A new risk indicator and stress testing tool: A multifactor n^{th}-to-default CDS basket

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Why a New Risk Indicator for LCFIs?

Need to develop a platform for stress test based on an arbitrage-free financial model with a coherent link to a macroeconomic framework.

Problems with existing indicators:

- Financial soundness indicators rely on (lagged) balance sheet information, which reflects a decreasing proportion of financial activities
- Distance to default indicators also rely on some balance sheet items
- (Spreads on) Bank subordinated debt suffers from insufficient liquidity in the underlying bond markets

Why a New Risk Indicator for LCFIs?

This paper develops, as market-based indicator, a CDS basket of LCFIs for financial sector surveillance

- LCFIs CDSs are liquid and in a basket represent a good credit risk indicator for a portfolio of institutions accounting also for their default correlations

- Multivariate latent factor structure which underpins the LCFI's correlations

The Agenda

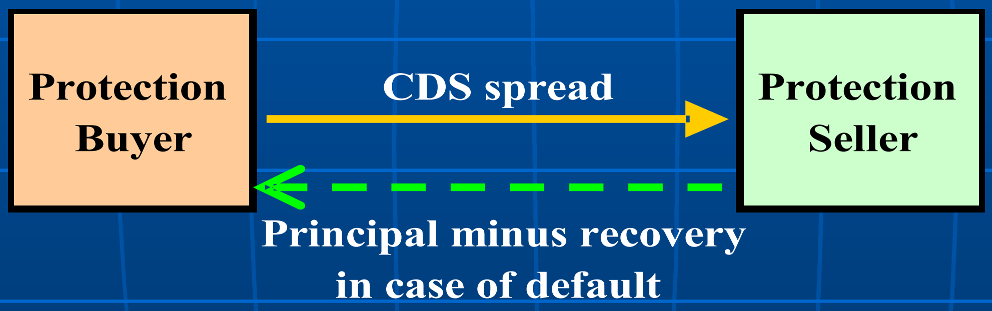
1. Description of the indicator
2. The Model: A Multivariate Approach
3. Estimation of the Model Parameters

   - The data: CDS and Equity Return Correlations
   - Maximum Likelihood Estimates

4. Computing the Probabilities of Default
5. Sensitivity Analysis
6. Stress Testing
7. Conclusions

I. Description of the Indicator

- An n^{th}-to-default CDS basket is the simplest form of a collateralized debt obligation (CDO)
- In our case, the (synthetic) CDO is composed by the actively traded CDSs of the 15 large complex financial institutions



- In an n^{th}-to-default CDS basket, the investor (seller of protection) provides protection on the basket only until the n^{th} name is subject to a credit event. After that, settlement takes place and the financial instrument ceases to exist
- Default correlations are the main driver of the basket value

I. Description of the Indicator

- An example: Default correlations are the main driver of the CDS basket’s value
  - A basket of five credits where each CDS pays a spread of 100 bps
  - In the case of zero correlation, the first-to-default swap would have a spread of 500 bps
  - If the correlation is one, the spread for the basket would be 100 bps

II. The Model: A Multivariate Approach

Consider a portfolio of N different companies each associated with a given CDS spread and a recovery rate.

We assume that the correlation of defaults within the portfolio is driven by *m* common factors. Creditworthiness for each company i depends on its asset value x_i :

$$x_i = a_{i1}M_1 + a_{i2}M_2 + \ldots + a_{im}M_m + Z_i\sqrt{1 - a_{i1}^2 - a_{i2}^2 - \ldots - a_{im}^2}$$

- The conditional default probability of company i :

$$prob(x_i < \bar{x}_i \mid M) = H_i\left[\frac{\bar{x}_i - (a_{i1}M_1 + \ldots + a_{im}M_m)}{\sqrt{1 - a_{i1}^2 - \ldots - a_{im}^2}}\right]$$

