis, this approach provides a formal and explicit quantitative framework for identifying and measuring major sources of fiscal vulnerability; in particular, it captures the effect of the volatility in underlying risk variables, as well as interactions among them. The application can be customized to any given country, incorporating specific behavioral and institutional features that characterize the country's environment.

A potentially serious shortcoming in many cases is that the vulnerability assessment is limited to the public sector. Indeed, such assessments should in many cases be conducted simultaneously for the public sector and the financial system (perhaps on an economy-wide basis), applying a uniform risk assessment methodology. In principle, an integrated risk assessment—treating the risk of systemic bank failures and government default as correlated events—seems a worthwhile and feasible future endeavor (see Barnhill and Souto (2006) for an example of this type of integrated risk assessment). The models developed in this paper have the potential to be incorporated into such a more comprehensive risk assessment.

References

Adrogue, R., 2005, Fiscal Sustainability: A Value-at-Risk Approach,

Barnhill, T., and W. Handorf, 2002, "Latin American Credit Ratings, the New Basel Capital Accord and Portfolio Risk," *Economía Aplicada*.

Barnhill, T., and G. Kopits, "Assessing Fiscal Sustainability Under Uncertainty", IMF Working Paper, 2003, Journal of Risk, Volume 6 / Number 4, Summer 2004.

Barnhill, T., and W. F. Maxwell, 2002, "Modeling Correlated Market and Credit Risk in Fixed Income Portfolios", *Journal of Banking and Finance*, vol. 26, pp. 347-374.

Barnhill, T, P. Papapanagiotou, and L. Schumacher, 2002, "Measuring Integrated Market and Credit Risk In Bank Portfolios: An Application to a Set of Hypothetical Banks Operating in South Africa," *Journal of Financial Markets, Institutions and Instruments.*

Barnhill, T., and M. Souto, 2006, "Systemic Bank Risk in Brazil: Assessing Correlated Market, Credit, and Sovereign Risk in an Economic Environment with Stochastic Volatilities and Correlations", GEFRI Working Paper, April 2006.

Engle, R., 2002, "Dynamic Conditional Correlation – A Simple Class of Multivariate GARCH Models." *Journal of Business and Economic Statistics* 20 (3), 339-350.

FinSoft, Inc., 1996-2005. ValueCalc 3.0: Global Portfolio Risk Management Software, FinSoft, Inc.

Goldfajn, I, and E. Guardia, (2004), "Fiscal Rules and Debt Sustainability in Brazil", in *Rules-Based Fiscal Policy in Emerging Markets: Background, Analysis and Prospects* (Palgrave MacMillan), edited by George Kopits.

Hull, J., 2000, *Options, Futures, and Other Derivatives* (Upper Saddle River, Prentice-Hall)

Hull, J. and A. White, 1994, "Numerical Procedures for Implementing Term Structure Models: Single-factor Models," *Journal of Derivatives* (Fall), pp. 7-16.

International Monetary Fund, 2002, "Assessing Sustainability" (unpublished, Washington, May 28) http://www.imf.org/external/np/pdr/sus/2002/eng/052802.htm

Jorion, P., 2001, *Value at Risk: The New Benchmark for Managing Risk* (New York, McGraw-Hill)

Kaminsky, G., Reinhart, C., and Vegh, C. (2004). When it rains it pours: procyclical capital flows and macroeconomic policies. *NBER Macroeconomic annual*. Edited by Mark Gertler and Kenneth Rogoff; Cambridge, MA: MIT Press.

Merton, R., 2976, "Option Pricing When Underlying Stock Returns Are Discontinuous", *Journal of Financial Economics*, 3 (March 1976), 125-44.

Mendoza, E. and P. Oviedo, "Fiscal Policy and Macroeconomic Uncertainty in Emerging Markets: The Tale of the Tormented Insurer", working paper December 2005.

Pattillo, C, H Poirson, and L Ricci, 2002, "External Debt and Growth," IMF working paper 02/69.

Reinhart, C, K. Rogoff, and M. Savastano, 2003, "Debt Intolerance," NBR Working Paper No. 9908.

APPENDIX 1: SIMULATION METHODOLOGY

Our model starts by simulating potential future financial and economic environments, including all of the required stochastic variables used to value assets and liabilities, over as long a time-step as necessary.[35] Subsequently, the full portfolio is revalued in the simulated environment. After many repetitions of the simulation, a distribution of portfolio values is created and analyzed to determine the value at risk at a given confidence level. Such simulations can allow for non-normal distributions of risk variables, non-linear option-like payoffs, and time-decay effects.[36]

For simulating the stochastic term structure of *interest rates* (i.e., interest rates denominated in domestic and foreign currency), it is assumed that interest rates follow a time-dependent mean-reversion process.[37] In this process, the risk-neutral[38] change in the continuously compounded short-term interest rate (r) is

$$\Delta r = a\left(\frac{\theta(t)}{a} - r\right)\Delta t + \sigma \Delta z \qquad (11)$$

where

a = the rate at which r reverts to its long-term mean
$\theta(t)$ = a time dependent parameter for consistency with the initial term structure
σ = instantaneous standard deviation of r
$\Delta z = \varepsilon\sqrt{\Delta t}$ = movement in term structure or r (Wiener process)
ε = random draw from a standardized normal distribution.