- The risk-neutral PD that i defaults before time t (forward default hazard rate λ_i) :

$$Q_i(t_i \leq t) \equiv 1 - e^{-\int_0^t \lambda_i(u)du}$$

- Under a copula model, the x_i are mapped to t_i (time of default), so that :

$$\bar{x}_i = F^{-1}\left(Q_i(t)\right) \quad ; \quad prob\left(x_i < \bar{x}_i \mid M\right) = Q_i\left(t \mid M\right)$$

II. The Model: A Multivariate Approach

Following Andersen, Sidenius and Basu (2003) and Gibson (2004), we can compute the distribution of a # of defaults k at time t, conditional on the common factors **M** in a k-name portfolio:

$$p^k(l,t\,|\,\mathrm{M}); \quad l = 0,\ldots,k$$

Adding one credit, with a conditional PD $Q_k(t\,|\,\mathrm{M})$, the default distribution can be obtained by recursion:

$$p^{k+1}(0,t\,|\,\mathrm{M}) = p^k(0,t\,|\,\mathrm{M})\left(1 - Q_{k+1}(t\,|\,\mathrm{M})\right)$$

$$p^{k+1}(l,t\,|\,\mathrm{M}) = p^k(l,t\,|\,\mathrm{M})\left(1 - Q_{k+1}(t\,|\,\mathrm{M})\right) + p^k(l-1,t\,|\,\mathrm{M})\,Q_{k+1}(t\,|\,\mathrm{M}); \quad l = 1,\ldots,k$$

$$p^{k+1}(k+1,t\,|\,\mathrm{M}) = p^k(k,t\,|\,\mathrm{M})\,Q_{k+1}(t\,|\,\mathrm{M})$$

Starting with the initial condition for the “degenerate” PD for k=0

$$p^0(0,t\,|\,\mathrm{M}) = 1$$

II. The Model: A Multivariate Approach

We can integrate out the factors to get the unconditional portfolio loss distribution

$$p(l,t) = \int_{R^m} p^N(l,t \mid \mathrm{M})\, g(\mathrm{M})\, d\mathrm{M}$$

- $p^N(l,t\,/\,\mathbf{M})$ is the probability that l defaults occur by t in our portfolio of N names, conditional on the common factors **M**
- The joint density of **M**, g(**M**), is the product of m independent standard Gaussian densities

III. Estimating the Model Parameters

- Risk-neutral PDs: CDS spreads and recovery rates
- Factor Loadings:
  - The copula default correlation of two companies is approximated by the correlation between their equity returns
  - A multifactor model with orthogonal factors is used to estimate the factor loading matrix (i.e., the default correlations of the portfolio)

$$X = \mu + AF + U$$

X vector of observed equity returns

μ constant vector of means

U idiosyncratic variable (mean = 0, cov = Ψ) independent from F

A (N x m) factor loadings matrix (a_{ij} is the generic element)

F column vector of m factors which are assumed independent Gaussian (0,1)

m number of factors based on a model-selection-criteria, such as LR test

III. Data: Equity Returns Correlations

We focus on the group of LCFIs as defined by the Bank of England

ABN Amro, Bank of America, Barclays, BNP Paribas, Citigroup, Credit Suisse, Deutsche Bank, Goldman Sachs, HSBC Holdings, JP Morgan Chase, Lehman Brothers, Merrill Lynch, Morgan Stanley, Societe Generale, and UBS.

Average Correlations of Daily Equity Returns (2003-05)

	SG	BNP	DB	ABN	HSBC	BARC	UBS	CS	BoA	CITI	JPM	LEH	ML	GS	MS
SG	1.00	0.87	0.70	0.80	0.65	0.66	0.73	0.67	0.01	0.08	0.06	0.04	0.06	0.05	0.08
BNP		1.00	0.69	0.73	0.62	0.67	0.67	0.59	0.04	0.11	0.08	0.04	0.09	0.04	0.13
DB			1.00	0.67	0.55	0.61	0.65	0.64	0.27	0.24	0.29	0.21	0.36	0.30	0.33
ABN				1.00	0.68	0.68	0.75	0.63	0.05	0.11	0.13	0.06	0.08	0.01	0.04
HSBC					1.00	0.78	0.68	0.63	-0.06	0.16	0.06	0.06	0.15	0.02	0.16
BARC						1.00	0.73	0.62	0.04	0.16	0.09	0.15	0.15	0.07	0.16
UBS							1.00	0.81	-0.01	0.04	0.08	0.05	0.09	0.03	0.00
CS								1.00	0.09	0.13	0.05	0.07	0.10	0.13	0.14
BoA									1.00	0.76	0.75	0.61	0.71	0.59	0.62
CITI										1.00	0.66	0.51	0.67	0.50	0.67
JPM											1.00	0.53	0.68	0.53	0.46
LEH												1.00	0.71	0.82	0.64
ML													1.00	0.67	0.75
GS														1.00	0.64
MS															1.00

III. MLE of the Factor Loadings

- MLE estimates based on 2005:QIV and the results of the LR test indicate that a 5-factor representation is the most adequate
- 5 factors explain 78 percent of the variance of the asset returns
- To provide interpretation of the factors, we followed BoE approach (exploratory analysis through PCA)
  - The first common factor: financial sector
  - The second: EUR regional effect
  - The third: US regional effect
  - The fourth: commercial banking business
  - The fifth: investment banking business
- We rotate the factors, while leaving the statistical properties of the MLE unchanged, to create a more easily interpretable A.

III. MLE of the Factor Loadings

Rotated Factor Loadings, ML Estimates

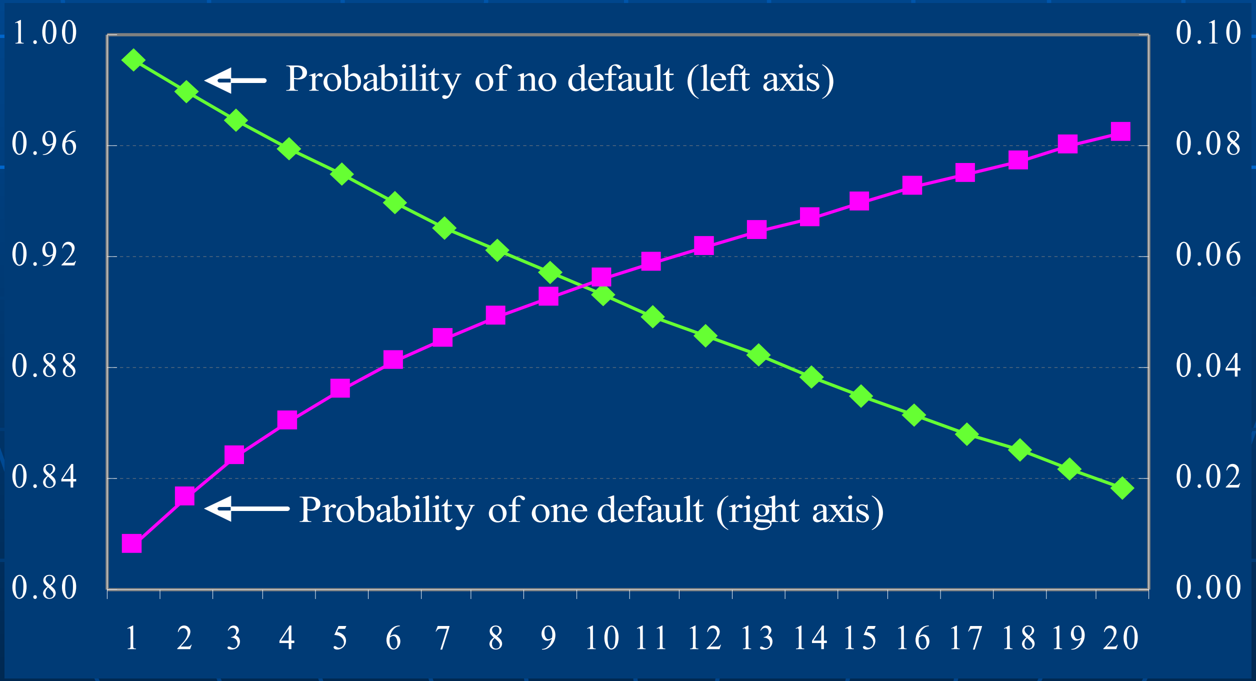
	1st. (Fin. Sect.)	2nd (EUR)	3rd (U.S.)	4th (Com. Bank)	5th (Inv. Bank)	Residual Variance
SG	0.645	0.456	-0.285	0.462	-0.076	0.076
BNP	0.615	0.435	-0.204	0.454	-0.117	0.172
DB	0.572	0.342	-0.024	0.420	0.188	0.344
ABN	0.437	0.497	-0.178	0.516	0.032	0.264
HSBC	0.227	0.836	0.148	0.346	-0.025	0.108
BARC	0.307	0.686	0.025	0.387	0.109	0.274
UBS	0.231	0.601	-0.331	0.538	0.417	0.014
CS	0.303	0.515	-0.181	0.450	0.298	0.319
BoA	0.359	-0.398	0.572	0.338	0.370	0.134
CITI	0.296	-0.150	0.671	0.355	0.229	0.262
JPM	0.299	-0.247	0.506	0.330	0.363	0.352
LEH	0.494	-0.046	0.464	-0.194	0.541	0.208
ML	0.394	-0.072	0.618	0.112	0.441	0.250
GS	0.552	-0.089	0.417	-0.238	0.542	0.163
MS	0.444	-0.028	0.621	0.018	0.274	0.341