The mean reversion and volatility rates can be estimated from a time series of short-term interest rates or as implied from cap and floor prices. Once the risk-free term structure (i.e., U.S. government rates) has been estimated, then other risky term structures are modeled as a stochastic lognormal spread over risk-free structure. This procedure ensures that simulated

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[35] The simulation of the financial environment can be viewed as a random draw from an n-dimensional joint density function, where n is the number of jointly stochastic variables.

[36] Barnhill and others have developed a portfolio simulation approach for assessing integrated market and credit risk for bond portfolios and banks. For an extension of this approach to estimate bank capital requirements and compare these to those required under the proposed new Basel Capital accord in the case of Latin America, see Barnhill and Handorf (2002).

[37] See the extension of the Vasicek model for stochastic risk-free interest rates in Hull and White (1994).

[38] In a risk neutral world, investors require no compensation for risk and the expected return on all assets is the risk-free interest rate.

rate spreads are always positive and that the simulated term structures are approximately arbitrage free.[39]

For the simulation of most *other financial and economic risk variables* (rates of return, exchange rate, prices, output), we adopt an approach often used for simulating asset prices and returns. Specifically, the value of the variable S is assumed to follow a geometric Brownian motion where the expected growth rate (m) and volatility (σ) are constant.[40] The expected growth rate is equal to the expected return on the asset (μ) minus its dividend yield (d). For a discrete time step Δt, it can be shown that[41]

$$S + \Delta S = S \exp\left[\left(m - \frac{\sigma^2}{2}\right)\Delta t + \sigma \varepsilon \sqrt{\Delta t}\right] \qquad (12)$$

All risk variables are simulated as correlated stochastic variables. This procedure requires the specification of correlations between each of the n stochastic variables.[42] Subsequently, n independent random samples x are drawn from standardized normal distributions, which allows for the calculation of a set of correlated random error terms for the n stochastic variables. For a bivariate normal distribution,

$$\varepsilon_1 = x_1 \qquad (13)$$

$$\varepsilon_2 = \rho x_1 + x_2\sqrt{1 - \rho^2} \qquad (14)$$

where

x_1, x_2 = independent random samples from standardized normal distributions
ρ = correlation between the two stochastic variables
ε_1, ε_2 = random drawings from a standardized bivariate normal distribution.

It can be shown that the simulated volatilities and correlations for all of the stochastic variables closely match values estimated from time series data.[43]

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[39] For a fuller discussion of modeling risk-free and risky interest rate term structures, along with the application of a diffusion-based methodology for assessing the *VaR* of a portfolio of fixed income securities (with correlated interest rate, interest yield spread, exchange rate, and credit risk), see Barnhill and Maxwell (2002).

[40] See, for example, Hull (2000, p. 408).

[41] If needed it is also possible to simulate the return on and value of state owned enterprises as well the value of revenue streams generated by non-renewable resources (e.g. oil) (see Barnhill and Kopits, 2004).

[42] See Hull (2000) for a procedure derived for an n-variate normal distribution.

[43] The current application of *VaR* to the government balance sheet abstracts from counterparty credit risk. However, Barnhill, Papapanagiotou, and Schumacher (2002) have shown that it is possible to simulate

Appendix 2: Distributions of Simulated Financial and Economic Variables

Tables A2:1 to A2:4 below give distributional analyses for the simulated values of selected variables. These simulation results are consistent with the model input assumptions which were estimated from historical data over the 1999 to 2004 period.

Table A2:1 Simulated IPCA Inflation Rates As of 6/30/05

Mean	0.0603
Std	0.0189
Max	0.1179
Min	0.0027
Percentile Distribution	
1%	0.0179
2.5%	0.0224
5%	0.0306
10%	0.0356
50%	0.0603
90%	0.0850
95%	0.0900
99%	0.1027

Table A2:2 Simulated Selic Interest Rates As of 6/30/05

Mean	0.1731
Std	0.0336
Max	0.3091
Min	0.0371
Percentile Distribution	
1%	0.0948
2.5%	0.1099
5%	0.1198
10%	0.1296
50%	0.1731
90%	0.2165
95%	0.2267
99%	0.2520

correlated market and credit risk and undertake risk assessments of banks using a contingent claims framework. In principle, this can be extended to conduct a simultaneous government and banking sector risk analysis.

Table A2:3 Simulated $R/USD Exchange Rate As of 6/30/05	
Mean	3.5760
Std	0.5326
Max	6.0242
Min	2.0767
Percentile Distribution	
1%	2.5202
2.5%	2.6343
5%	2.7572
10%	2.9015
50%	3.5396
90%	4.3116
95%	4.5409
99%	4.9664

Table A2:4 Simulated Cost of GOB Debt (%) 6/30/04 to 6/30/05	
Mean	0.1524
Std	0.0396
Max	0.2729
Min	0.0342
Percentile Distribution	
1%	0.0627
2.5%	0.0795
5%	0.0894
10%	0.1006
50%	0.1522
90%	0.2041
95%	0.2163
99%	0.2450

Based on Interest Paid plus Change in Market Value