III. Data: CDS Spreads

ABN Amro, Bank of America, Barclays, BNP Paribas, Citigroup, Credit Suisse, Deutsche Bank, Goldman Sachs, HSBC Holdings, JP Morgan Chase, Lehman Brothers, Merrill Lynch, Morgan Stanley, Societe Generale, and UBS.

IV. Computing the PDs

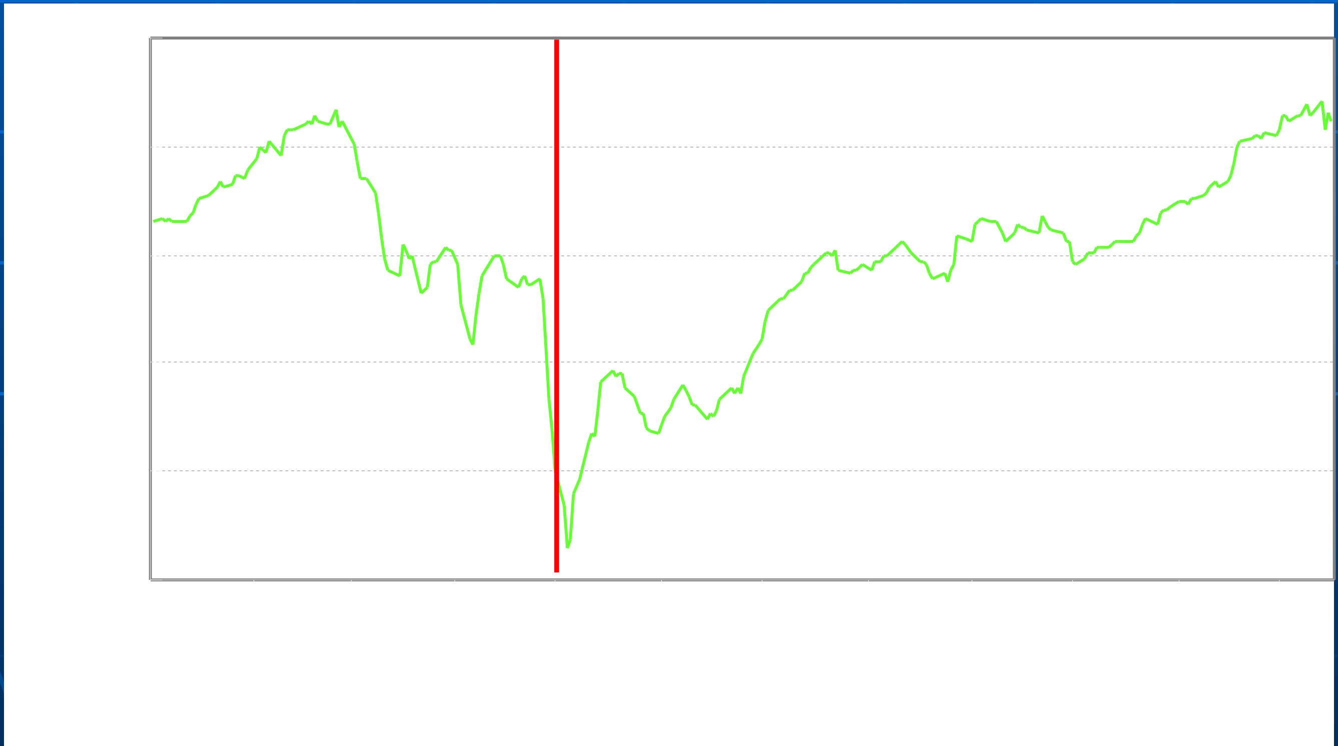
- The parameters that go into the forward PDs:
  - Factor loadings (A)
  - Default hazard rate: ratio of CDS spread to the LGD
- The PDs are computed every period in which there is a payment:
  - For example, 5 year-CDS and quarterly payments => 20 payment dates
  - We compute the PD in 1Q, 2Q,…, 20Q (i.e., 5 year)



IV. The PDs: Recent Events

The PDs provide a good indication of the market's views on the underlying credit quality of the financial institutions in the basket

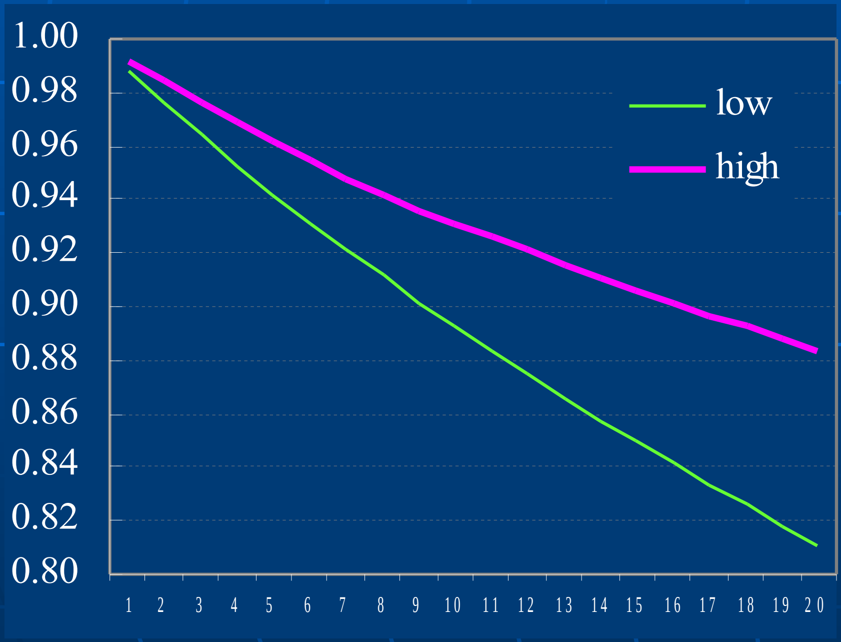
Two-year ahead Probability of no Default



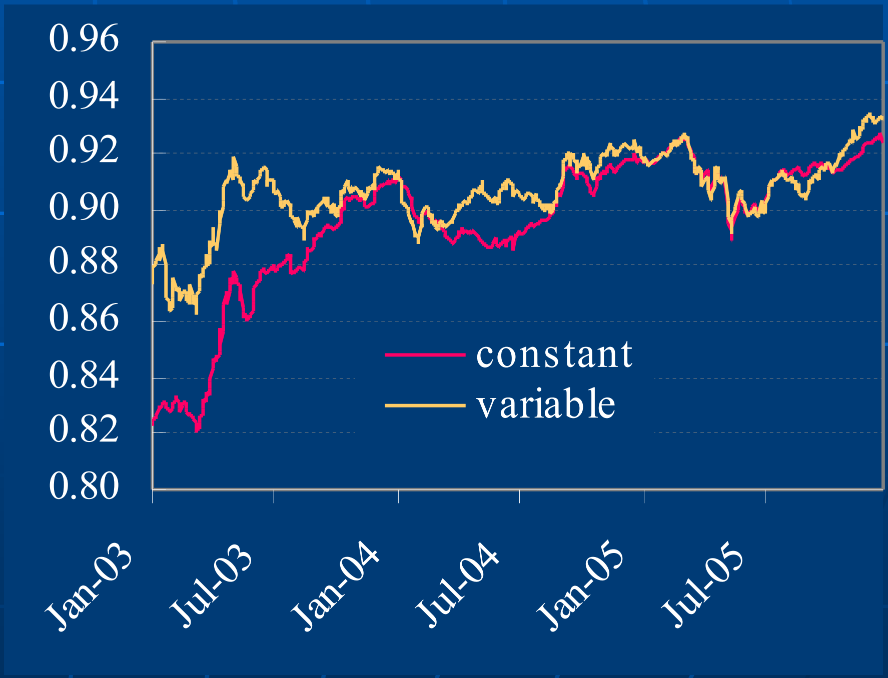
V. Sensitivity Analysis

Impact on the PDs of different correlations and their multifactor representations

Probability of no default under “high” and “low” correlations (Dec 05)



Two-year-ahead probability of no default (2003-05)

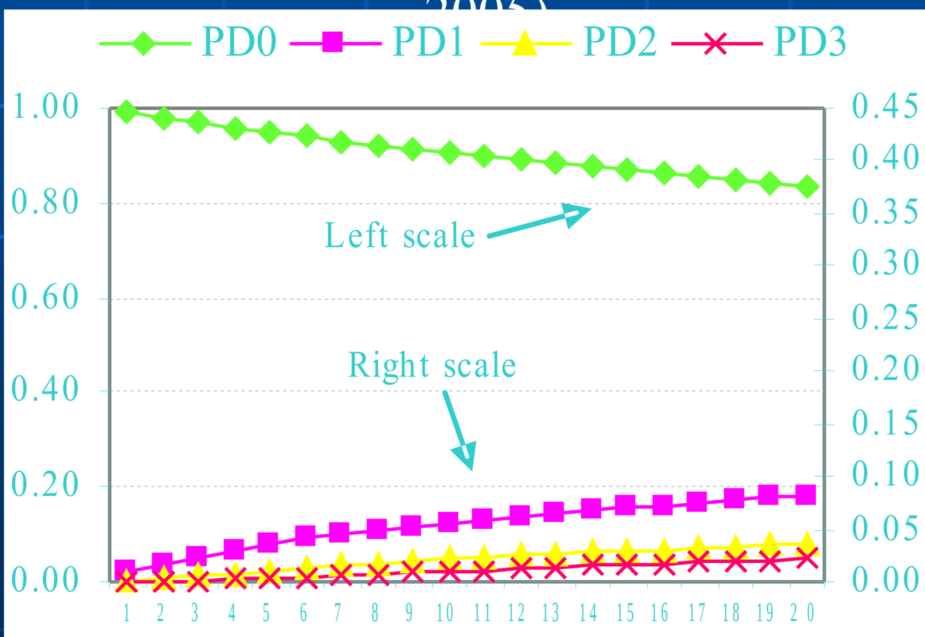


VI. Stress Tests

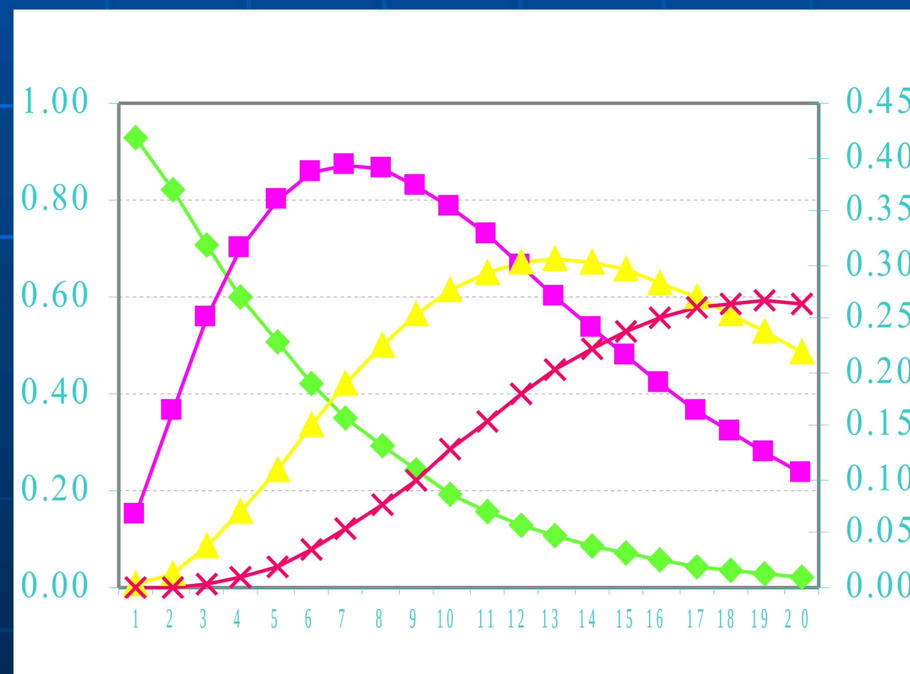
To study the effect of a recession (boom) on a given factor, we can integrate over the in the left (right) tail of the factor's distribution (Gibson, 2004). Recall that:

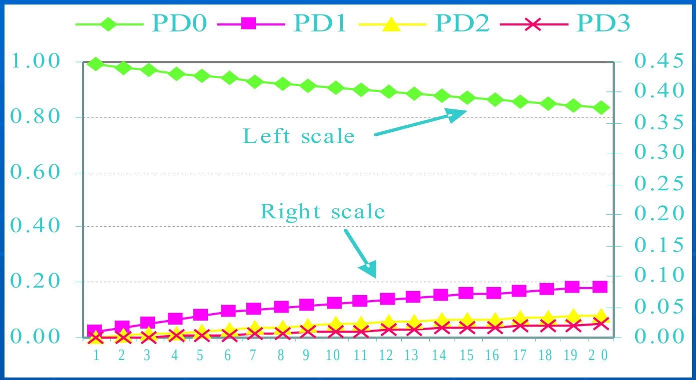
$$p(l,t) = \int_{R^m} p^N(l,t \mid \mathrm{M})\, g(\mathrm{M})\, d\mathrm{M}$$

Baseline, unconditional PDs (Dec 2005)

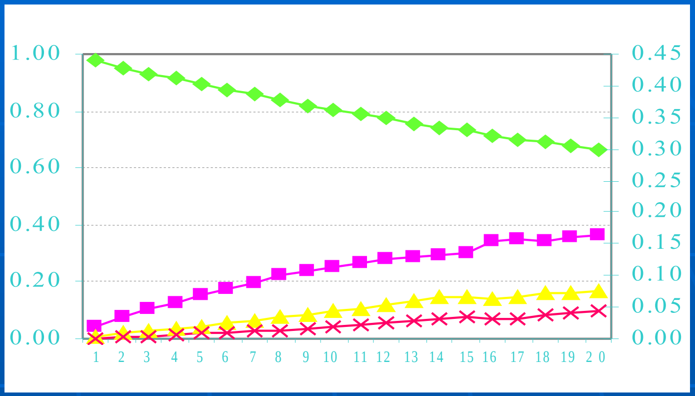


All factors in recession (Dec 2005)

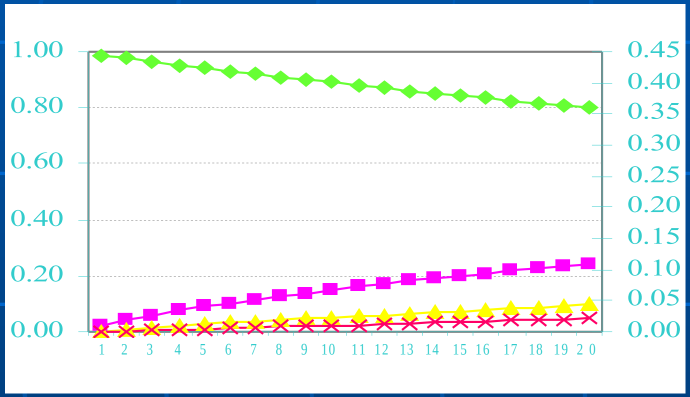




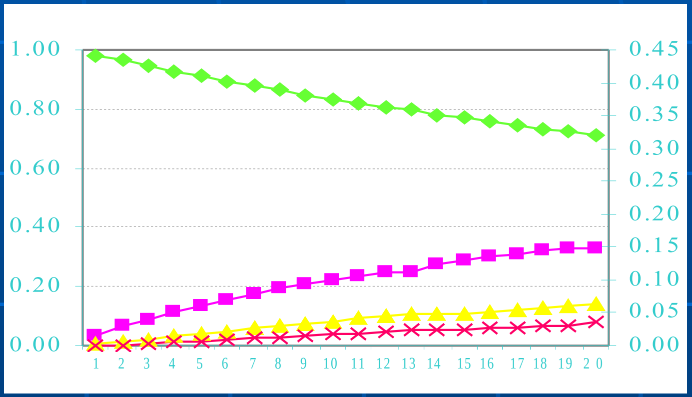
Baseline



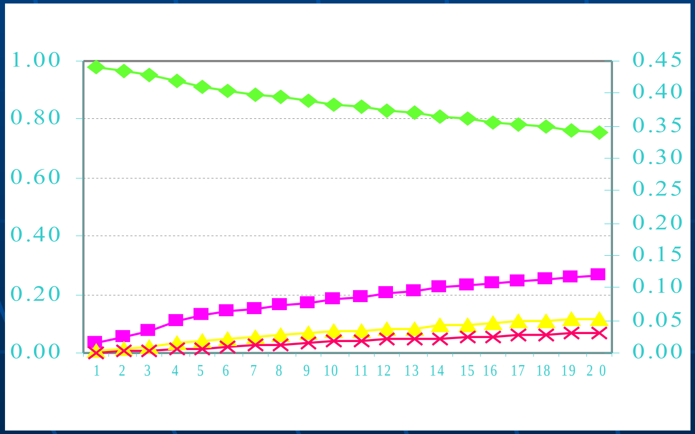
"Financial sector" factor in recession



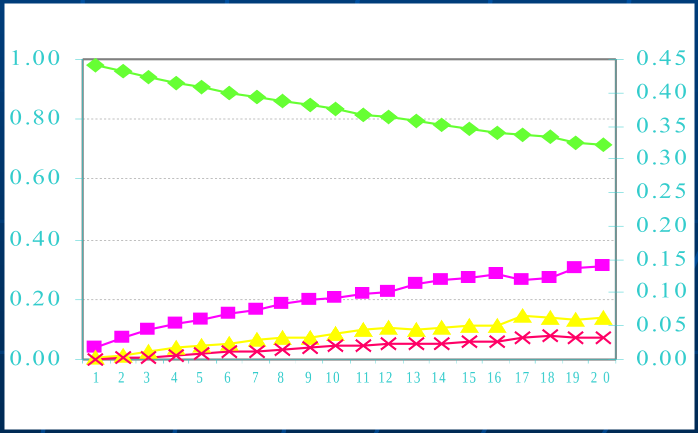
"European" factor in recession



"U.S." factor in recession



"Commercial banks" factor in recession



"Investment banks" factor in recession

Conclusions

- A platform for stress test based on an arbitrage-free financial model with a coherent link to a macroeconomic framework
- A CDS basket: a market-based indicator for financial surveillance
- A multifactor latent structure in the determination of the correlation dynamics between LCFIs (critical for the PDs and pricing)